PRZEGLĄD ELEKTROTECHNICZNY

Ukazuje się od 1919 roku 9b'12

Organ Stowarzyszenia Elektryków Polskich • Wydawnictwo SIGMA-NOT Sp. z o.o.

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The Dynamic Model of Closed-Loop Supply Chain with Product Recovering and its Robust Control

Abstract. *In recent years, closed-loop supply chain (CLSC) operation has been an important method to recover waste materials and products for resource conservation and environmental protection and paid more attention in industry and academia. We establish a dynamic model for closed-loop supply chain with product recovering with the consideration of some uncertainties including remanufacturing rate, disposal rate, operation costs and customers' demand in this paper. Furthermore, we provide some insights about robust operation of closed-loop supply chain and propose a robust H∞ control strategy based on linear matrix inequality (LMI) arithmetic. Some simulations are executed to validate the effectiveness of our control strategy, and the results show that through state feedback control of supply chain inventories, it can not only make the CLSC achieve the goal of restraining uncertainty disturbances, but also result in an ideal operation cost.*

Streszczenie. *Closed loop supply chain (CLCP – zamknięta pętla łańcucha zasobów) jest dziś ważną metodą odzyskiwania zmarnowanego materiału i produktu w celu ochrony środowiska i konserwacji zasobów. Zaproponowano model dynamiczny uwzględniający niepewności typu tempo produkcji, tempo usuwania, koszty operacyjne i żądania klienta. (**Model dynamiczny łańcucha CLSC uwzględniający odzyskiwanie produktów i odporne sterowanie**)*

Keywords: closed-loop supply chain; uncertainty; robust control; dynamic model; linear matrix inequality(LMI)
Słowa kluczowe: CLCP – zamknięta pętla łańcucha zasobów, odporne sterowanie

1. Introduction

Developing a conserving and environmentally friendly society is an important development trend in the future. However, the complexity of recycling makes it must use closed-loop supply chain(CLSC) operation [1]. Closed-loop supply chain, which is an important method to recycle waste materials and products for resource conservation and environmental protection, has been paid more attention in industry and academia. Traditionally, the general supply chain is defined as a forward network composed of factories, suppliers, retailers and so on that supply each other with raw materials, components, products and service. Whereas closed-loop supply chain is a closed-loop system including both forward supply chain and reverse one whose logistics operation is in contrary to the former. Closed-loop supply chain is currently emerging as vital logistical structures for many discrete-part manufacturers whose products are amenable to remanufacturing or refurbishing practices [2]. For example, most of electronics and automotive products are always recovered because of their relatively high recoverable value and long product life cycles. Companies such as Dell, HP, GM are embracing the practice of product recovery for its potential to improve cost-effectiveness and its environmental benefits created by the reuse of resources there by saving on raw material requirements.

In the last decades, the literatures about CLSC relate to inventory control, network structure, recovery management, remanufacturing, and so on [3]-[7]. Specially, product recovering and remanufacturing have attracted extensive attention worldwide [8]-[9]. In recent years, robust control, as an effective method to deal with uncertainties in dynamic systems, has been paid much attention in many field, some scholars study CLSC by using control method [10]-[14]. However, little uncertainty is considered. In fact, a CLSC has more uncertainty because of its complexity. Under this circumstance, Huang develop dynamic models of closed-loop supply chain and provide corresponding robust H∞ control strategies [15].

In this paper, we will emphasize the uncertainty in CLSC operation, and develop a closed-loop dynamic model with uncertain remanufacturing rate, disposal rate and external demand. Besides, an effective robust H∞ strategy will be put forward by using Linear Matrix Inequality (LMI). A simulation will be done to test the effectiveness of our control strategy.

2. Closed-loop supply chain dyanmic model with product recovring

A discrete time supply chain dynamic system is taken into account in this paper with the consideration of uncertain remanufacturing rate, disposal rate of recovered products, and customers' demand. Fig.1 shows the operation of a CLSC system with product recovering.

On the basis of Figure 1, dynamic inventory models of CLSC with product recovering can be formulated as two scalar equations:

(1) $$x_{1,k+1} = x_{1,k} + \alpha x_{2,k} + p_k - d_k$$

(2) $$x_{2,k+1} = x_{2,k} + w_k - \alpha x_{2,k} - \beta x_{2,k}$$

Note that, both above two formulas are scalar equations. That is, the dynamic system is described by using deviation. State variables $x_{1,k}$ and $x_{2,k}$ are usable and recovered inventories at time k, control variables p_k and w_k are production and recovery quantities at time k. Uncertain parameters α and β represent remanufacturing and disposal rates separately with $0 \le \alpha \le 1$, $0 \le \beta \le 1$, and $0 < \alpha + \beta \le 1$. Uncertain external input variable dk is customers' demand at time k. Note that, the inventory, production and demand can be positive or negative because of the use of deviation values.

