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Using a conformal mapping method for calculation of a multipolar system magnetic field

***Abstract**. A conformal mapping method was used to obtain formulas for calculation of magnetic field strength in the operating areas of multipolar cylindrical systems of magnetic separators. The offered expressions allow calculation of the field at any point of the initial area in an explicit form according to the coordinates of this point.*

***Streszczenie.** Metoda odwzorowań konforemnych została zastosowana do uzyskania wzorów na natężenie pola magnetycznego w obszarach aktywnych wielobiegunowych systemów cylindrycznych w separatorach magnetycznych. Przedstawione wzory pozwalają obliczać pole w każdym punkcie obszaru wyjściowego. (**Zastosowanie metody odwzorowania konforemnego w obliczaniu pola magnetycznego w systemach wielobiegunowych**).*

Keywords: magnetic field, magnetic separation, conformal mapping.
Słowa kluczowe: pole magnetyczne, separacja magnetyczna, odwzorowanie konforemne

Introduction

The practice of magnetic separators design requires a quantitative estimation of magnetic intensity with the purpose of choosing such their parameters that will make it possible to create the necessary intensity value at minimum costs [1-7]. This estimation is made relatively easily for electromagnetic separators whose magnetic field in the operating area can be regarded as plane-parallel, as in this case a conformal mapping method can be used for calculation. In particular, such a type of separators includes devices presenting multipolar cylindrical systems with poles polarity interlacing along the rotation axis (Fig. 1, a) and with poles polarity interlacing along the circle of cylindrical cross-section (Fig. 1, b). This paper contains found analytical expressions allowing one to calculate the magnetic intensity for such magnetic separators types.

Mathematical models for calculation of the field in operating areas of magnetic separator multilopar systems

Application of the conformal mapping method implies that calculated area of a plane-parallel field is limited by a polygon whose sides are equipotential or (and) evident field lines. Magnetic field in operating areas of the considered electromagnetic systems (Fig. 1) is usually assumed to be plane-parallel [8-15], as interpolar air gap δ in both structures is considerably smaller than the external diameter D ($D/\delta \geq 5$), and for structure (Fig. 1, b) it is also considerably smaller (10 – 15 times) than the length of the poles. Besides, some other assumptions, accepted in the engineering practice of electromagnets calculation, are introduced [8-17]: absence of the magnetizing coils geometry influence on the distribution of the field in the operating area; infinitely small thickness of electromagnetic system pole pieces. Due to a relatively big number of interlacing polarity poles, the field within the pole pitch is assumed symmetrical for multipolar structures of magnetic separators.

Taking the accepted assumptions into account, it is possible to pass to consideration of two-dimensional idealized systems (models) of infinitely big external diameter D ($D/\delta \to \infty$) and infinitely thin ($d/\delta \to 0$) pole pieces. These systems poles are under magnetic potential difference $2U_0$, and their polarity interlaces along the rotation axis (Fig.1, a) or along the cylinder cross-section circle (Fig.1, b). It is known from experience [8, 12] that analytical expressions, obtained for such models, make it possible to compare adequately analogous structures and choose their optimum variants.

As operating interpolar zones (Fig. 1) are noted for repetition of characteristic areas, instead of the analysis of field distribution across the whole interpolar zone, it is possible to limit oneself with the research of the field in a characteristic, periodically repeated area. These areas (Figs. 2, 3) are formed taking the following into consideration. The lines between infinitely thin pole pieces are field lines. The lines perpendicular to them and crossing the centers of interpolar gaps δ meet the condition of equipotentiality U=0. The lines crossing the poles centers perpendicular to their surface are field lines under the assumption of poles steel infinite permeability.

a)

b)

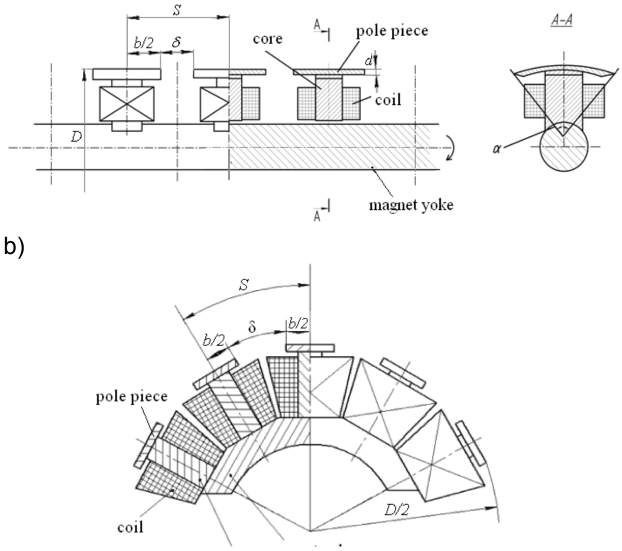
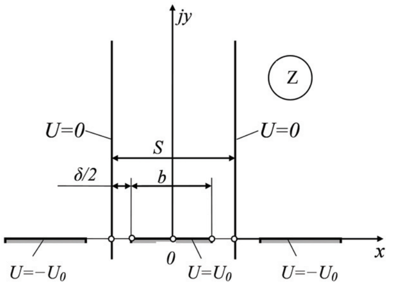


