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Controlling Chaotic Systems Using Aggregated Linear Quadratic Regulator

***Abstract**. A systematic design method for controlling chaotic systems is presented in this paper. The aggregated multiple local models are adopted to express chaotic systems. The Linear Quadratic Regulator (LQR) theory is proposed to design state feedback control system for each local model. Multi-model control strategy has come into being by combining of T-S fuzzy model and LQR. The global stability of closed loop control system can guarantee and it is illustrated with several chaotic systems as examples.*

***Streszczenie.** Zaproponowano projekt sterowania systemem chaotycznym. Zaadaptowano wypadkowe (aggregated) połączenie wielu lokalnych modeli do opisu system chaotycznego. Każdy lokalny model jest sterowany z wykorzystaniem teorii LQR – linear quadratic regulator. (**Sterowanie systemem chaotycznym z wykorzystaniem wypadkowego regulatora LQR**)*

Keywords: Chaotic System, Aggregated Control, T-S Fuzzy Model, Linear Quadratic Regulator.
Słowa kluczowe: systemy chaotyczne, LQR – linear quadratic regulator

Introduction

During recent years, controlling of chaotic systems have absorbed many researchers because of the potential applications in some specific fields. And lots of control methods have been presented. For instance, OGY method [1], differential geometric method [2], feedback and non-feedback control [3-6], inverse optimal control [7], adaptive control [8,9] ,back-stepping design technique [10], variable structure control [11], low pass filter design [12] , H_∞ design [13], PID design [14-16] and fuzzy design [17-19], Decentralized control method [20], Multi-Model method [21], optimal control design [22], Invariant-Manifold Method [28] and LaSalle's invariance principle [30].

Much work has shown that both fuzzy modeling method and multi-model based modeling method are very suitable for modeling complex nonlinear systems such as chaotic systems [23,24]. And multi-model controller design technique is very effective for complex nonlinear systems [25], e.g. the decentralized control is adopted to stabilize chaotic networks where each subsystem is controlled by a local feedback which uses the subsystem's states [20]. The self-organizing map is employed to quantize the operating regions of chaotic system such that a local linear model is built for each region in which a sliding mode controller is designed based on each model [21]. The use of multiple radial basis function neural net model approach to reconstruct piecewise chaos dynamics [25]. In this letter, we use T-S fuzzy model[23] to express chaotic dynamics. As regards the controller design, we use Linear Quadratic Regulator(LQR) [26], which is a very traditional but applicable control method, to design state feedback system.

The rest of the letter is organized as follows: In section 2, Linear quadratic regulator with and without expected decay rate for time invariant system are presented; T-S fuzzy model expression for complex system is introduced in section 3, and it is followed by Parallel Distributed Compensation (PDC) technique that is effective for controller design; Section 4 presents stability analysis of the closed-loop control system; And in section 5, five chaotic dynamics are used as examples to show control effects of the method proposed; Some conclusions are drawn in section 6.

Linear quadratic optimal control

Linear Quadratic Regulator of Time Invariant System

Considering the following LTI continuous system.

(1) $$\dot{x} = Ax + Bu, x(0) = x_0$$

where, A, B are constant matrices with the dimension of $n\times n$ and $n\times p$ respectively, and (A, B) is controllable.

The performance index to be considered here is

(2) $$J(u) = \frac{1}{2}\int_0^\infty [x^T Q x + u^T R u]\mathrm{d}t$$

where, R is a constant symmetric positive definite matrix, Q is a constant symmetric positive or semi-positive definite matrix.

The optimal control of (1) and (2) is

(3) $$u^*(t) = -Kx(t)$$

in which, $K=R^{-1}B^TP$, and P is the unique symmetric positive definite matrix solution of the equivalence (4)

(4) $$A^T P + PA - PBR^{-1}B^T P + Q = 0$$

Under the controlling of formula (3), the controlled system (1) takes the strong stability and robustness, which can be expressed in Theorem 1 and Theorem 2.

