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Analysis of impact of nonlinear loads on losses in power network elements

***Abstract**. The problems of exploitation of nonlinear receivers in electrical power networks are discussed in this paper. Their common applicability results in distortion of voltages and currents from sinusoid, which in turn causes a number of unfavourable phenomena. The presented analysis is aimed at determining losses brought about by nonlinear receivers in lines and transformers and in a chosen low-voltage distribution network.*

***Streszczenie.**. Artykuł dotyczy problemów eksploatacji odbiorników nieliniowych w sieciach elektroenergetycznych. Ich coraz powszechniejsze stosowanie skutkuje odkształceniem od sinusoidy przebiegów napięć i prądów, co jest przyczyną wielu niekorzystnych zjawisk. Przedstawiona została analiza, której celem było określenie wielkości strat powodowanych przez odbiorniki nieliniowe w liniach i transformatorach oraz w wybranej sieci rozdzielczej niskiego napięcia. (**Analiza wpływu odbiorników nieliniowych na straty w elementach sieci**).*

Keywords: power network, nonlinear loads, power losses.
Słowa kluczowe: sieci elektroenergetyczne, odbiorniki nieliniowe, straty mocy.

Introduction

Common applicability of nonlinear receivers in electrical power networks results in distortion of voltages and currents from a sine wave. This causes numerous unfavourable phenomena, e.g. increased power and energy losses in particular elements of the network, mainly in lines and transformers.

Losses in lines are caused by additional heating up of conductors – a consequence of presence of higher harmonics in current. Moreover, increased frequencies result in the growth of resistance of conductors. The loading of neutral conductor with "triple harmonics" in low-voltage lines is also important for the distortion of current.

Additional load losses in transformers caused by distorted courses of current are mainly losses resulting from eddy currents in conductor windings, formed as a consequence of the dissipation flux, and also in other construction elements of the transformer.

Those issues are important not only economically but also technically; additional losses caused by distortion of currents directly affect the process of selecting (or limiting) the power of transformers and the conductor's cross-section. The presented analysis is aimed at defining losses caused by nonlinear receivers in lines and transformers as well as in a model low voltage network.

Power losses in lines

The flow of current in a four-conductor low-voltage line results in power losses ΔP_L, being a sum of losses in phase conductors ΔP_P and losses in neutral conductor ΔP_N:

$$\Delta P_L = \Delta P_P + \Delta P_N \tag{1}$$

$$\Delta P_P = \left(I_{L1}^2 + I_{L2}^2 + I_{L3}^2\right) R_P \tag{2}$$

$$\Delta P_N = I_N^2 R_N \tag{3}$$

where: I_{L1}, I_{L2}, I_{L3} – rms value of currents in phase conductors, R_P – resistance of phase conductor, I_N – rms value of current in neutral conductor, R_N – resistance of neutral conductor.

Dependences (1) to (3) hold true also for distorted current. Then rms values of currents are defined as follows:

$$I_P = \sqrt{\sum_{n=1}^{\infty} I_{Pn}^2} \tag{4}$$

$$I_N = \sqrt{\sum_{n=1}^{\infty} I_{Nn}^2} \tag{5}$$

where: I_{Pn} – rms value of *n*-th harmonic of current in phase conductors, I_{Nn} – rms value of *n*-th harmonic of current in neutral conductor.

Analyzing the influence of distorted currents on power losses one should also account for the increase of resistance of conductors caused by the skin effect and the conductors' proximity. In the case of cables equipped with metal shields or armour, attention should be also paid to the influence of those construction elements. Generally, the increase of resistance R_{AC} in low-voltage lines at AC vs. resistance R_{DC} of conductor at DC, may be expressed on the basis of [1, 2, 3] with the following formula:

$$\frac{R_{AC}(f)}{R_{DC}} = 1 + \xi_s(f) + \xi_p(f) \tag{6}$$

where: ξ_s – coefficient determining increase of resistance caused by skin effect, ξ_p – coefficient determining increase of resistance caused by conductors proximity, f – current frequency.

Coefficient ξ_s is determined by the dependence:

$$\xi_s(f) = \mathrm{Re}\left[\frac{\hat{X}}{2} \cdot \frac{\mathrm{J}_0(\hat{X})}{\mathrm{J}_1(\hat{X})}\right] - 1 \tag{7}$$

$$\hat{X} = \hat{X}(f) = \sqrt{\frac{2 f \mu k_s}{R_{DC}}} \exp\left(\mathrm{j}\frac{3\pi}{4}\right) \tag{8}$$

where: J_0, J_1 – Bessel functions of the first kind of zero and one order, respectively, μ – magnetic permeability of conductor material, R_{DC} – unit resistance of conductor, k_s – coefficient depending on conductors' design.