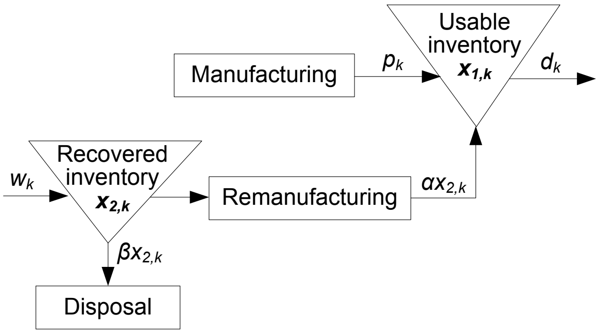

Fig.1. The closed-loop supply chain with product recovering

The total operation cost of closed-loop supply chain can be described as:

(3) $$z_k = c_{h1}x_{1,k} + c_{h2}x_{2,k} + c_r\alpha x_{2,k} + c_n p_k + c_0\beta x_{2,k} + c_p w_k$$

where, z_k is an output variable. c_{h1} and c_{h2} are usable and recovered inventory cost separately. c_r is remanufacturing cost, c_0 is disposal cost, c_n is production cost of new product, and c_p is product recovery cost.

Eq.1 to Eq.3 can be rewrite as following matrix form.

(4) $$x_{k+1} = (A + \Delta A)x_k + Bu_k + Ed_k$$

(5) $$z_k = (C + \Delta C)x_k + Du_k$$

where, $x_k^T = (x_{1,k}, x_{2,k})$, $u_k^T = (p_k, w_k)$, $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $\Delta A = \begin{bmatrix} 0 & \alpha \\ 0 & -\alpha-\beta \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $E = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$, $C = (c_{h1}, c_{h2})$, $\Delta C = (0, c_r\alpha + c_0\beta)$, $D = (c_n, c_p)$. ΔA and ΔC are uncertain parameters matrix vectors.

3. Robust H∞ control

The operational implications of robust H∞ control of the CLSC system is to restrain the uncertain disturbances through the inventory state variables x_k, control variables u_k, and make the total operation cost z_k relatively ideal. The restraint degree of the controller to the above disturbances can be described as parameter γ, namely, $||z||_2/||d||_2 \le \gamma$. the smaller the parameter γ is, the better the performance would be. $||\cdot||_2$ is L_2 norm, which measures the cost deviation and demand deviation in operation process. $||z||_2/||d||_2$ describes the gain of supply chain system output energy to external input energy ratio, which is actually the bullwhip effect from external demand d_k to output cost z_k. Namely, the gain when the amplification effect of demand moves to output cost. Robust control will minimize $||z||_2/||d||_2$ [16]. Using robust control method, we can obtain a controller about production and recovery quantities under the uncertain disturbances of internal parameters and external input which can restrain the bullwhip effect and make the operation cost be ideal.

In this paper, we will use LMI to obtain a robust H∞ control strategy for CLSC.

The dynamic model developed in this paper is a linear discrete time system with uncertain remanufacturing rate, disposal rate, operation costs and customer's demand. According to Kim and Park(1999) [17], we can rewrite Eq.4 and Eq.5 by introducing additional disturbance input, control output and a positive real number to increase the dimensions of input and output equations in original system, and then get a equivalent system as the following equations.

(6) $$x_{k+1} = Ax_k + Bu_k + E_1\tilde{d}_k$$

(7) $$\tilde{z}_k = C_1x_k + D_1u_k$$

where, uncertain matrix satisfies$[\Delta A\ \Delta C]^T=[G_x\ G_z]^T F_k H_x$. H_x, G_z and H_x are known optimal matrixes, F_k is unknown matrix with $F_k^T F_k \le I$, $\tilde{d}_k^T = (d_k, \hat{d}_k)$, $\tilde{z}_k^T = (z_k, \hat{z}_k)$, $C_1^T = (C, H_x/\lambda)$, $D_1^T = (D, H_u/\lambda)$, H_u is known optimal real matrix, $E_1=(E,\ \gamma\lambda G_x)$. $\hat{d}_k$, $\hat{z}_k$ and λ are additional disturbance input, control output and real number separately. That is, the impacts of parameters uncertainty can be eliminated by add additional uncertain external input and control output. Then, the robust H∞ controller u_k of (6) and (7) is equivalent to that of (4) and (5).