Theorem 1(Stability) [29]

The closed-loop controlled system of (1) and (3) is

(5) $$\dot{x} = (A - BK)x, K = R^{-1}B^T P, t \ge 0$$

Closed-loop system (5) is asymptotically stable in the large, i.e. its eigenvalues satisfy

(6) $$\mathrm{R_e}\left[\lambda_i(A - BK)\right] < 0, i = 1, 2, \cdots, n$$

in which, $\lambda_i(\cdot)$ denotes ith eigenvalue of a matrix , $R_e(\cdot)$ denotes the real part of a complex.

Theorem 2(Robustness) [29]

If we let weighting matrixes satisfy the following condition for multi-input LQR problems described in (1), (2), (3) and (4),

$R=diag\{r_1,r_2,...,r_p\}$, $r_i>0$

then each feedback controlled loop has the performances as follows:

I. At least ±60 degrees' phase margin;

II. Gain margin is from 1/2 to ∞.

Remark 1. Theorem 2 aims at multi-input systems directly, i.e. the case of $P>1$ is directly considered. But, it's obviously that the robustness conclusion also satisfies single input system.

Remark 2. In Theorem 2, it is easy to satisfy the condition that R should be diagonal positive definite. And R will be simply set to be a unit matrix in this letter.

Linear Quadratic Regulator with Expected Decay Rate

In regard to formula (6), sometimes we are not satisfied with it. That is to say that sometimes we need a faster convergence speed of $x(t)$, which is expressed as decay rate α in the following improved performance index of LQR.

(7) $$J_\alpha(u)=\frac{1}{2}\int_0^\infty e^{2\alpha t}[x^{\mathrm{T}}Qx+u^{\mathrm{T}}Ru]\mathrm{d}t$$

The optimal control of (1) and (7) is

(8) $$u_\alpha^*(t)=-K_\alpha x(t)$$

in which, $K_\alpha=R^{-1}B^{\mathrm{T}}P_\alpha$, and P_α is the unique symmetric positive definite matrix solution of the equivalence (9)

(9) $$(A+\alpha I)^{\mathrm{T}}P_\alpha+P_\alpha(A+\alpha I)-P_\alpha BR^{-1}B^{\mathrm{T}}P_\alpha+Q=0$$

Accordingly, the closed-loop system is

(10) $$\dot{x}=(A-BK_\alpha)x, K_\alpha=R^{-1}B^{\mathrm{T}}P_\alpha, t\ge 0$$

and we have

(11) $$\mathrm{R_e}\left[\lambda_i(A-BK_\alpha)\right]<-\alpha, i=1,2,\cdots,n$$

It's clear that $-\alpha$ is the upper decay limit of $x(t)$, and decay rate is faster with increasing of α, so the regulating performance and robust performance will be improved.

Multiple T-S fuzzy models and design of controllers

The continuous time T-S fuzzy system

The T-S fuzzy system is described by a set of fuzzy IF-THEN rules which have linear models in the consequent parts. A single input T-S fuzzy system used in this paper is of the following form:

Rule i: IF $z_1(t)$ is M_{i1} ⋯ and $z_n(t)$ is M_{in}

THEN

(12) $$\dot{x}(t)=A_i x(t)+B_i u(t)$$

where $x(t)\in R_n$, $u(t)\in R$, $A_i\in R^{n*n}$, $B_i\in R_n$, i=1,2, ⋯,r, in which r is the number of IF-THEN rules, M_{ij} are fuzzy sets, $z_1(t)$, $z_2(t)$,⋯, $z_n(t)$ are premise variables that may be functions of the state variables, $\dot{x}(t)=A_i x(t)+B_i u(t)$ is the ith subsystem of the fuzzy system, and (A_i, B_i) is assumed to be controllable, i=1,2, ⋯,r.