Coefficient ξ_p is determined by the dependence:

$$\xi_p(f) = F(\hat{Y})\left(\frac{2r}{a}\right)^2 \left[\frac{1{,}18}{F(\hat{Y}) + 0{,}27} + 0{,}312\left(\frac{2r}{a}\right)^2\right] \tag{9}$$

$$\hat{Y}=\hat{Y}(f)=\sqrt{\frac{2f\mu k_p}{R_{DC}}}\exp\left(\mathrm{j}\frac{3\pi}{4}\right) \tag{10}$$

$$F\left(\hat{Y}\right)=\mathrm{Re}\left[\frac{\hat{Y}}{2}\cdot\frac{\mathrm{J}_0\left(\hat{Y}\right)}{\mathrm{J}_1\left(\hat{Y}\right)}\right]-1 \tag{11}$$

where: r – radius of conductor, a – distance between axes of conductors, k_p – coefficient depending on conductors' design.

Power losses in transformers

Power losses ΔP_T in transformers [4, 5, 6] are a sum of no-load losses ΔP_{nl} and load losses ΔP_{ll}.

$$\Delta P_T=\Delta P_{nl}+\Delta P_{ll} \tag{12}$$

No-load losses do not depend on the load of the transformer. Load losses can be viewed as a sum of fundamental losses ΔP_f and additional losses ΔP_{ad}:

$$\Delta P_{ll}=\Delta P_f+\Delta P_{ad} \tag{13}$$

Fundamental losses are related with emission of heat in transformer windings in load conditions. Those losses are proportional to the rms value of current I_{RMS}. If the distortion of load current contributes to the increase of rms value, then the basic losses grow as well:

$$\Delta P_{fH}=\left(1+THD_I^2\right)\Delta P_f \tag{14}$$

where: P_{fH} – losses caused by flow of distorted current, ΔP_f – losses caused by flow of the first harmonic of current, THD_I – total harmonic distortion of current:

$$THD_I=\frac{\sqrt{\sum_{n=2}^{\infty}I_n^2}}{I_1} \tag{15}$$

where: I_1 – rms value of fundamental harmonic of current, I_n – rms value of *n*-th harmonic of current, n – order of harmonic.

Additional losses are mainly caused by eddy currents in winding conductors, generated by the dissipation flux, and also in other construction elements of the transformer within the magnetic field of the dissipation flux, e.g. transformer tank. The distortion of load current contributes to increasing additional load losses in the transformer. Those increased load losses can be determined from the dependence:

$$\Delta P_{adH}=K\,\Delta P_{ad} \tag{16}$$

$$K=\sum_{n=1}^{\infty}\left[\left(\frac{I_n}{I_{RMS}}\right)^2 n^2\right] \tag{17}$$

where: I_{RMS} – rms value of distorted current.

Basing on (13)÷(17) the load losses ΔP_{llH} at distorted current can be expressed as:

$$\Delta P_{llH}=\left(1+THD_I^2\right)\Delta P_f+K\,\Delta P_{ad} \tag{18}$$

The coefficient of load loss increase K_H can be described as:

$$K_H=\frac{\Delta P_{llH}}{\Delta P_{ll}}=\frac{\left(1+THD_I^2\right)\Delta P_f+K\,\Delta P_{ad}}{\Delta P_f+\Delta P_{ad}} \tag{19}$$

Assuming that the additional to fundamental losses ratio β is:

$$\beta=\frac{\Delta P_{ad}}{\Delta P_f} \tag{20}$$

then the coefficient K_H expressed as (19) can assume the form:

$$K_H=\frac{1+THD_I^2+K\beta}{1+\beta} \tag{21}$$

Results of simulation calculations

A fragment of low-voltage network was analyzed (fig. 1). The system consists of a transformer 15/0.4 kV with rated power 100 kVA and overhead power line 600 m long.

