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Calculating steady-state non-sinusoidal fields in nonlinear conducting media by analytical-numerical method

Streszczenie. *Przedstawiona została analiza pola elektromagnetycznego z uwzględnieniem nieliniowej konduktywności materiału przewodzącego. W pracy wzięto również pod uwagę odkształcone przebiegi wielkości pola. W celu otrzymania rozkładu pola, zastosowano metodę analityczno-numeryczną opartą na metodzie małego parametru. Przedstawionym przykładem jest przewód rurowy o nieliniowej konduktywności. Rozwiązanie sprawdzono poprzez wprowadzenie dwóch kryteriów: błędu całkowego oraz błędu wartości całkowitego prądu. (**Obliczanie pól o przebiegach odkształconych w środowisku nieliniowym poprzez wykorzystanie metody analityczno-numerycznej**).*

Abstract*. An analysis of electromagnetic fields inside a nonlinear conducting material has been presented. Non-sinusoidal periodic fields have also been taken into account. In order to obtain the field distributions, an analytical-numerical method based on the method of small parameter has been used. The presented example is a tubular conductor of nonlinear conductivity. The solution was verified with the use of two criteria: an integral error and a total current error.*

Słowa kluczowe: analiza pola elektromagnetycznego, przebiegi odkształcone, nieliniowa konduktywność, rozwiązanie analityczne.
Keywords: electromagnetic field analysis, non-sinusoidal field, nonlinear conductivity, analytical solution.

Introduction

The application of an analytical-numerical method for calculating periodic electromagnetic fields is presented. The method is based on the method of small parameter [1]. Its specific feature is a partially symbolic solution. The partially symbolic solution is a relationship between the boundary field and parameters common to the inertial field harmonics. The first stage reduces the boundary value problem to a system of nonlinear equations. The system of equations can be solved with a chosen numerical method. The presented method is designed to solve nonlinear problems for forced periodic fields with higher harmonics taken into account. The method utilizes analytical solutions hence the paper considers a boundary value problem in an axially symmetric structure of a tubular conductor (fig.1). Some simplifications have been made – for example the base frequency is the industrial $50\mathrm{Hz}$, which means that displacement currents can be omitted. Several other simplifications regarding the conductor geometry and field directions have been taken into account. The proceedings, by which the symbolic solution is obtained, are of analytical nature and most operations (like several Fourier series multiplications) have been achieved with a custom written C++ program.

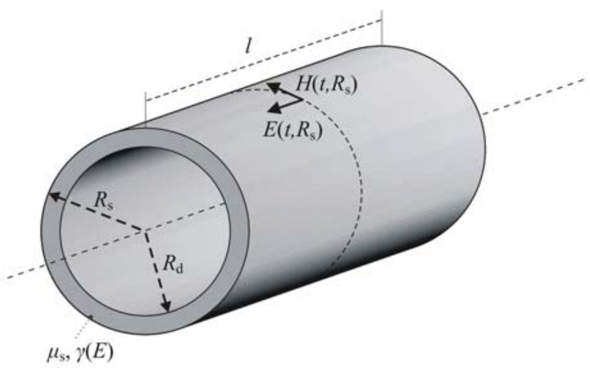

Fig.1. Boundary value problem of a tubular conductor with nonlinear conductivity

Problem formulation

A theoretical example is presented in order to explain the method. A tubular conductor of homogenous, isotropic and nonlinear conductivity is taken into account. The conductivity is chosen to obey a relationship similar to that of a superconductor J-E curve at constant temperature of $77\mathrm{K}$ [2], [3] (fig. 2). The J-E relationship has been approximated with an odd power series to reflect the model curve at least up to $1{,}5$ of the critical value of electric field strength. The power series can be expressed by the formula:

$$J(E) = \sum_{k=1,3,5...}^{m} \gamma_{sk} E^k . \tag{1}$$

The critical value is often assumed as $1\ \mu\mathrm{V/cm}$ [4]. Following diamagnetic properties of a superconductor, its relative permeability takes values of $\mu_{rs} << 1$. In this work the value chosen is $\mu_{rs} = 10^{-3}$.