Fig. 1. Multipolar electromagnetic system designs: a) with poles polarity interlacing along the rotation axis; b) with poles polarity interlacing along the circle of cylindrical cross-section (S – interpolar pitch; D – electromagnetic system diameter; δ – operating gap; b – poles width; d – pole pieces thickness; α – sector angle)

Thus, a semi-infinite fringe presents such calculation zone for structure (Fig. 1, a). In this case equipotential plates, replacing electromagnet poles, can be situated in plane Z in two ways in relation to the half-fringe (Fig. 2, a, b): in the first case the vertical boundaries of the areas present equipotential lines (Fig. 2, a); in the second case the vertical boundaries of the area are field lines (Fig. 2, b). It should be taken into consideration that the air gap lines

(Fig. 2) can be regarded as field lines with a sufficient degree of accuracy. It should be mentioned that for the considered areas (Fig. 2) their halves are reflectively symmetrical and present two new variants of the calculation area of the examined separator.

a)



b)

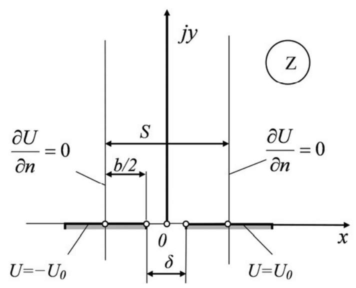
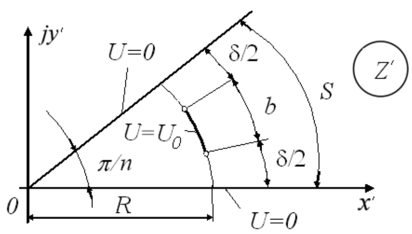


Fig. 2 Mathematical model of the searched area according to Fig. 1, a: a) area vertical boundaries – equipotential lines; b) area vertical boundaries – field lines

The calculation area for structure (Fig. 1, b) is a semi-infinite sector of the form $Z' = z'e^{j\varphi}$, where module z' meets the condition $z' \geq R$, and angle φ is measured from 0 to π/*n*, where *n* is a number of magnetic system ports. In this case two ways of operating area limitation are possible (Fig. 1, b): in the first case the area boundaries present equipotential lines (Fig. 3, a), in the second case – field lines (Fig. 3, b).

a)



b)

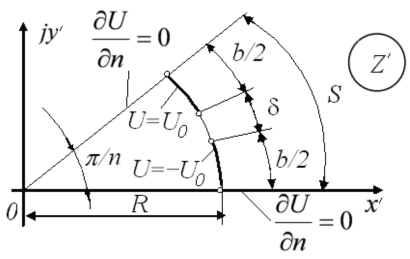


Fig.3 Mathematical model of the searched area according to Fig.1, b: a) area boundaries – equipotential lines;
b) area boundaries – field lines

In their turn, calculation areas (Fig. 3) can be reduced to the form analogous to the one shown in Fig. 2, using transformation

$$Z^* = j\ln\frac{Z'}{R} + \frac{S}{2R}\,, \qquad (1)$$

which presents a known logarithmic transformation with a turn and shift [18]. Here coordinates of the mapped area Z^* are non-dimensional values and connected with dimensional coordinates of *Z* area (Fig. 2) by means of relation

$$Z^* = Z/R. \qquad (2)$$

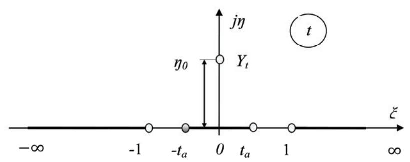
As a result of mapping the operating areas in the form of semi-infinite fringes on planes *Z* (Fig. 2) and Z^* on the upper half-plane $t=\xi+j\eta$ in a standard manner on the basis of the use of Kristoffel-Schwarts integral, it is possible to obtain the following systems of plates-poles relative position [18]:

a symmetrical system of semi-infinite plates-poles with a central pole (Fig. 4, a);

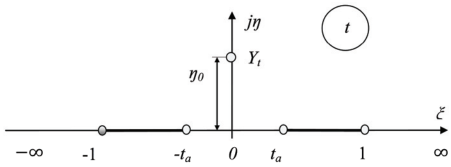
a symmetrical system of two poles-plates (Fig. 4, b);

two systems of a semi-infinite plate-pole and a finite-width plate-pole, different from each other due to the position of the plates-poles in relation to the origin of coordinates (Fig. 4, c, d).