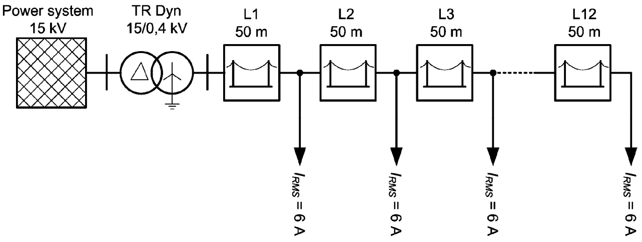

Fig. 1. Scheme of analyzed low-voltage network

The analyzed line feeds receivers distributed every 50 meters; they have the same power and the same character. The rms value of current for each receiver is 6 A. The load of the transformer feeding the line is about 50% of rated power. It was assumed that insulated conductors of AsXSn 4×50 mm^2 type (fig. 2a) were used.

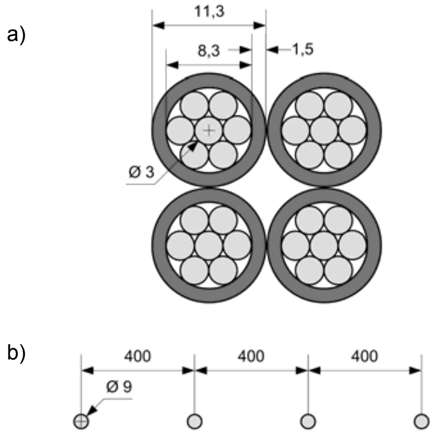

Fig. 2. Low-voltage line: a) with insulated conductors of AsXSn 4×50 mm^2 type, b) with bare conductors of AL 50 mm^2 type

Fig. 3 illustrates the change of resistance of line with insulated conductors evoked by the skin effect and proximity for various frequencies of current. The following values were assumed for calculations k_s = 0.5 and k_p = 0.6. In fig. 5 the resistance for insulated and bare conductors was compared.

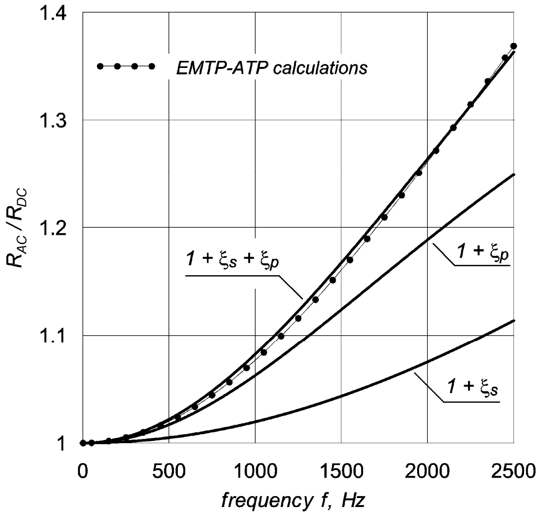

Fig. 3. Resistance of insulated conductor of AsXSn type vs. frequency of current

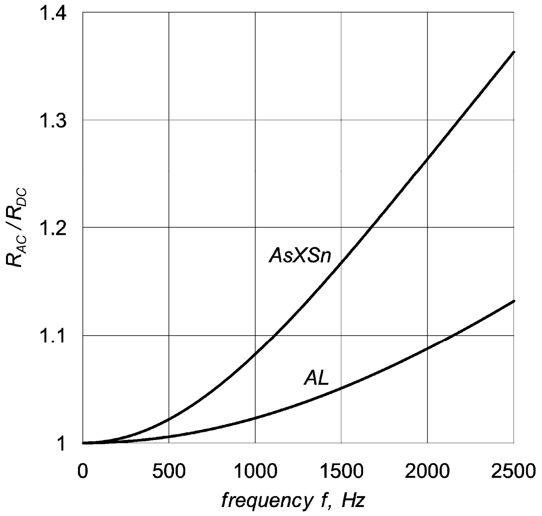

Fig. 4. Resistance of insulated conductor of AsXSn type and bare conductor of AL. type vs. frequency of current

The active power losses in a line at nonsinusoid current were determined with a simulation program *ElectroMagnetic Transients Program EMTP-ATP*. A model of a power line with feedbacks between conductors and the variability of line parameters as a function of frequency has been employed.

The calculations were performed for nine variants differing in the shape of the load current, with the rms value maintained at I_{RMS} = 6 A. Specific variants differed in the content of the constituent harmonics. The shapes of load currents and their spectrums in specific variants have been presented in fig. 5. Calculations for variants *V-1* ÷ *V-7* were aimed at showing the influence of the harmonic's order on the increase of power losses in elements of power networks. Variants *V-8* and *V-9* correspond to loads of mixed type and of various level of current distortion (connection of linear and nonlinear receivers).