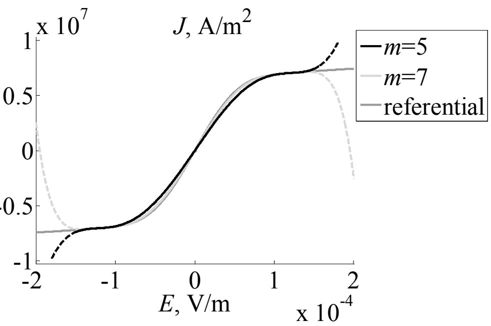

Fig.2. Comparison of model J-E curve and its power series approximations (dotted lines are values above $1.5\ \mu\mathrm{V/cm}$)

An approximation for $m = 5$ and $m = 7$ does not differ significantly, hence in further analysis, a power series of 3 terms is used:

$$J(E) = \gamma_{s1}E + \gamma_{s3}E^3 + \gamma_{s5}E^5 . \tag{2}$$

The object has been placed in a cylindrical coordinate system (fig.3).

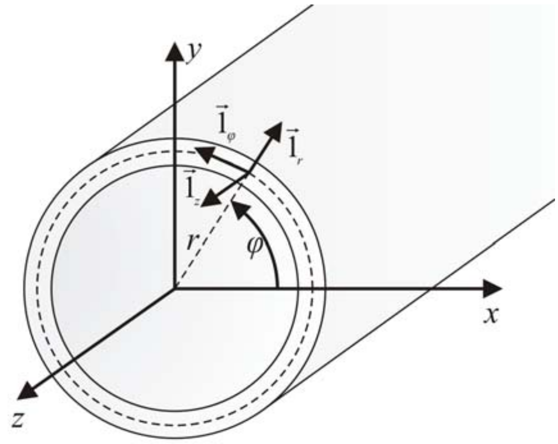

Fig.3. Nonlinear tubular conductor in cylindrical coordinates

Longitudinal analysis has been omitted ($l >> R_s$) and only changes along the r axis are assumed. Following the simplifications mentioned above, electromagnetic field values E, J, B and H each only appear as one axial component:

(3) $$\frac{1}{\gamma(E)}\vec{J}(t,r) = \vec{E}(t,r) = E_z(t,r)\vec{1}_z,$$

(4) $$\frac{1}{\mu_s}\vec{B}(t,r) = \vec{H}(t,r) = H_\varphi(t,r)\vec{1}_\varphi.$$

Their relations to the magnetic vector potential are as follows:

(5) $$E_z(t,r) = -\frac{\partial A(t,r)}{\partial t},$$

(6) $$\mu_s H_\varphi(t,r) = -\frac{\partial A(t,r)}{\partial r},$$

where the potential consists only of its z axis component:

(7) $$A(t,r) = A_z(t,r).$$

A boundary condition, imposed on the edge of the conductor $r=R_s$, takes form of a Dirichlet problem for electric field strength:

(8) $$E_z(t,R_s) = -\frac{\partial A(t,R_s)}{\partial t} = \Xi_D(t).$$

When imposing the magnetic field strength angular component value, a Neumann problem needs to be solved:

(9) $$H_\varphi(t,R_s) = -\frac{1}{\mu_s}\frac{\partial A(t,R_s)}{\partial r} = \Xi_N(t).$$

The latter can also be used to calculate the distribution for a given time function of the total current flowing through the conductor:

(10) $$I(t) = -\frac{2\pi R_s}{\mu_s}\frac{\partial A(t,R_s)}{\partial r} = \Xi_I(t),$$

which is also a Neumann problem. The Ξ symbol represents boundary values of different units depending on the given physical quantity.

