

## Measurement errors in efficiency estimation of squirrel-cage motor

**Abstract.** Two methods of estimating efficiency of a squirrel-cage induction motor are presented. Both are based on determination of electromagnetic torque in the motor's air gap. Boundary errors of these methods are determined and results of efficiency estimations are compared to those measured directly in accordance with a reference standard.

**Streszczenie.** W artykule zaprezentowano dwie metody estymacji współczynnika sprawności silnika indukcyjnego klatkowego. Obydwie metody powstały w oparciu o wyznaczanie momentu elektromagnetycznego w szczelinie silnika. Wyznaczono błędy graniczne dla omówionych metod oraz porównano otrzymane wyniki estymacji współczynnika sprawności ze współczynnikiem sprawności wyznaczonym metodą bezpośrednią zgodnie z przywołaną normą. (Wpływ błędów pomiarowych na charakterystykę estymacji współczynnika sprawności silnika indukcyjnego klatkowego).

**Keywords:** efficiency estimation error, NAGT method, non-intrusive efficiency estimation, squirrel-cage induction motor efficiency.

**Słowa kluczowe:** błąd estymacji współczynnika sprawności, metoda NAGT, nieinwazyjna metoda estymacji sprawności, sprawność silnika klatkowego.

### Wstęp

Increasing energy efficiency of industrial processes is continually addressed and analysed by research and development institutions. Approximately 69% of industrial electricity is consumed by electric motors, most of them low-voltage squirrel-cage induction motors [1]. Increasingly strict standards are introduced of efficient energy processing in electric motors in order to reduce energy consumption of industrial processes [2]. Introduction of new technologies and materials to production of electric motors to improve their rated efficiency may prove insufficient where a control system allows for energy processing in counter-productive regions of motor operation. Energy-efficient control systems should therefore have access to information on in-service motor's efficiency. There are a number of methods of estimating efficiency of a motor, including those based on electric measurements across stator winding of a machine. Boundary errors introduced by measurement devices are determined and methods of estimating efficiency of a squirrel-cage induction motor are compared in the present paper.

### Selected methods of estimating efficiency of a squirrel-cage motor

Impact of measurement errors is determined for methods that provide for estimating in-service motor efficiency  $\eta$  without mechanical interventions, i.e. fitting of additional measurement devices on the motor shaft. The selected estimation methods are based on determination of the motor's air gap torque.

#### NAGT method

Non-intrusive air-gap torque method – non-intrusive determination of  $\eta$  of an in-service motor – has been covered in [3,4,5,6]. The method is a modification of AGT method, in its original version applied to determine the torque  $T_{ag}$  in the air gap of synchronous machines [7]. This method rests on the following assumptions:

- stator winding is three-phase and symmetrical,
- equivalent three-phase and symmetrical winding of the rotor is introduced,
- motor supply voltage is sinusoidal and variable,
- magnetic fluxes generated by the particular phase windings of the stator and rotor along the air gap are sinusoidal,
- effects of anisotropy, hysteresis and magnetic loop saturation, as well current displacement cross the winding wires are ignored,
- the motor is powered in a three-wire system.

In this method,  $\eta$  is determined on the basis of the vector form of motor's voltage equation, formulated as [8,9]:

$$(1) \quad \mathbf{u}_s = R_s \mathbf{i}_s + \frac{d\boldsymbol{\psi}_s}{dt}$$

where:  $R_s$  – resistance of a phase of the stator's winding,  $\mathbf{u}_s$  – vector of momentary phase voltages supplied to the motor, defined as:

$$(2) \quad \mathbf{u}_s = [u_U \quad u_V \quad u_W]^T$$

$\mathbf{i}_s$  – vector of momentary phase currents of the stator, defined as:

$$(3) \quad \mathbf{i}_s = [i_U \quad i_V \quad i_W]^T$$

$\boldsymbol{\psi}_s$  – vector of momentary phase fluxes of the stator, defined as:

$$(4) \quad \boldsymbol{\psi}_s = [\psi_U \quad \psi_V \quad \psi_W]^T$$

The vector of momentary phase fluxes  $\boldsymbol{\psi}_s$  of the machine stator is determined on the basis of (1):

$$(5) \quad \boldsymbol{\psi}_s = \int (\mathbf{u}_s - R_s \mathbf{i}_s) dt$$

The resistance  $R_s$  in (5) is measured by current injection method [3,10]. Momentary value of torque  $t_{ag}$  is computed as  $\boldsymbol{\psi}_s$  times  $\mathbf{i}_s$  according to [9]:

$$(6) \quad t_{ag} = p |\boldsymbol{\psi}_s \times \mathbf{i}_s|$$

where:  $p$  – number of motor pole pairs.

The dependences (2), (3), (5) and (6) imply  $t_{ag}$  is defined as:

$$(7) \quad t_{ag} = \frac{\sqrt{3}p}{3} \left[ (i_U - i_V) \int (u_{WU} + R_s(2i_U + i_V)) dt \right. \\ \left. + (2i_U + i_V) \int (u_{UV} + R_s(i_U - i_V)) dt \right]$$

On the basis of momentary value of  $t_{ag}$  the average value of  $T_{ag}$  in the air gap is determined:

$$(8) \quad T_{ag} = \frac{1}{t_T} \int_0^{t_T} t_{ag} dt$$

where:  $t_T$  – duration of  $t_{ag}$  under analysis, a total multiple of its period.

Values of  $T_{ag}$  and speed  $n_s$  of magnetic field rotations help to determine power  $P_\psi$  (fig.1) of the motor's rotating field in line with:

$$(9) \quad P_\psi = \frac{2\pi T_{ag} n_s}{60}$$

where:  $n_s$  – synchronous speed of a machine, defined as:

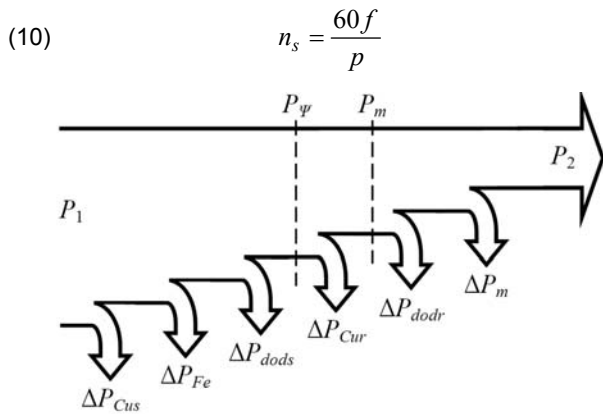


Fig.1. A simplified spread of power losses across a squirrel-cage induction motor

The spread of power losses across an induction motor (fig. 1) produces a dependence between shaft power  $P_2$  and flux power  $P_\psi$ :

$$(11) \quad P_2 = P_\psi - \Delta P_{Cur} - \Delta P_{dodr} - \Delta P_m$$

where:  $\Delta P_{dodr}$  – additional losses across the rotor, defined as a percentage of  $P_2$  [3,4,5]:

$$(12) \quad \Delta P_{dodr} = 1,8\%P_2$$

$\Delta P_m$  –mechanical motor losses, defined as a percentage of  $P_2$  [3,5]:

$$(13) \quad \Delta P_m = 1,2\%P_2$$

$\Delta P_{Cur}$  – losses across the rotor winding, defined as  $P_\psi$  times the slip  $s$  [3,10]:

$$(14) \quad \Delta P_{Cur} = sP_\psi$$

Considering (9) through (14),  $P_2$  yielded by a motor is defined as:

$$(15) \quad P_2 = \frac{\pi}{30,9} T_{ag} n$$

where:  $n$  – rotor's rotational speed.