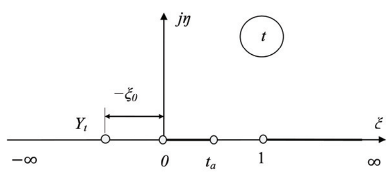
a)



b)



c)



d)

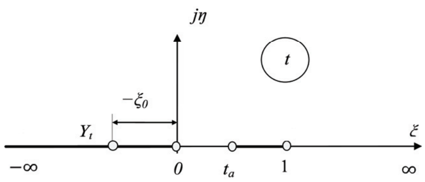


Fig. 4. Conformal mappings on plane t: a) of initial area *Z*, according to Fig. 2, a ($t_a = \cos(\delta\pi/2S)$); b) of initial area *Z*, according to Fig. 2, b ($t_a = \cos(b\pi/2S)$); c) of a half of initial area *Z*, according to Fig.2, a ($t_a = \cos^2(\pi\delta/2S)$); d) of a half of initial area Z, according to Fig. 2, b ($t_a = \cos^2(\pi b/2S)$)

Kristoffel-Schwarts differential relations for the mentioned systems are of the forms:
model (Fig. 4, a, b)

(3) $$\frac{dZ}{dt}=\frac{jSdt}{\pi\sqrt{t^2-1}} \text{ and } \frac{dZ^*}{dt}=\frac{jSdt}{\pi R\sqrt{t^2-1}};$$

model (Fig. 4 c, d)

(4) $$\frac{dZ}{dt}=\frac{jSdt}{2\pi\sqrt{t(t-1)}} \text{ and } \frac{dZ^*}{dt}=\frac{jSdt}{2\pi R\sqrt{t^2-1}}.$$

For the obtained pole systems (Fig. 4) their mapping on the uniform field area *W*=φ+*j*ψ (rectangle interior) is also applied with the use of Kristoffel-Schwarts integral. As a result of application of the said transformations, conformal mapping functions were obtained for all areas. These functions make it possible to determine correspondence of the points on the mapped areas explicitly.

Calculation formulas and their discussion

For the areas shown in Fig. 4, a, b the coordinates $Y_t=0+j\eta_0$ and the initial area *Z* (Fig. 2) coordinates $Y_Z=0+jy_0$ are connected by the following relation

(5) $$Y_t=0+j\eta_0=0{,}5j(e^{\frac{\pi y_0}{S}}-e^{-\frac{\pi y_0}{S}}).$$

For areas, shown in Fig. 4 c, d, relationship between coordinates $Y_t=-\xi_0+j0$ and coordinates $Y_Z=0+jy_0$ of area *Z* (Fig. 2) is of the form, respectively

(6) $$Y_t=-\xi_0+j0=-0{,}5(e^{\frac{\pi y_0}{S}}-e^{-\frac{\pi y_0}{S}})^2.$$

Using expressions (5) and (6) for magnetic field strength H_Z in areas *Z* (Fig. 2), the following expressions have been obtained.

For the analysis of area *Z* shown in Fig. 2, a

(7) $$H_Z=U_0\frac{\pi}{SK(k')}\frac{\cos\frac{\pi\delta}{2S}\sqrt{1-t^2}}{\left(\sqrt{(\cos\frac{\pi\delta}{2S})^2-t^2}\sqrt{(\cos\frac{\pi\delta}{2S})^2-k^2t^2}\right)},$$

where $K(k')$ – complete elliptic integral of the first kind with an additional module $k'=\sqrt{1-k^2}$. Here $k=\cos\pi\delta/2S$ – module of a complete elliptic integral of the first kind.

For the analysis of area *Z* shown in Fig. 2, b

(8) $$H_Z=U_0\frac{\pi}{SK(k_1)}\frac{\cos\frac{\pi b}{2S}\sqrt{1-t^2}}{\left(\sqrt{(\cos\frac{\pi b}{2S})^2-t^2}\sqrt{(\cos\frac{\pi b}{2S})^2-k^2t^2}\right)},$$

where $K(k_1)$ – complete elliptic integral of the first kind with module $k_1=\cos\pi b/2S$.