Partially symbolic solution

The problem deals with periodic non-sinusoidal fields hence the partially symbolic solution, obtained in this chapter, is a relationship between each harmonic of the inertial fields and each of the boundary time function. Because of the simplifications listed in the previous chapter, the problem's mathematical basis becomes the following nonlinear differential equation of the magnetic vector potential [3]:

(11) $$\frac{\partial^2 A(t,r)}{\partial r^2} + \frac{1}{r}\frac{\partial A(t,r)}{\partial r} - \gamma_{s1}\mu_s\frac{\partial A(t,r)}{\partial t} = \\ = \gamma_{s3}\mu_s\left(\frac{\partial A(t,r)}{\partial t}\right)^3 + \gamma_{s5}\mu_s\left(\frac{\partial A(t,r)}{\partial t}\right)^5.$$

The method of small parameter is used to obtain the solution of this equation. It is assumed that the potential is expanded into a power series of the form:

(12) $$A(t,r) = \sum_{i=1}^{n}\kappa^{i-1}A_i(t,r), \quad n \to \infty,$$

where κ is called the "small parameter". In most cases, only the first two terms are required. However, the more significant the nonlinear terms of (11) become, the greater the value of n is required to achieve an accurate solution. It is best to apply certain substitutions so that all nonlinear terms are multiplied by κ. First, the "small parameter" is defined as:

(13) $$\kappa = \gamma_{s5}\mu_s,$$

next, an auxiliary parameter γ_{s3}' is defined, which follows:

(14) $$\kappa\gamma_{s3}' = \gamma_{s3}\mu_s,$$

therefore (11) becomes:

(15) $$\sum_{i=1}^{n}\left(\kappa^{i-1}\frac{\partial^2 A_i(t,r)}{\partial r^2}\right) + \frac{1}{r}\sum_{i=1}^{n}\left(\kappa^{i-1}\frac{\partial A_i(t,r)}{\partial r}\right) - \\ - \gamma_{s1}\mu_s\sum_{i=1}^{n}\left(\kappa^{i-1}\frac{\partial A_i(t,r)}{\partial t}\right) = \\ = \kappa\gamma_{s3}'\left(\sum_{i=1}^{n}\kappa^{i-1}\frac{\partial A_i(t,r)}{\partial t}\right)^3 + \\ + \kappa\left(\sum_{i=1}^{n}\kappa^{i-1}\frac{\partial A_i(t,r)}{\partial t}\right)^5, \quad n \to \infty.$$

Comparing terms of the same power of κ on both sides of the above equation, a system of n linear differential equations can be formulated. Each of these equations can be expressed by the formula:

$$\frac{\partial^2 A_i(t,r)}{\partial r^2}+\frac{1}{r}\frac{\partial A_i(t,r)}{\partial r}-\gamma_{s1}\mu_s\frac{\partial A_i(t,r)}{\partial t}=W_i(t,r), \tag{16}$$

where the right-hand side terms are dependent on all solutions of the previous equations, hence for $i>1$ one obtains:

$$W_i(t,r)=W_i\left(\frac{\partial A_1(t,r)}{\partial t},\ldots,\frac{\partial A_{i-1}(t,r)}{\partial t}\right), \tag{17}$$

for $i=1$ the right-hand side of (17) is zero. The above states that the differential equations (16) need to be solved in sequence one after the other. However, first there is the task of obtaining the W terms. Since steady state solutions are assumed, the magnetic vector potential can be expressed by means of a Fourier series:

$$A(t,r)=A_0(r)+\sum_{h=1}^{h_{\max}}A_h(r)\cos\left(h\omega_0 t+\alpha_h(r)\right), \tag{18}$$

where $h_{\max}$ denotes the maximum higher harmonic number. ω_0 is the base pulsation and is related to the industrial frequency $\omega_0/2\pi=f_0=50\mathrm{Hz}$. Hence, terms of (12) can also be expressed as periodic functions:

$$A_i(t,r)=A_{i,0}(r)+\sum_{h=1}^{h_{i\max}}A_{i,h}(r)\cos\left(h\omega_0 t+\alpha_{i,h}(r)\right). \tag{19}$$