This computed shaft power  $P_2$  helps to determine the efficiency  $\eta_H$  of an in-service induction motor according to the dependence:

$$(16) \quad \eta_H = \frac{P_2}{P_1}$$

where:  $P_1$  – active power consumed by the motor, defined as [11]:

$$(17) \quad P_1 = \frac{1}{t_T} \int_0^{t_T} (2u_U i_U + 2u_V i_V + u_U i_V + u_V i_U) dt$$

### Modified NAGT method

NAGT was utilised to develop a modified method of estimating efficiency of squirrel-cage induction motors below 2.2kW.

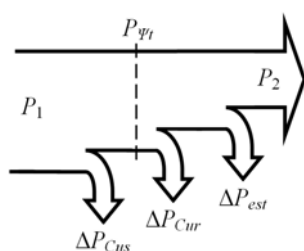


Fig.2. A simplified spread of power losses across a squirrel-cage induction motor for the modified NAGT

The modifications involved mainly introduction of estimated losses  $\Delta P_{est}$  dependent on the rotor's  $n$  in consideration of core losses  $\Delta P_{Fe}$ . The simplified spread of power losses for the modified NAGT (fig. 2) comprises two types of power losses: stator copper losses  $\Delta P_{Cus}$ , rotor copper  $\Delta P_{Cur}$  and  $\Delta P_{est}$ . The estimated losses  $\Delta P_{est}$  have been defined as the sum total of core losses  $\Delta P_{Fe}$ , stator stray load losses  $\Delta P_{dods}$ , windage and friction losses  $\Delta P_m$  and rotor stray load losses  $\Delta P_{dodr}$ :

$$(18) \quad \Delta P_{est} = \frac{n}{n_s} (\Delta P_{Fes} + \Delta P_{dods}) + \Delta P_{dodr} + \Delta P_m$$

As the mathematical apparatus defining partial losses in line with (18) is extensive,  $\Delta P_{est}$  have been defined as a function of rotational speed  $n$  [12,13]:

$$(19) \quad \Delta P_{est}^* = \beta (n^*)^\alpha$$

where:  $\Delta P_{est}^*$  – relative estimated losses;  $n^*$  – relative rotational speed of the rotor;  $\beta$  – correction factor;  $\alpha$  – power's exponent, where  $\alpha \in \mathbb{R}$ , with:

$$(20) \quad \Delta P_{est}^* = \frac{\Delta P_{est}}{\Delta P_{estN}}$$

$$(21) \quad n^* = \frac{n}{n_N}$$

where:  $\Delta P_{est}$  – estimated losses,  $\Delta P_{estN}$  – estimated losses at rated load of the motor,  $n$  – rotational speed of the motor,  $n_N$  – rated rotational speed of the motor.

The way of defining theoretical power  $P_{\psi t}$  of the rotating field has also been modified and based on the spread of motor power losses (fig. 2) in accordance with:

$$(22) \quad P_{\psi t} = P_1 - \Delta P_{Cus}$$

where:  $\Delta P_{Cus}$  – stator copper losses power, defined by:

$$(23) \quad \Delta P_{Cus} = \frac{1}{t_T} \int_0^{t_T} (R_s i_U^2 + R_s i_V^2 + R_s (i_U + i_V)^2) dt$$

The shaft power  $P_{2m}$  is computed according to:

$$(24) \quad P_{2m} = P_{\psi t} \frac{n}{n_s} - \Delta P_{est}$$

The efficiency  $\eta_m$  of an induction motor is defined by:

$$(25) \quad \eta_m = \frac{P_{2m}}{P_1}$$

### Effect of measurement errors on efficiency estimation

Accuracy of NAGT and the modified NAGT depends on their assumptions and simplifications as well as precision of measurement devices. Both the methods base on measuring momentary values of two voltages ( $u_{UV}$ ,  $u_{WU}$ ), two phase currents ( $i_U$ ,  $i_V$ ), resistance  $R_s$  of stator winding, frequency  $f$  of the supply voltage, and estimation of rotational speed  $n$ . A boundary error  $\delta_g$  of the estimated motor efficiency has been determined for both the methods employing the total differential according to accuracy classes of measurement devices.  $\delta_{gH}$  of NAGT is defined by:

$$(26) \quad \delta_{gH} = \left( \frac{2|I_s R_s n \Delta U| + 2|U_s R_s n \Delta I| + |I_s U_s n \Delta R|}{|U_s^2 n \cos \varphi - U_s I_s R_s n|} + \frac{|U_s^2 \cos \varphi \Delta n - U_s I_s R_s \Delta n|}{|U_s^2 n \cos \varphi - U_s I_s R_s n|} \right)$$

where:  $\Delta U$  – absolute error of voltage measurement  $U_s$ ,  $\Delta I$  – absolute error of current measurement  $I_s$ ,  $\Delta R$  – absolute measurement error of stator winding resistance  $R_s$ ,  $\Delta n$  – absolute measurement error of rotational speed  $n$ .  
 A boundary error  $\delta_{gm}$  for the modified NAGT is defined according to:

$$(27) \quad \delta_{gm} = \left( \begin{array}{l} 2 \left| \frac{(I_s^2 R_s p + 20 \beta f) \Delta U}{U_s (-I_s U_s p \cos \varphi + I_s^2 R_s p + 20 \beta f)} \right| + \\ + 2 \left| \frac{(I_s^2 R_s p + 20 \beta f) \Delta I}{I_s (-I_s U_s p \cos \varphi + I_s^2 R_s p + 20 \beta f)} \right| + \\ + \left| \frac{I_s^2 p \Delta R}{-I_s U_s p \cos \varphi + I_s^2 R_s p + 20 \beta f} \right| + \frac{\Delta n}{n} + \\ + \left| \frac{I_s p (U_s \cos \varphi + I_s R_s) \Delta f}{f (-I_s U_s p \cos \varphi + I_s^2 R_s p + 20 \beta f)} \right| \end{array} \right)$$

where:  $\Delta f$  – absolute measurement error of frequency  $f$  of the motor supply voltage;  
 The calculations were executed in respect of four motors (tab. 1) and frequencies  $f=25\text{Hz}$  and  $f=50\text{Hz}$  of the supply voltage.

Table 1. Data of tested motors

No.	Manufacturer	$P_N$ [kW]	$U_N$ [V]	$I_N$ [A]	$n_N$ [rpm]	$\cos$ [-]	$\eta$ [-]
1	TAMEL	1,50	380	3,7	1420	0,80	0,77
2	INDUKTA	2,20	400	5,0	2870	0,77	0,82
3	BESEL	0,75	400	2,7	670	0,61	0,66
4	BESEL	1,50	400	4,2	900	0,70	0,71

The results were compared to efficiency  $\eta_r$  (fig. 3-6) measured directly in compliance with the standard PN-EN 60034-2-1-2010 [14].  $\eta_r$  was termed the real value.  $\delta_{gr}$  for the real value of  $\eta_r$  was defined as follows:

$$(28) \quad \delta_{gr} = 2 \left| \frac{\Delta U}{U_s} \right| + 2 \left| \frac{\Delta I}{I_s} \right| + \left| \frac{\Delta T}{T} \right| + \left| \frac{\Delta n}{n} \right|$$