For half of searched area Z, according to Fig. 2, a

(9) $$H_Z=U_0\frac{\pi}{SK(k_2)}\frac{1}{\sqrt{t-\cos^2(\frac{\pi\delta}{2S})}},$$

where $K(k_2)$ – complete elliptic integral of the first kind with module $k_2=\sin(\pi\delta/2S)$.

For half of searched area *Z*, according to Fig. 2 b

(10) $$H_Z=U_0\frac{\pi}{SK(k_1)}\frac{1}{\sqrt{t-cos^2(\frac{\pi b}{2S})}}.$$

Calculation of magnetic field strength H_Z^* in searched areas Z^* is performed using relation

(11) $$H_Z^*=H_ZR,$$

which follows from the connection of dimensional area *Z* with non-dimensional area Z^*, according to formula (2).

Transition to magnetic field strength $H_{Z'}$ for initial calculation areas z' (Fig. 3) is performed by means of the following transformation chain [18]

(12) $$H_{Z'}=H_Z^*\frac{dZ^*}{dZ'}=H_Z\frac{R}{Z'}.$$

Thus, expressions for magnetic field strength calculation have been obtained for areas *Z* (Fig. 2) and Z' (Fig. 3). The peculiar feature of the mentioned formulas consists in the fact that they enable one to calculate magnetic field strength H_Z (or $H_{Z'}$) at any point of the initial area in an explicit form according to the coordinates of this point.

As expected, the calculation of magnetic field relative magnetic intensity H_Z according to formulas (7), (9) for the points above the center of the pole, when δ/*S*=0,1…0,5, and according to formulas (8), (10) for the point above the interpolar gap, when *b/S*=0,5…0,9, revealed practically complete coincidence of results. Carrying out preliminary calculations aiming at estimation of magnetic field strength H_Z, it is expedient to use formulas (9) and (10), as their form is simpler.

Experimental verification of the obtained relations (9) and (10) was performed on a physical model of a multipolar electromagnetic pulley of ESH5/6,3 type, whose design corresponds to Fig. 1, a. It was a one-fifth scale physical model, as compared with the production piece, and had the parameters: *D*=126 mm; *b*=30 mm; δ = 20 mm; *S*=50 mm. When physical modeling was carried out, magnetic induction *B* was measured at characteristic points situated above the pole center and above the center of interpolar gap at different distances y_0 from poles surfaces [19]. Magnetic field strength *H* at characteristic points of the physical model operating area was calculated according to magnetic induction *B* measured values as $H=B/\mu_0$, where $\mu_0=4\pi\cdot10^{-7}$ H/m – permeability of free space.

The tables below contain magnetic field strength values obtained on the basis of experimental data and computed according to formulas (9) and (10) for points situated above the pole center (Fig. 2, a) and above the center of interpolar gap (Fig. 2, b), respectively. In this case, in order to take magnetic field real 3D character into consideration, according to [20], magnetic field strength H_Z values, obtained analytically, were multiplied by coefficient k_δ equal to 1.1 for formula (9) and equal to 1.15 for formula (10). As the tables show, calculations errors are quite admissible.

Table 1. Verification of magnetic intensity calculation according to (9) for points above the pole center

Distance y_0 from the pole surfaces, mm	y_0/S	Magnetic intensity, A/m		Error %
		according to formula (9)	experiment	
5	0,1	109 154	111 400	−2,0
10	0,2	86 287,8	87 540	−1,4
20	0,4	50 204,7	51 010	−1,6
30	0,6	27 457,6	27 830	−1,3
40	0,8	14 036,4	14 300	3,0

Table 2. Verification of magnetic intensity calculation according to (10) for points above the center of interpolar gap

Distance y_0 from the pole surfaces, mm	y_0/S	Magnetic intensity, A/m		Error %
		according to formula (10)	experiment	
5	0,1	140 460,5	137 800	1,9
10	0,2	97 245,5	95 690	1,7
20	0,4	50 473,5	49 420	2,1
30	0,6	26 523,6	26 980	−1,7
40	0,8	14 724,7	14 870	−0,97

Conclusion

Formulas for calculation of magnetic field strength in cylindrical multipolar systems of magnetic separators with poles polarity interlacing both along the rotation axis and along the cylinder cross-section circle have been obtained. The peculiar feature of the mentioned formulas consists in the possibility of carrying out the calculation at any point of the initial area in an explicit form according to the coordinates of this point. In this case application of expressions (9) and (10) is preferable, as they are simpler. Validity of the obtained formulas is confirmed by comparison of calculation results with experimental data for industrial magnetic separators.

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