A_i time derivatives, which appear on the right-hand side of (15), take the form:

$$\frac{\partial A_i(t,r)}{\partial t}=-\sum_{h=1}^{h_{i\max}}h\omega_0 A_{i,h}(r)\sin\left(h\omega_0 t+\alpha_{i,h}(r)\right), \tag{20}$$

With the use of trigonometric identities, it is possible to obtain the W terms of (16) for finite values of harmonic terms considered for each term A_i. Equations of the form (16) are linear equations, which means that by superposition rule each harmonic component can be calculated separately – hence the complex form is used:

$$\frac{\mathrm{d}^2\underline{A}_{i,h}(r)}{\mathrm{d}r^2}+\frac{1}{r}\frac{\mathrm{d}\underline{A}_{i,h}(r)}{\mathrm{d}r}-\underline{\Gamma}_{sh}{}^2\underline{A}_{i,h}(r)=\underline{W}_{i,h}(r), \tag{21}$$

where $\underline{\Gamma}_{sh}$ is the linear propagation constant of harmonic h:

$$\underline{\Gamma}_{sh}=\sqrt{\mathrm{j}h\omega_0\mu_s\gamma_{s1}}. \tag{22}$$

By solving the equations of the form (21), the full solution can be obtained. For the harmonic function h, the solution of the first term ($i=1$) consists of Bessel functions as follows:

$$\underline{A}_{1,h}(r)=\underline{c}_{1,h}\underline{I}_0(\underline{\Gamma}_{sh}r)+\underline{c}_{2,h}\underline{K}_0(\underline{\Gamma}_{sh}r), \tag{23}$$

$$\frac{\mathrm{d}\underline{A}_{1,h}(r)}{\mathrm{d}r}=\underline{\Gamma}_{sh}\underline{c}_{1,h}\underline{I}_1(\underline{\Gamma}_{sh}r)-\underline{\Gamma}_{sh}\underline{c}_{2,h}\underline{K}_1(\underline{\Gamma}_{sh}r). \tag{24}$$

$\underline{c}_{1,h}$ and $\underline{c}_{2,h}$ are unknown constant coefficients of the potential distribution. The solution of (21), for $i>1$ takes the form:

$$\begin{bmatrix}\underline{A}_{i,h}(r)\\[2mm] \dfrac{\mathrm{d}\underline{A}_{i,h}(r)}{\mathrm{d}r}\end{bmatrix}=\mathbf{M}_{i,h}(r), \tag{25}$$

where the $\mathbf{M}$ vector can be expressed as the result of the following matrix multiplication:

$$\mathbf{M}_{i,h}(r)= \\ =\mathbf{N}_h(r)\begin{bmatrix}\int_{R_d}^{r}\underline{K}_0(\underline{\Gamma}_{sh}r)\underline{W}_{i,h}(r)r\mathrm{d}r\\ -\int_{R_d}^{r}\underline{I}_0(\underline{\Gamma}_{sh}r)\underline{W}_{i,h}(r)r\mathrm{d}r\end{bmatrix}. \tag{26}$$

$\mathbf{N}$ is the Wronsky matrix of harmonic h:

$$\mathbf{N}_h(r)=\begin{bmatrix}\underline{I}_0(\underline{\Gamma}_{sh}r) & \underline{K}_0(\underline{\Gamma}_{sh}r)\\ \underline{\Gamma}_{sh}\underline{I}_1(\underline{\Gamma}_{sh}r) & -\underline{\Gamma}_{sh}\underline{K}_1(\underline{\Gamma}_{sh}r)\end{bmatrix}. \tag{27}$$