Now, given a pair of ($x(t)$, $u(t)$), the final output of the fuzzy system is inferred by

(13) $$\dot{x}(t)=\frac{\sum_{i=1}^{r}\omega_i\left(z(t)\right)\left\{A_i x(t)+B_i u(t)\right\}}{\sum_{i=1}^{r}\omega_i\left(z(t)\right)}=\sum_{i=1}^{r}\mu_i\left(z(t)\right)\left\{A_i x(t)+B_i u(t)\right\}$$

where

$$\omega_i\left(z(t)\right)=\prod_{j=1}^{n}M_{ij}\left(z_j(t)\right),$$

$$\omega_i\left(z(t)\right)\ge 0, i=1,2,\cdots,r,$$

$$\mu_i\left(z(t)\right)=\frac{\omega_i\left(z(t)\right)}{\sum_{i=1}^{r}\omega_i\left(z(t)\right)}, \sum_{i=1}^{r}\mu_i\left(z(t)\right)=1,$$

$$\mu_i\left(z(t)\right)\ge 0, i=1,2,\cdots,r.$$

Designing controller using parallel distributed compensation (PDC) technique

A PDC controller is composed of the same set rules of T-S model for the system, and shares the same fuzzy sets with the same premise parts. The PDC controller for the fuzzy T-S system (13) will have the form

Rule i: IF $z_1(t)$ is M_{i1} ⋯ and $z_n(t)$ is M_{in}

THEN $u(t)=-K_i x(t)$

where K_i are the controller gain matrices to be designed. In this paper, the aforementioned LQR is used to design K_i. The overall fuzzy controller is

(14) $$u(t)=-\sum_{i=1}^{r}\mu_i\left(z(t)\right)K_i x(t)$$

Stability analysis of the closed-loop control system

We need design r controller gain matrices K_i according to LQR theory which is described in formula (2)-(4) or (7)-(9). Though each K_i can guarantee the stability and robustness of each subsystem as expressed in theorem 1 and theorem 2, can we say that they can guarantee the global stability of the whole system? Now let's analyze this problem. By substituting (14) into (13), one obtains the overall closed-loop controlled system as

(15) $$\dot{x}(t)=\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\left(z(t)\right)\mu_j\left(z(t)\right)\left[A_i-B_iK_j\right]x(t)$$

Define $G_{ij}=A_i-B_iK_j$ and rewrite the closed-loop system as

(16) $$\dot{x}(t)=\sum_{i=1}^{r}\mu_i\left(z(t)\right)\mu_i\left(z(t)\right)G_{ii}x(t)+\sum_{i=1}^{r}\sum_{i<j}^{r}\mu_i\left(z(t)\right)\mu_j\left(z(t)\right)\left[\frac{G_{ij}+G_{ji}}{2}\right]x(t)$$

where the second term on the right is for all pairs (i, j) such that the product $\mu_i\left(z(t)\right)\mu_j\left(z(t)\right)\ne 0$.

Applying the Lyapunov approach to the closed-loop system (16) yields the following stability conditions[27].

Theorem 3 The zero equilibrium of the continuous time fuzzy controlled system (16) is globally asymptotically stable in the large if there exists a common positive definite matrix P such that

(17) $$G_{ii}^T P+PG_{ii}<0, i=1,2,\cdots,r.$$

and

(18) $$\left(\frac{G_{ij}+G_{ji}}{2}\right)^T P+P\left(\frac{G_{ij}+G_{ji}}{2}\right)\le 0$$

for $i<j$, except the pairs (i,j) such that $\mu_i\left(z(t)\right)\mu_j\left(z(t)\right)=0$.

So, according to Theorem 3, we can give the control design steps for chaotic systems as follows:

Step 1, do T-S fuzzy model partition according to (12);

Step 2, design feedback gain matrix for each subsystem of (12) according to LQR theory (2-4) or (7-9);

Step 3, check stability conditions in Theorem 3. If conditions are satisfied, end; otherwise, return to step 2, rechoose matrixes Q, R and/or α till stability conditions in Theorem 3 are satisfied.