Here, we will provide a theorem which can be used to solve (6) and (7) without proof based on Kim and Park (1999) as follows.

Theorem. For a given constant $\gamma>0$, if there exist positive definite matrix Q, S_1, S_2 and M such that

(8) $$\begin{bmatrix} -Q & AQ+BM & E_1 & 0 & 0 & 0 \\ QA^T+M^TB^T & -Q & 0 & QC_1^T+M^TD_1^T & M^T & Q \\ E_1^T & 0 & -\gamma^2 I & 0 & 0 & 0 \\ 0 & C_1Q+D_1M & 0 & -I & 0 & 0 \\ 0 & M & 0 & 0 & -S_2 & 0 \\ 0 & Q & 0 & 0 & 0 & -S_1 \end{bmatrix} < 0$$

then the linear discrete time system described by (6) and (7) is quadratically stable with an H∞ norm bound γ. The corresponding state feedback controller is $u_k = Kx_k$, $K = MQ^{-1}$.

4. Simulation Calculations

In this section, we will carry out simulation calculations to validate the robust H_∞ control strategy for CLSC under uncertainties. The parameters are set as: c_{h1}=0.15, c_{h2}=0.15, c_r=1, c_n=4, c_0=0.3, λ=10^-5^. Using the *feasp solver* in LMI Toolbox of MATLAB and after seven iterations, we can obtain corresponding results which satisfy the condition of LMI Eq.8 as: $t_{\min}$=-0.2071, $M = \begin{bmatrix} -1.9324 & 0.0285 \\ 0.0472 & -0.7571 \end{bmatrix}$, $Q = \begin{bmatrix} 49.8793 & -0.0138 \\ -0.0138 & 0.7600 \end{bmatrix}$. The state feedback controller is

$$u_k = Kx_k = \begin{bmatrix} -0.0387 & 0.0369 \\ 0.0007 & -0.9962 \end{bmatrix} x_k$$

with the restrain parameter γ equals 1.2.

To verify the restraint effect of designed robust control strategy to uncertainty in CLSC system (4) and (5), we will carry out the simulations under following four cases. Before that, we set the nominal values of $x_{1,k}$, $x_{2,k}$, p_k, w_k, z_k as 30, 20, 35, 10, 30 separately. The initial values of both $x_{1,0}$ and $x_{2,0}$ are 50. The simulation results are actual numbers which equal deviation values plus nominal values.

Case 1: We suppose the external uncertain demand follows normal distribution $d_k \sim N(\mu, \sigma^2)$ with $\mu = 20$ and $\sigma^2 = 0.1$. The remanufacturing and disposal rates follow uniform distributions $\alpha \sim U(0, 0.5)$ and $\beta \sim U[0, 0.4]$.

Case 2: The external uncertain demand is the same as case 1. The remanufacturing and disposal rates follow normal distributions, i.e. $\alpha \sim N(\mu_1, \sigma_1^2)$, with $\mu_1 = 0.5$, $\sigma_1^2 = 0.02$; $\beta \sim N(\mu_2, \sigma_2^2)$, $\mu_2 = 0.4$, $\sigma_2^2 = 0.02$.

Case 3: The external uncertain demand follows sine function $d_{1,k} = 10 + 0.1 \sin k$. The distributions of remanufacturing and disposal rates are as large as they are in case 1.

Case 4: The external uncertain demand is the same as it is in case 3. The distributions of remanufacturing and disposal rates are as large as they are in case 2.

The simulation results are depicted in Fig. 2 to Fig. 5. From the simulation results, we find that using the robust H∞ control strategy u_k proposed can restrain the uncertain demand and parameters disturbance no matter what kind of distribution forms they are, and meanwhile, it can make the closed-loop supply chain operation cost be stable, which provides a means of settlement for enterprises to cope with various of uncertainty in actual operation.