Fig. 6 illustrates results of calculations of the total value of power losses in a line, and the power losses in neutral conductors. The obtained values can be directly compared on the basis of assumed constant rms value of load current. The increasing of the order of the harmonic in the current, with its percent share maintained (*V-1* ÷ *V-7*), slightly affects the growth of power losses in phase conductors. Special cases are loads with harmonics of order being a multiple of 3; flowing through the neutral conductor they cause significant additional losses (*V-2* and *V-5*). The share of losses in the neutral conductor is considerable, i.e. about 30% of losses in phase conductors.

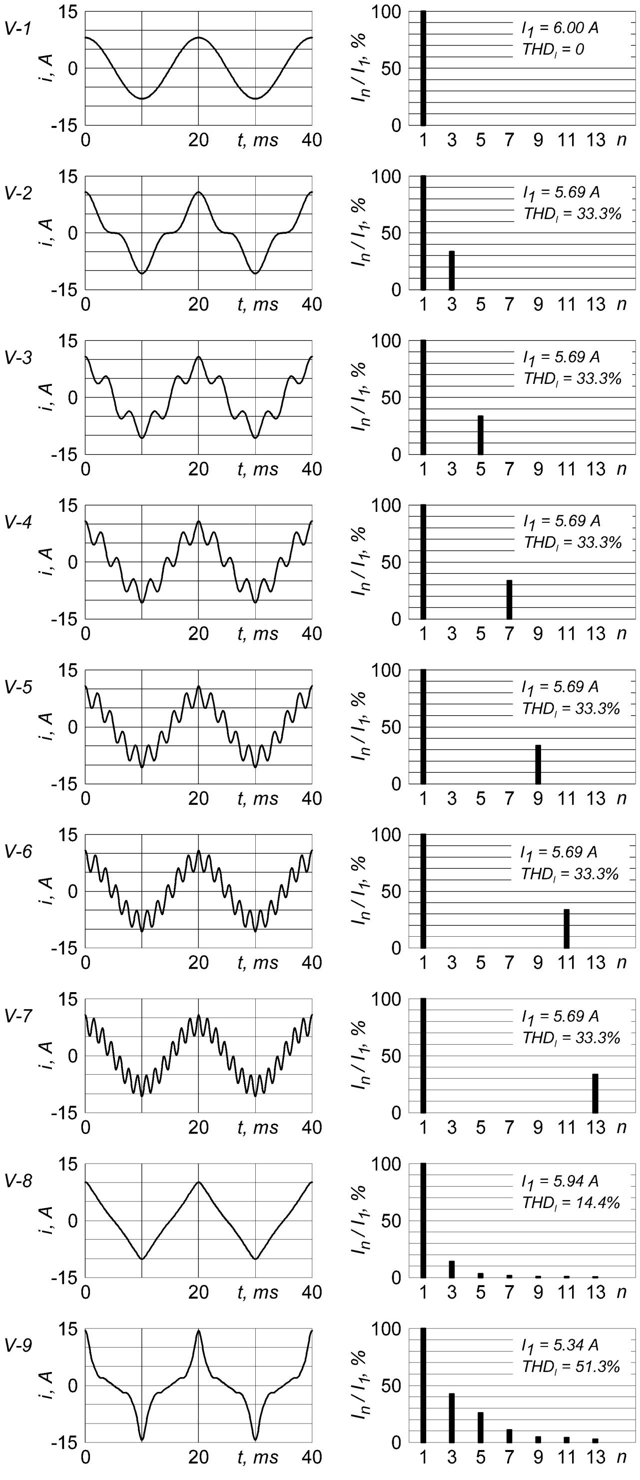

Fig. 5. Load currents and their spectrums for analyzed variants

Variants *V-8* and *V-9* clearly reveal that power losses in the feeding line greatly depend on the share of nonlinear receivers in the total loading of the line. Fig. 7 illustrates power losses in a line, brought about by specific harmonic's currents (*V-8* and *V-9*). The main increase of power losses in the feeding line is generated by the flow of harmonic currents of third order.