Illustrative examples

The T-S fuzzy system is described by a set of fuzzy IF-THEN rules which have linear models in the consequent parts. A single input T-S fuzzy system used in this paper is of the following form with rule number N.

(19) Rule i: If x_1 is M_i, then $\dot{x}=A_i x+Bu$ (i=1,2,…, N).

In this section, five chaotic dynamics are used as examples to show control effects of the method proposed in this paper. The five chaotic dynamics are Chen's chaotic system, Chua chaotic system, Lorenz chaotic system, Rössler chaotic system, and Duffing chaotic system. The LQR-based feedback gain for the former four chaotic systems are simply based on $Q=diag(1,1,1)$ and R=1 according to formula (2). But for the fifth chaotic system, i.e. Duffing chaotic system, a stable feedback gain cannot be obtained simply according to formula (2) with $Q=diag(1,1,1)$ and R=1.

We solved it according to formula (9) with $Q=diag(1,1,1)$, $R=1$, and decay rate $\alpha=1$.

Example 1 Chen's chaotic system

Chen's chaotic system is as follows:

$$\begin{cases} \dot{x}_1 = a(x_2 - x_1) + u \\ \dot{x}_2 = (c-a)x_1 - x_1x_3 + cx_2 \\ \dot{x}_3 = x_1x_2 - bx_3 \end{cases}$$

The system is of chaotic status with $a=35$, $b=3$, $c=28$ and $u=0$. Corresponding to (19), rule number N is two, and

$$A_1 = \begin{bmatrix} -a & a & 0 \\ c-a & c & -d \\ 0 & d & -b \end{bmatrix}, A_2 = \begin{bmatrix} -a & a & 0 \\ c-a & c & d \\ 0 & -d & -b \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix};$$

$$M_1(x_1) = \frac{1}{2}\left(1 + \frac{x_1}{d}\right), M_2(x_1) = \frac{1}{2}\left(1 - \frac{x_1}{d}\right), d=30.$$

According to LQR theory, i.e. formula (4), we can obtain the feedback gain as follows: $k_1=[44.6, -365.4, 210]$, $k_2=[44.6, -365.4, -210]$. The common positive definite matrix P exists in formula (17-18). Initial value is (-18,3,4) in the simulation and multi-model LQR controller acts after $t=10$s. The control results are shown in Fig.1.

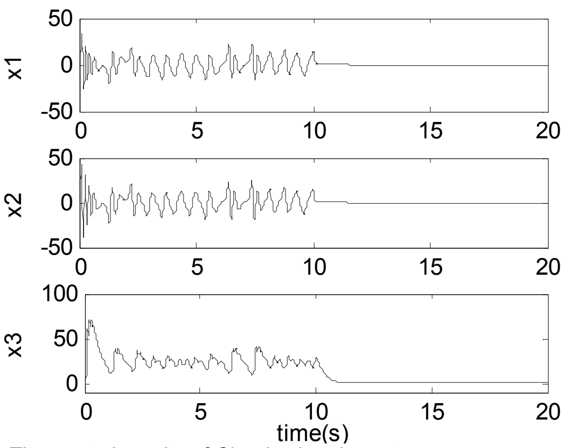

Fig.1 The control results of Chen's chaotic system

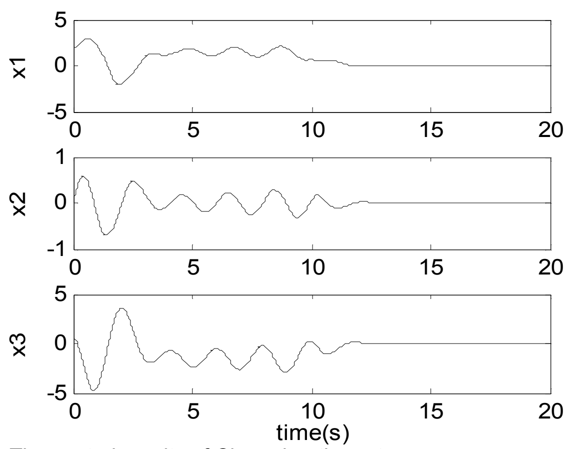

Fig.2 The control results of Chua chaotic system

Example 2 Chua chaotic system

Chua chaotic system is as follows:

$$\begin{cases} \dot{x}_1 = ax_1 + bx_2 + c\left(\left|x_1+1\right| - \left|x_1-1\right|\right) + u \\ \dot{x}_2 = dx_1 + dx_2 + ex_3 \\ \dot{x}_3 = fx_2 \end{cases}$$

The system is of chaotic status with $a=-2.5167$, $b=8.7997$, $c=1.8875$, $d=1$, $e=1$, $f=-16$ and $u=0$. Corresponding to (19), rule number N is three, and

$$A_1 = \begin{bmatrix} a & b & 0 \\ d & -d & e \\ 0 & f & 0 \end{bmatrix}, A_2 = \begin{bmatrix} a+2c & b & 0 \\ d & -d & e \\ 0 & f & 0 \end{bmatrix}, A_3 = A_1, B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix};$$

$$M_1(x_1) = \frac{x_1}{2}, M_2(x_1) = 1 - \frac{|x_1|}{2}, M_3(x_1) = -\frac{x_1}{2}.$$

According to LQR theory, i.e. formula (4), we can calculate the feedback gain: $k_1=[1.5115, 4.4465, 1.1414]$, $k_2=[5.0493, 5.8942, 1.8981]$, $k_3=k_1$. The common positive definite matrix P exists in formula (17-18). Initial value is (2,0,0.5) in the simulation and multi-model LQR controller acts after $t=10$s. The control results are shown in Fig.2.

Example 3 Lorenz chaotic system

Lorenz chaotic system is as follows:

$$\begin{cases} \dot{x}_1 = a(x_2 - x_1) + u \\ \dot{x}_2 = cx_1 - x_1x_3 - x_2 \\ \dot{x}_3 = x_1x_2 - bx_3 \end{cases}$$

The system is of chaotic status with $a=10$, $b=8/3$, $c=28$ and $u=0$. Corresponding to (19), rule number N is three, and

$$A_1 = \begin{bmatrix} -a & a & 0 \\ c & -1 & d \\ 0 & d & -b \end{bmatrix}, A_2 = \begin{bmatrix} -a & a & 0 \\ c & -1 & 0 \\ 0 & 0 & -b \end{bmatrix}, A_3 = \begin{bmatrix} -a & a & 0 \\ c & -1 & -d \\ 0 & -d & -b \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix};$$

$$M_1(x_1) = \frac{x_1}{d}, M_2(x_1) = 1 - \frac{|x_1|}{d}, M_3(x_1) = -\frac{x_1}{d}, d=10.$$

According to LQR theory, i.e. formula (4), we can calculate the feedback gain: $k_1=[15.56, 9.86, -7.85]$, $k_2=[23.72, 18.49, 0]$, $k_3=[15.56, 9.86, 7.85]$. The common positive definite matrix P exists in formula (17-18). Initial value is (0.1, 0.1, –0.1) in the simulation and multi-model LQR controller acts after $t=10$s. The control results are shown in Fig.3.