No currents flow inside $r<R_d$, thus the following inertial condition can be formulated:

$$\frac{\partial A(t,R_d)}{\partial r}=-\mu_s H_\varphi(t,R_d)=0. \tag{28}$$

The definite integral in (27) for $i>1$ causes:

$$\mathbf{M}_{i,h}(R_d)=\begin{bmatrix}0\\0\end{bmatrix}. \tag{29}$$

The condition (28) determines the value of the first term of (12) giving:

$$\frac{\partial A_1(t,R_d)}{\partial r}=0. \tag{30}$$

The above and (24) yield:

$$\underline{c}_{2,h}=\frac{\underline{c}_{1,h}\underline{I}_1(\underline{\Gamma}_{sh}R_d)}{\underline{K}_1(\underline{\Gamma}_{sh}R_d)}, \tag{31}$$

which reduces the number of complex unknowns per harmonic to one. The magnetic vector potential takes the form:

$$\underline{A}_{1,h}(r)=\underline{c}_{1,h}\left(\underline{I}_0(\underline{\Gamma}_{sh}r)+\frac{\underline{I}_1(\underline{\Gamma}_{sh}R_d)}{\underline{K}_1(\underline{\Gamma}_{sh}R_d)}\underline{K}_0(\underline{\Gamma}_{sh}r)\right). \tag{32}$$

An imposed boundary condition does not lead to a direct relationship between its time function Ξ and the unknown coefficients $\underline{c}_{1,h}$. Therefore, the next task becomes finding that direct relationship. The boundary condition can also be imposed in complex form, for each harmonic separately, so in the case of Dirichlet boundary condition one obtains:

$$\underline{A}_h(R_s)=-\frac{1}{\mathrm{j}h\omega_0}\underline{\Xi}_{Dh}, \tag{33}$$

and Neumann boundary conditions in complex form are:

(34) $$\frac{\mathrm{d}\underline{A}_h(R_\mathrm{s})}{\mathrm{d}r} = -\mu_\mathrm{s}\underline{\Xi}_{\mathrm{N}h}.$$

Further on, the complex coefficients are represented as vectors of their magnitudes:

(35) $$\mathbf{c} = [|\underline{c}_{1,1}|, |\underline{c}_{1,2}|, \ldots |\underline{c}_{1,\eta_1}|],$$

and arguments:

(36) $$\boldsymbol{\theta} = [\arg(\underline{c}_{1,1}), \arg(\underline{c}_{1,2}), \ldots \arg(\underline{c}_{1,\eta_1})],$$

hence for all harmonics of the boundary condition, the required relationship is:

(37) $$\underline{\Xi}_h = \underline{f}_h(\mathbf{c}, \boldsymbol{\theta}).$$

In this work a set of the above, for $h = 1, 3, \ldots h_{1\,\max}$, is called the partially symbolic solution in complex form. In terms of (35) and (36), the solution for the first term of (12), for harmonic h becomes:

(38) $$\begin{aligned} &A_{1,h}(t,r) = \\ &= |\underline{c}_{1,h}| A_{1,h}'(r)\cos\left(h\omega_0 t + \alpha_{1,h}'(r) + \arg(\underline{c}_{1,h})\right), \end{aligned}$$

where:

(39) $$A_{1,h}(r) = |\underline{c}_{1,h}| A_{1,h}'(r),$$

(40) $$\alpha_{1,h}(r) = \alpha_{1,h}'(r) + \arg(\underline{c}_{1,h}).$$

For further terms, the relationship is not direct and requires analytical calculations. The solutions for $i > 1$ can be expressed by the general formula:

(41) $$\begin{aligned} &\mathbf{M}_{i,h}(\mathbf{c},\boldsymbol{\theta},r) = \\ &= \mathbf{N}_h(r)\begin{bmatrix} \int_{R_\mathrm{d}}^{r} \underline{K}_0(\underline{\Gamma}_{\mathrm{s}h} r)\underline{W}_{i,h}(\mathbf{c},\boldsymbol{\theta},r) r\,\mathrm{d}r \\ -\int_{R_\mathrm{d}}^{r} \underline{I}_0(\underline{\Gamma}_{\mathrm{s}h} r)\underline{W}_{i,h}(\mathbf{c},\boldsymbol{\theta},r) r\,\mathrm{d}r \end{bmatrix}, \end{aligned}$$

which in consequence allows to present a relation in time domain:

(42) $$\begin{aligned} &A_i(\mathbf{c},\boldsymbol{\theta},t,r) = A_{i,0}(\mathbf{c},\boldsymbol{\theta},r) + \\ &+ \sum_{h=1}^{h_{i\max}} A_{i,h}(\mathbf{c},\boldsymbol{\theta},r)\cos\left(h\omega_0 t + \alpha_{i,h}(\mathbf{c},\boldsymbol{\theta},r)\right), \end{aligned}$$

and a full solution for the magnetic vector potential:

(43) $$A(\mathbf{c},\boldsymbol{\theta},t,r) = \sum_{i=1}^{n} \kappa^{i-1} A_i(\mathbf{c},\boldsymbol{\theta},t,r).$$

The radial derivative is:

(44) $$\frac{\partial A(\mathbf{c},\boldsymbol{\theta},t,r)}{\partial r} = \sum_{i=1}^{n} \kappa^{i-1}\frac{\partial A_i(\mathbf{c},\boldsymbol{\theta},t,r)}{\partial r}.$$

This allows to formulate the partially symbolic solution. A set of nonlinear equations is obtained for a Dirichlet boundary condition:

(45) $$\underline{A}_h(\mathbf{c},\boldsymbol{\theta},R_\mathrm{s}) = -\frac{1}{\mathrm{j}h\omega_0}\underline{\Xi}_{\mathrm{D}h},$$

and for a Neumann boundary condition:

(46) $$\frac{\mathrm{d}\underline{A}_h(\mathbf{c},\boldsymbol{\theta},R_\mathrm{s})}{\mathrm{d}r} = -\mu_\mathrm{s}\underline{\Xi}_{\mathrm{N}h}.$$

Note that the number of harmonics in the first term ($h_{1\,\max}$) is also the number of the complex unknowns $\underline{c}_{1,h}$ which means that $h_{1\,\max} = h_{\max}$.

Field distribution in a nonlinear tubular conductor, error calculation

The study involves a theoretical J-E curve and constant magnetic permeability of the nonlinear conductor. A real superconductor has much more nonlinear properties [5], [6]. The aim was only to introduce a method of calculating distributions in a nonlinear conductor. Even though the solution is only in part symbolic, the amount of parametric terms is huge. In this case some limits needed to be taken into account in order to increase computer calculation performance (this however reduced the accuracy of the results). Not only can the resulting amount of harmonics per i-th term $h_{i\max}$ be reduced but also intermediate higher harmonic results when calculating the W terms can be omitted. In this paper, only 3 harmonic terms were taken into account in every term $h_{i\max} = 3$. When calculating the W terms the results have been limited at a chosen $h_{\mathrm{interm.}}$ per every multiplication of two non-sinusoidal time functions. The results presented here have been obtained for $h_{\mathrm{interm.}} = 11$. To obtain the W terms is not as difficult of a task as their evaluation in partially symbolic form which is why the chosen number of maximum terms is $n = 5$. The previous chapter presents the proceedings in case of boundary conditions of the first or second kind. An example presented (results on fig.4, 5) considers electromagnetic field distribution calculations for a Neumann boundary condition:

(47) $$\frac{\mathrm{d}\underline{A}_h(R_\mathrm{s})}{\mathrm{d}r} = \begin{cases} h = 1, & 7\cdot 10^{-5}\,\mathrm{T}, \\ h > 1, & 0. \end{cases}$$

In terms of magnetic field strength, this boundary condition can be interpreted as the following forced values of time function harmonics:

(48) $$\underline{H}_h(R_\mathrm{s}) = \begin{cases} h = 1, & 5.57\cdot 10^{4}\,\mathrm{A/m}, \\ h > 1, & 0. \end{cases}$$