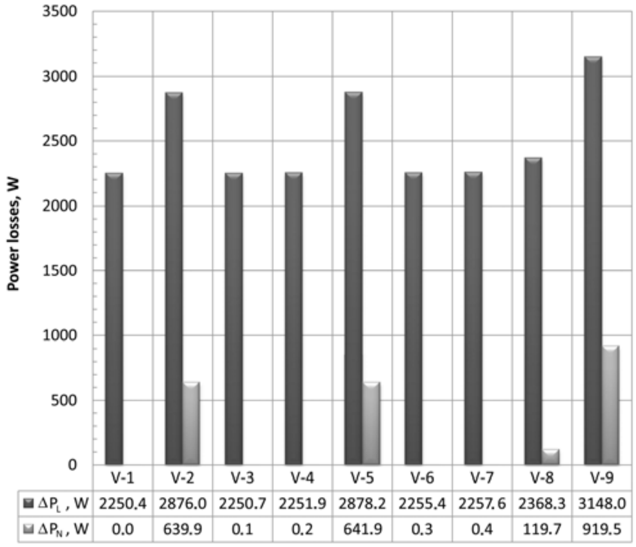

Fig. 6. Total power losses in low-voltage lines and power losses in neutral conductors for variants *V-1 ÷ V-9*

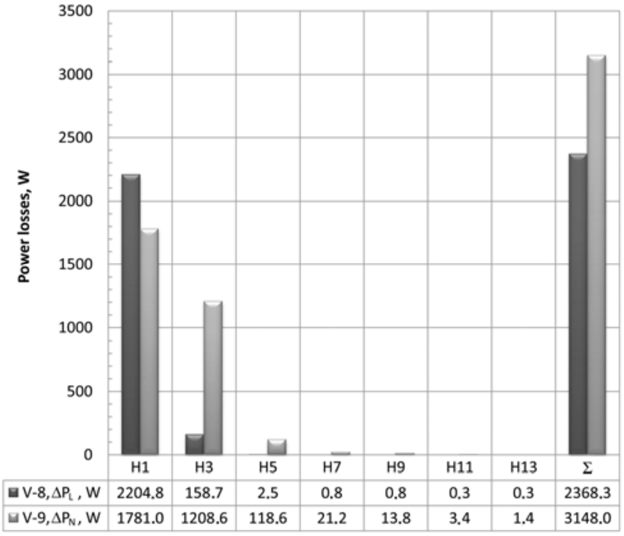

Fig. 7. Total power losses in a line caused by specific harmonic currents for variants *V-8* and *V-9*

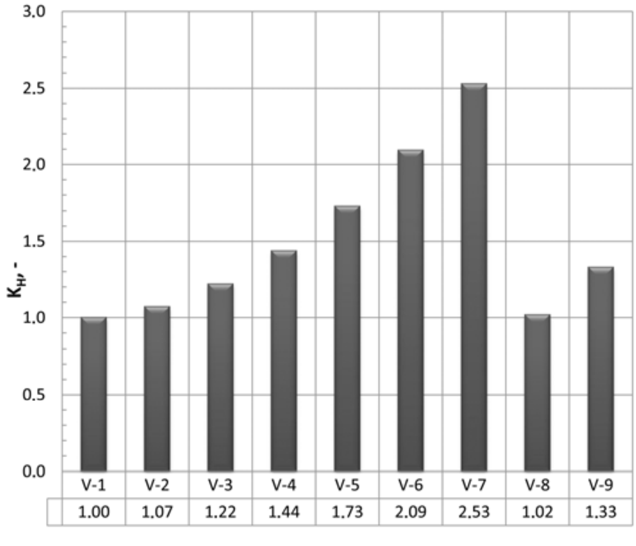

Fig. 8. Coefficient of load loss increase in transformer 15/0.4 kV for analyzed variants of load

The increase of load losses in a transformer 15/0.4 kV were determined without accounting for the increase of fundamental losses (14), which results from assuming the same rms value for all analyzed variants. The results of calculations of the coefficient of load loss increase presented in fig. 8 were obtained for the assumed ratio β = 10%.

One can observe a strong dependence of load losses in a transformer on the order of load current harmonic (*V-1 ÷ V-7*). In variant *V-7* the losses in a transformer increase by above 2.5 times. In real operation conditions the load current distortion can significantly limit the load capacity of the power transformer.

Conclusions

Attention should be paid to the fact that various factors influence the magnitude of change of power loss in lines and transformers. As far as lines are concerned, losses caused by harmonics of order of magnitude of 3 are most important. In the transformers, additional losses strongly depend on the degree of current distortion and it is the harmonics of higher orders which can be indicated as the main cause of increase of additional losses.

The discussed issues become more and more important not only because of the economic aspect, but also for the technical reasons. They are closely connected with designing of power networks, especially selection of feeding lines and transformers in view of expected work conditions. The risk of overloading the neutral conductor and also increase of temperature of the transformer above admissible value may turn up when phenomena discussed in this paper have not been accounted. In both cases the reliability of power supply will be deteriorated and the lifetime of equipment shortened.

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