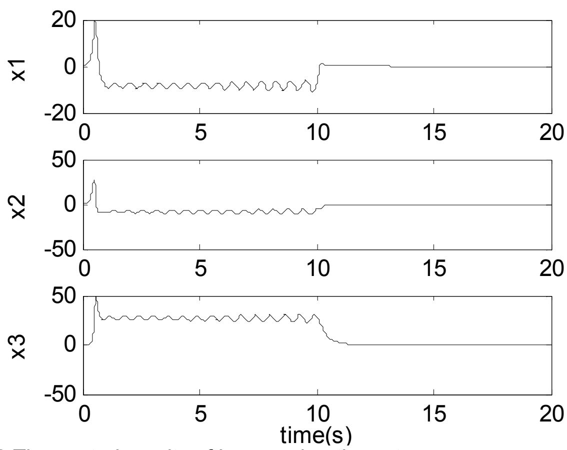

Fig.3 The control results of Lorenz chaotic system

Example 4 Rössler chaotic system

Rössler chaotic system is as follows:

$$\begin{cases} \dot{x}_1 = -x_2 - x_3 \\ \dot{x}_2 = x_1 + ax_2 \\ \dot{x}_3 = bx_1 - (c - x_1)x_3 + u \end{cases}$$

The system is of chaotic status with a=0.34, b=0.4, c=4.5 and u=0. Corresponding to (19), rule number N is three, and

$$A_1 = \begin{bmatrix} 0 & -1 & -1 \\ 1 & a & 0 \\ b & 0 & -d \end{bmatrix}, A_2 = \begin{bmatrix} 0 & -1 & -1 \\ 1 & a & 0 \\ b & 0 & 0 \end{bmatrix}, A_3 = \begin{bmatrix} 0 & -1 & -1 \\ 1 & a & 0 \\ b & 0 & d \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix};$$

$$M_1(x_1) = \frac{4.5 - x_1}{d}, \ M_2(x_1) = 1 - \frac{|4.5 - x_1|}{d}, \ M_3(x_1) = -\frac{4.5 - x_1}{d},$$

d=10.

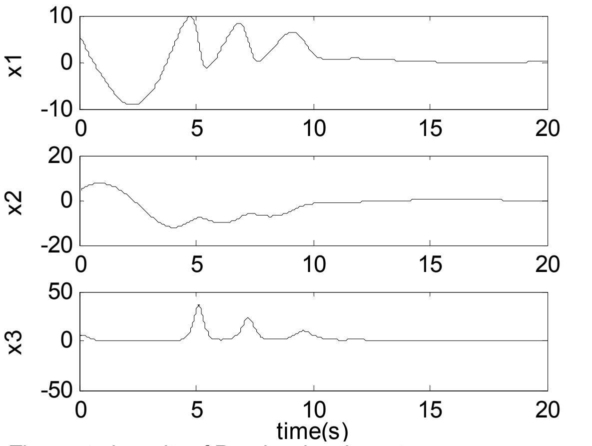

Fig.4 The control results of Rössler chaotic system

According to LQR theory, i.e. formula (4), we can calculate the feedback gain as follows: k_1=[–6.569, –1.657, 0.684], k_2=[–1.853, –0.057, 2.169], k_3=[–7.341, –2.116, 20.755]. The common positive definite matrix P exists in formula (17-18). Initial value is (6,4,4) in the simulation and multi-model LQR controller acts after t=10s. The control results are shown in Fig.4.

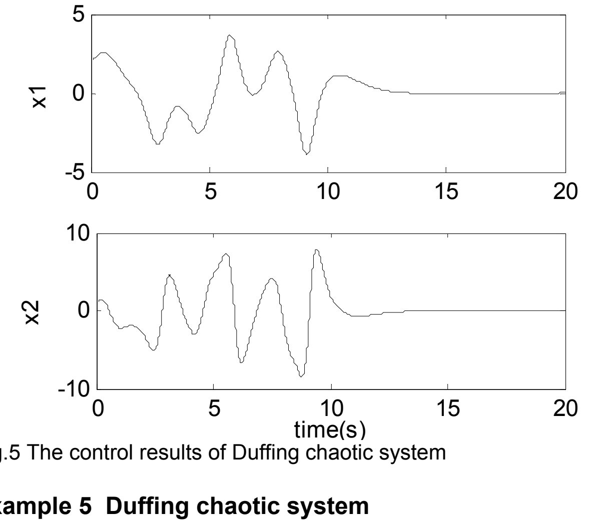

Fig.5 The control results of Duffing chaotic system

Example 5 Duffing chaotic system

Duffing chaotic system is as follows:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -x_1^3 - 0.1x_2 + 12\cos(t) + u \end{cases}$$