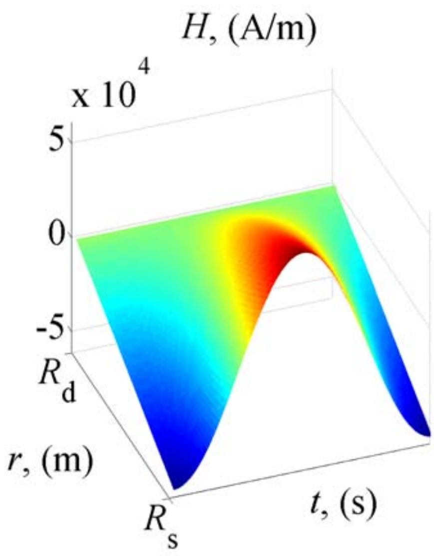

Fig.4. Magnetic field distribution along the radius of the nonlinear tubular conductor

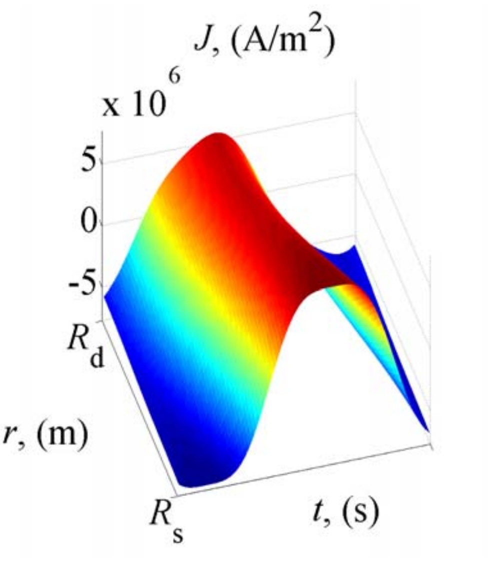

Fig.5. Current density along the radius of the nonlinear tubular conductor

The imposed Neumann boundary condition points out the total current applied to the conducting region. $H(t,R_\mathrm{s})$ expresses only the first harmonic which proves that the estimation of the $\underline{c}_{1,h}$ coefficients has been done correctly. However, the method is not without error as the current density time function (fig.5) is clearly non-sinusoidal and harmonics above 3 have been omitted in the result. A total current error is defined with respect to amplitude:

$$e_{I\ \mathrm{amp.}} = \left|1 - \left|\frac{\underline{I}_J}{\underline{I}_H}\right|\right| \cdot 100\%, \tag{49}$$

and for phase we calculate a value according to the following formula:

$$e_{I\ \mathrm{ph.}} = \left|\frac{1}{\pi}\arg\left(\frac{\underline{I}_J}{\underline{I}_H}\right)\right| \cdot 100\%. \tag{50}$$

Additionally, an error expressing the accuracy of the result is defined – in the case of this paper it is the integral error used in the authors' previous article [3]. It exists in alternate forms of amplitude error:

$$e_{\int\mathrm{amp.}} = \frac{1}{R_\mathrm{s} - R_\mathrm{d}} \int_{R_\mathrm{d}}^{R_\mathrm{s}} \left|1 - \left|\frac{-\mu_\mathrm{s}\underline{J}_h(r)}{\dfrac{\partial^2 \underline{A}_h(r)}{\partial r^2} + \dfrac{1}{r}\dfrac{\partial \underline{A}_h(r)}{\partial r}}\right|\right| \mathrm{d}r \cdot 100\%, \tag{51}$$

and phase error:

$$e_{\int\mathrm{ph.}} = \frac{1}{R_\mathrm{s} - R_\mathrm{d}} \int_{R_\mathrm{d}}^{R_\mathrm{s}} \left|\frac{1}{\pi}\arg\left(\frac{-\mu_\mathrm{s}\underline{J}_h(r)}{\dfrac{\partial^2 \underline{A}_h(r)}{\partial r^2} + \dfrac{1}{r}\dfrac{\partial \underline{A}_h(r)}{\partial r}}\right)\right| \mathrm{d}r \cdot 100\%. \tag{52}$$

Taking into account that the potential consists of the first and third harmonic function, the integral error for these harmonics has been calculated. Results of (51) and (52) for n=1÷5 are presented on figure 6 for the first time harmonic and on figure 7 for the third time harmonic.