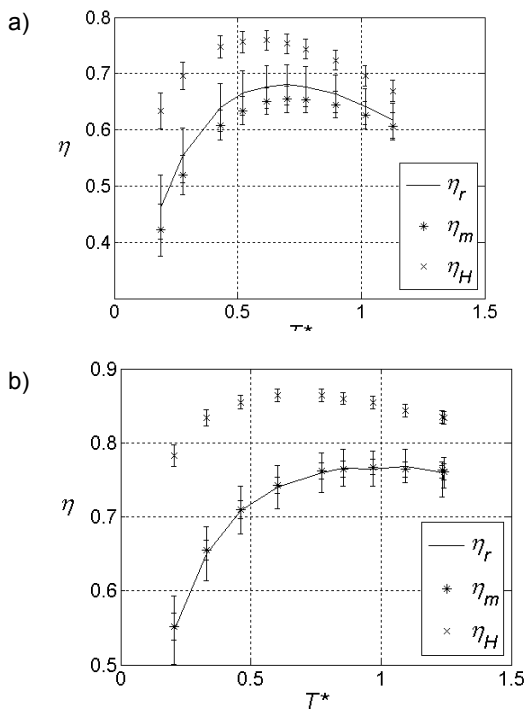


Fig.3. Efficiency  $\eta$  as a function of relative shaft motor 1 torque  $T^*$ : a)  $f=25\text{Hz}$ , b)  $f=50\text{Hz}$

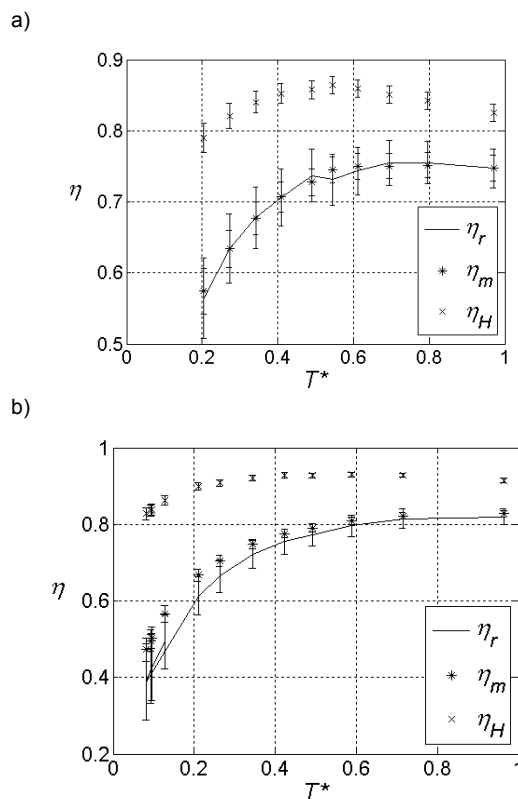


Fig.4. Efficiency  $\eta$  as a function of relative shaft motor 2 torque  $T^*$ : a)  $f=25\text{Hz}$ , b)  $f=50\text{Hz}$

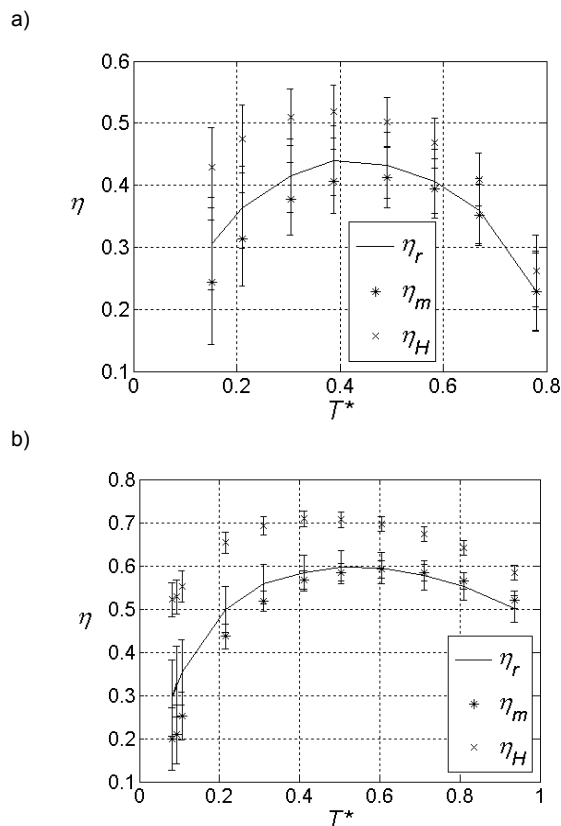


Fig.5. Efficiency  $\eta$  as a function of relative shaft motor 3 torque  $T^*$ : a)  $f=25\text{Hz}$ , b)  $f=50\text{Hz}$