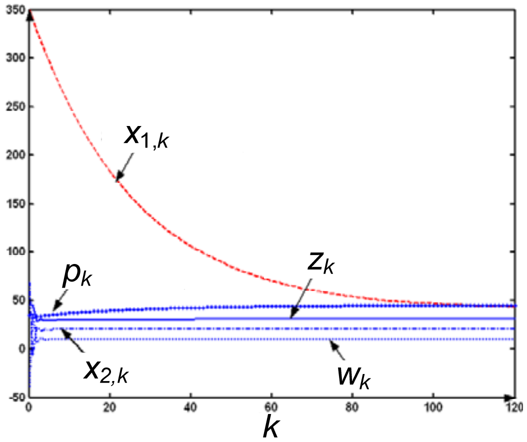

Fig.2. The simulation of closed-loop supply chain under normal disturbance with α~[0, 0.5] and β~[0, 0.4].

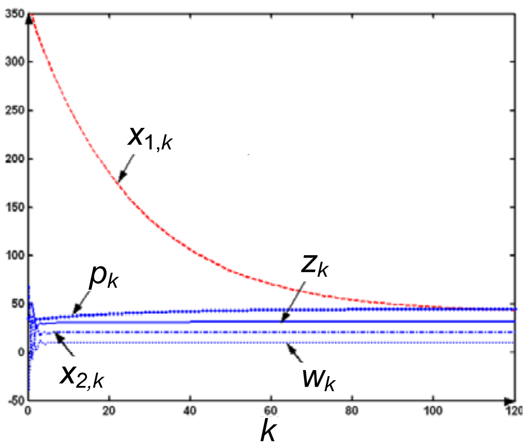

Fig.3. The simulation of closed-loop supply chain under sinusoid disturbance with α~[0, 0.5] and β~[0, 0.4].

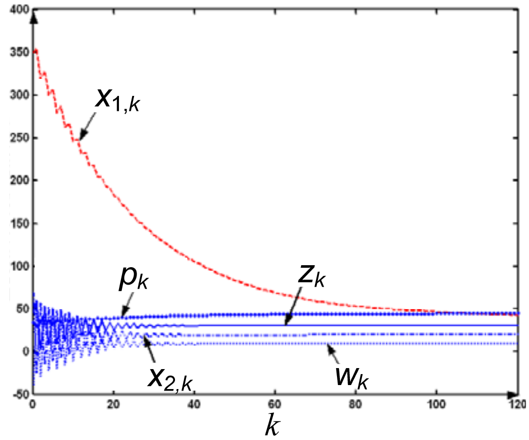

Fig.4. The simulation of closed-loop supply chain under normal disturbance with N~[0.5, 0.02] and N~[0.4, 0.02].

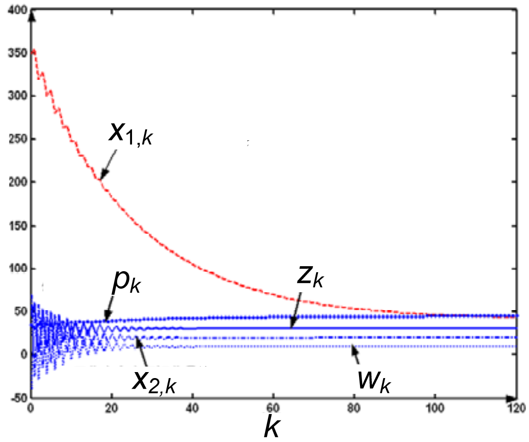

Fig.5. The simulation of closed-loop supply chain under sinusoid disturbance with N~[0.5, 0.02] and N~[0.4, 0.02].

5. Conclusion

In this paper, we develop a dynamic model of closed-loop supply chain with uncertain remanufacturing rate, disposal rate and customers' demand, which involves usable and recovered inventory state equations and system operation cost output equation. Using robust H∞ control method, we present an optimal control strategy which actually is a state feedback controller. The simulation results show that the robust H∞ control strategy proposed can effectively restrain the uncertainty disturbances in closed-loop supply chain, and make the total system operation cost be ideal. In the future, we should explore the value of information in the context of uncertainties with respect to demands, returns and recoveries, that is, the dynamic operation of closed-loop supply chain based on RFID may be the possible future research direction.

Acknowledgments

The authors thank referees for their very constructive comments, and Professor Haifei Yu and Guoqi Feng for English editing assistance. This research is partly supported by the National Nature Science Foundation of China (70702037), Special Fund of the Central Collegiate Basic Scientific Research Bursary Foundation (N090406004), and The Ministry of Education General Research Project in Humanities and Social Sciences (11YJC630165).

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