The system is of chaotic status with u=0. Corresponding to (19), rule number N is two, and

$$A_1 = \begin{bmatrix} 0 & 1 \\ 0 & -0.1 \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 1 \\ -d^2 & -0.1 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix};$$

$$M_1(x_1) = 1 - \frac{x_1^2}{d^2}, M_2(x_1) = \frac{x_1^2}{d^2}, d=5.$$

According to LQR theory, i.e. formula (4), we can calculate the feedback gain: k_1=[1.0000, 1.6349], k_2=[0.0002, 0.9052].The common positive definite matrix P doesn't exist in formula (17-18). Then according to formula (9), we simply let Q=$diag$(1,1,1), R=1, and decay rate α=1, the following feedback gain can be obtained: k_1=[5.1588, 4.3825], k_2=[4.1036, 4.0650]. And then the common positive definite matrix P exists in formula (17-18). Initial value is (2,1) in the simulation and multi-model LQR controller acts after t=10s. The control results are shown in Fig.5.

Verification of robustness

As to illustration of robustness verification for the proposed method, we only give the results of Chen's chaotic system. Assume that the perturbation of parameters a, b and c in Chen's chaotic system are all 25%, whereas the feedback coefficients of the controllers keep invariable, and initial value of the system is the same as that in **Example 1**, the control results with 25% positive perturbation and 25% negative perturbation are shown in Fig.6 and Fig.7 respectively.

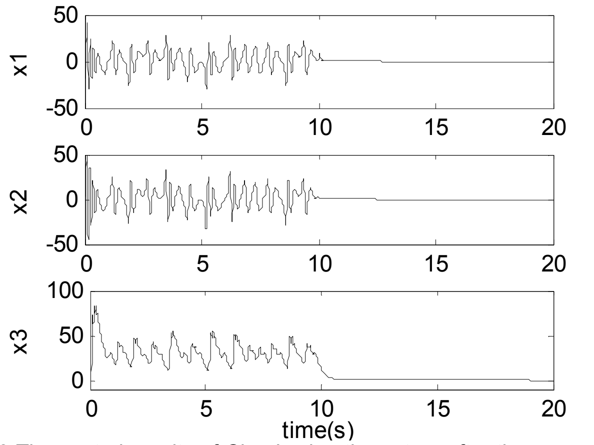

Fig.6 The control results of Chen's chaotic system after the parameters perturbation: Case 1

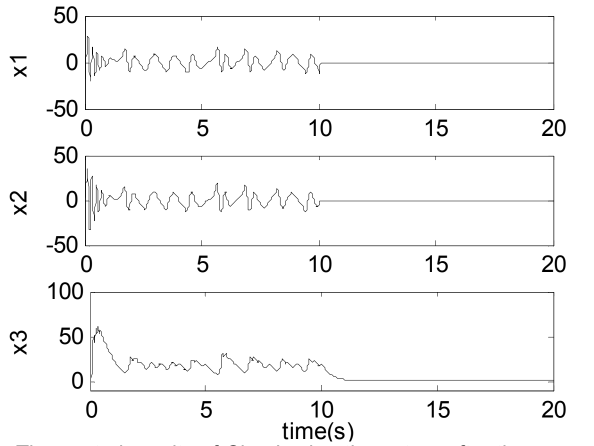

Fig.7 The control results of Chen's chaotic system after the parameters perturbation: Case 2

Conclusions

A multiple T-S fuzzy models and LQR theory based approach for controlling of chaotic systems was presented. The main contribution of this paper is the development of a systematic and effective framework for fuzzy model and LQR based controller design for control of chaotic systems. The validity of the new approach is confirmed through theoretical analysis and numerical simulations.

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