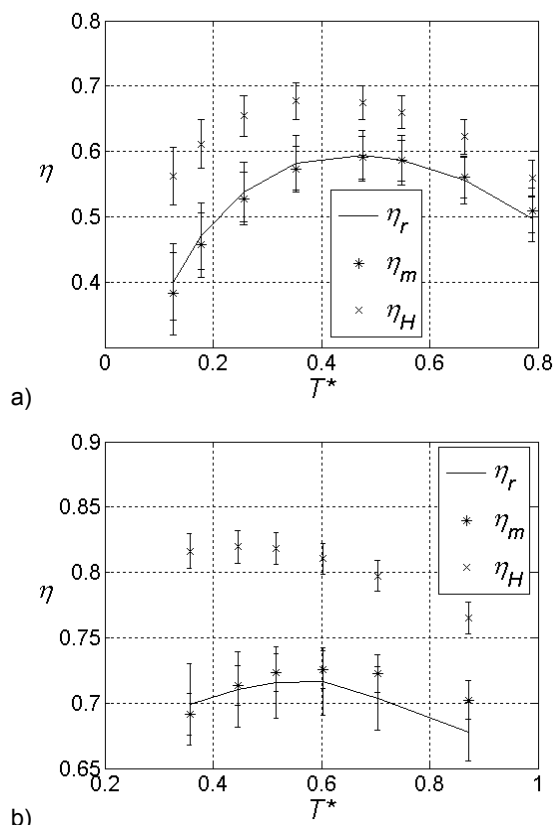


Fig.6. Efficiency  $\eta$  as a function of relative shaft motor torque  $T^*$ : a)  $f=25\text{Hz}$ , b)  $f=50\text{Hz}$

Figures 3 - 6 illustrate estimated efficiencies  $\eta_H$  and  $\eta_m$  and real efficiency  $\eta_r$ . Boundary error bars according to classes of measurement devices have been plotted in each point. Calculations imply the order of magnitude of  $\delta_{gH}$  (26) for NAGT is comparable to  $\delta_{gm}$  (27) for the modified NAGT. The maximum  $\delta_{gH} = 6.5\%$  was noted in the case of motor 3 at  $f=25\text{Hz}$  of the supply voltage. In the same case,  $\delta_{gm}$  reached 10% and measurement error of the real value was estimated as 7.5%. Such great errors occur for torque  $T^*$  below 0.5, where  $T^*$  is defined by:

$$(29) \quad T^* = \frac{T}{T_N}$$

This is caused by too high measurement ranges of the devices compared to such measured values as the current  $I_s$  of the stator winding or  $T$  across the motor shaft. This effect is most conspicuous in motor 3 (fig. 5a).

Friction and windage losses and stray load losses according to (12) and (13), adopted for purposes of NAGT, caused the estimated efficiency  $\eta_H$  to become excessive in relation to the real  $\eta_r$ . NAGT can only be applied to specific types of motors, referred to in [3,4,5]. The method proved unsatisfactory with regard to the motors under examination (tab. 1). Modifying NAGT by defining estimated losses as a function of rotational speed caused the bars of estimated efficiency  $\eta_m$  and bars of real efficiency  $\eta_r$  to overlap.

## Conclusion

Our laboratory testing helped to compare the estimation methods of efficiency of squirrel-cage induction motor. Different boundary errors for each estimation method were obtained on the basis of the same measurement data. The differences resulted from adoption of various assumptions. In the case of NAGT,  $\delta_{gH}$  is lower than  $\delta_{gm}$  of the modified NAGT. The lower measurement error does not translate into a reduced error of efficiency estimation, however. The modified NAGT is characterised by a measurement error  $\delta_{gm}$  greater than  $\delta_{gH}$ , yet the error of efficiency estimation is clearly lower in the case of tested motors. Continued testing of induction motors is intended to verify the modified NAGT.

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