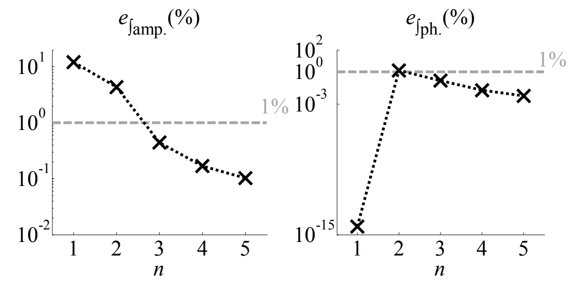

Fig.6. First harmonic integral error calculation results for different n

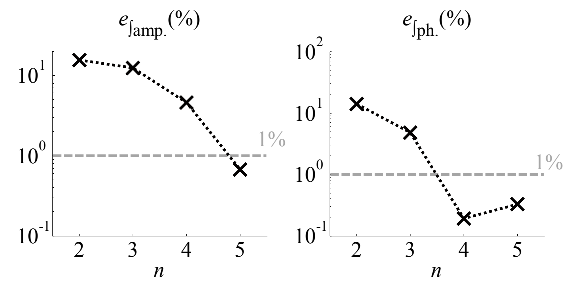

Fig.7. Third harmonic integral error calculation results for different n

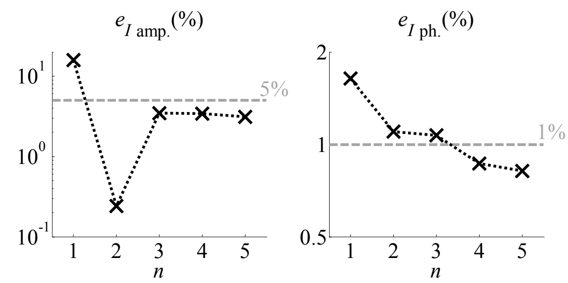

Fig.8. Total current error calculation results for different n

Further on, calculations have been performed for both variants of the total current error (49) and (50). The results for n=1÷5 are presented on figure 8.

Mostly, the chosen error criteria gain decreased values when more terms of (12) are taken into account. Some exceptions exist like the case for n=2, where the amplitude total current error is the smallest. However, the integral errors are not as satisfying for this specific case, which means that in order to complete a more reliable verification, more criteria need to be taken into account. The method's accuracy can be further improved with bigger $h_{\mathrm{interm.}}$ and $h_{i\max}$ values. However, assuming that $h_{i\max}$ are all the same, the maximum value of $h_{\mathrm{interm.}}$ is $5\,h_{i\max}$.

Conclusions

Boundary value electromagnetic field problems involving a nonlinear conducting region were solved. The obtained solution features non-sinusoidal periodic fields, which were presented in the form of Fourier series.

The applied method bases on the method of small parameter. It also consists of essential numerical procedures, which lead to a partially symbolic solution. This solution keeps only the essential parameters, required to evaluate the distribution, in symbolic form. This part of the method was written in C++. The numerical part was done with the use of Matlab software, where a least squares nonlinear estimation was performed.

Electromagnetic field distributions for a boundary condition of the second kind were evaluated. The methodology is very similar for a Dirichlet boundary condition, which was also presented.

The chosen error criteria were decreasing when more terms (presented as the value n) had been included in the symbolic solution. The error values were that of both amplitude and phase of the solution harmonics. This work presented an integral error and a total current error. All values of the integral errors (including those of the third harmonic) fell beneath 1% for $n = 5$. For an increasing n, the amplitude total current error has taken values beneath 5%, while the phase error has decreased to values beneath 1%.

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