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Horizontal dipole antenna very close to lossy half-space surface

***Abstract**. Input impedance of a horizontal dipole antenna in close proximity of a lossy half-space is determined in this paper. Integrals of Sommerfeld's type are approximately solved applying a simple model for the spectral reflection coefficient that is a part of their integrand. The system of integral equations of Hallén's type is numerically solved using the point-matching method, assuming the polynomial approximation for the current along the antenna conductors. Obtained results will be graphically illustrated and compared to corresponding ones of other authors.*

***Streszczenie.** W artykule wyznaczono wejściową impedancję anteny dipolowej horyzontalnej w bliskim sąsiedztwie powierzchni stratnej półprzestrzeni. Rozwiązano w tym celu całki Sommerfelda oraz układ równań całkowych Hallena. Otrzymane wyniki zostały przedstawione w formie graficznej i porównane z otrzymanymi w innych badaniach. (**Antena dipolowa horyzontalna w bliskim sąsiedztwie powierzchni stratnej półprzestrzeni**).*

Keywords: Horizontal dipole antenna, Input impedance/admittance, Lossy half-space, Sommerfeld`s integral.
Słowa kluczowe: antenna dipolowa horyzontalna, impedancja/admitancja wejściowa, stratna półprzestrzeń

Introduction

Many antennas and antenna systems, depending on their purpose, are installed in close proximity to the ground or even directly on it. Exact evaluation of the influence of real ground on characteristics of wire structures above or inside of it, presents a well-known and widely researched problem, [1-15]. In [11], the first author proposed a very efficient, simple, and accurate approximate model for calculation of a type of Sommerfeld`s integrals that occurs in the case of vertical wire structures placed above a lossy half-space (LHS). The methodology proposed in [11], and later applied in [13], can be also successfully used for solving a more complex, but practically more important case of a horizontal dipole antenna (HDA), [14, 15]. The goal is to form a simple model, whose application will lead to a satisfyingly accurate and generally applicable closed-form solution of the Sommerfeld`s integral kernel (SIK), regardless of the type of ground underneath the observed wire antenna structure.

Problem setting

Let's consider an asymmetric HDA in the air ($\sigma_0=0$, ε_0, μ_0) at height $h>0$ above the LHS that is treated as a homogeneous and isotropic medium of known electrical parameters (σ_1 - specific conductivity, $\varepsilon_1=\varepsilon_{r1}\varepsilon_0$ – permittivity, $\mu_1=\mu_0$ - permeability, $\underline{\sigma}_1=\sigma_1+j\omega\varepsilon_1=j\omega\underline{\varepsilon}_1$ - complex specific conductivity, $\underline{\varepsilon}_1\approx\varepsilon_{r1}-j60\sigma_1\lambda_0$ - relative complex permittivity, λ_0 - wavelength in the air, $\underline{\gamma}_i=\alpha_i+j\beta_i=j\omega(\underline{\varepsilon}_i\,\mu_i)^{1/2}=(j\omega\underline{\sigma}_i\mu_i)^{1/2}$– propagation constant in the air, i=0, and semi-conducting ground, i=1, $\underline{n}=\underline{\gamma}_1/\underline{\gamma}_0=\sqrt{\underline{\varepsilon}_1}$ - refractive index, and $\omega=2\pi f$ – angular frequency). The HDA is observed in the direction parallel to the x-axis of Descartes' right-angled coordinate system, Fig. 1, and is fed by an ideal Dirac`s δ- generator of voltage U. The

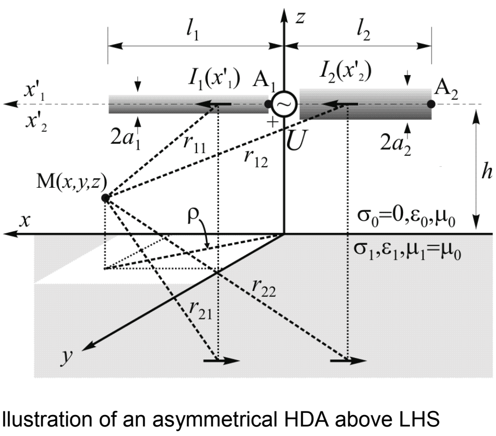

Fig.1. Illustration of an asymmetrical HDA above LHS

current is localized along antenna conductors` axes and is denoted by $I_k(x`_k)$, $0\le x`_k \le l_k$, k=1, 2.

Hertz's vector and electric scalar potential

In the case of the HDA, the Hertz's vector potential calculated in the air at point M(x,y,z) has two components, $\Pi_0=\Pi_{x0}+\Pi_{zx0}$, which are here expressed by their modified equivalents $\Pi^*_{x0}=\Pi_{x0}+\Pi^*_{zx0}$ and Π^*_{zx0}:

$$\text{(1a)}\quad \Pi^*_{x0}=\frac{1}{4\pi\underline{\sigma}_0}\sum_{k=1}^{2}\int_0^{l_k} I_k(x'_k)\begin{bmatrix}K_0(r_{1k})+\\ +(\underline{n}^{-2}-1)K_0(r_{2k})+\\ +\underline{n}^{-2}S^v_{00}(r_{2k})\end{bmatrix}\mathrm{d}x'_k\,,$$

$$\text{(1b)}\quad \Pi^*_{zx0}=\frac{1}{4\pi\underline{\sigma}_0}\sum_{k=1}^{2}\int_0^{l_k} I_k(x'_k)\begin{bmatrix}(\underline{n}^{-2}-1)K_0(r_{2k})+\\ +\underline{n}^{-2}S^v_{00}(r_{2k})-\\ -S^h_{00}(r_{2k})\end{bmatrix}\mathrm{d}x'_k\,.$$

Thus, the electric scalar potential is:

$$\text{(2)}\quad \varphi_0=-\mathrm{div}\vec{\Pi}_0=-\frac{\partial}{\partial x}\Pi_{x0}-\frac{\partial}{\partial z}\Pi_{zx0}=-\frac{\partial}{\partial x}\Pi^*_{x0}\,.$$

In previous expressions, $K_0(r_{ik})=\exp(-\underline{\gamma}_0 r_{ik})/r_{ik}$ denotes the standard form of the potential kernel, whereas $S^h_{00}(r_{2k})$ and $S^v_{00}(r_{2k})$ represent Sommerfeld`s integral kernels:

$$\text{(3a)}\qquad S^h_{00}(r_{2k})=\int_{\alpha=0}^{\infty}\widetilde{R}_{\eta10}(\alpha)\,K^*_0(\alpha,r_{2k})\,\mathrm{d}\alpha\,,$$

$$\text{(3b)}\qquad S^v_{00}(r_{2k})=\int_{\alpha=0}^{\infty}\widetilde{R}_{z10}(\alpha)\,K^*_0(\alpha,r_{2k})\,\mathrm{d}\alpha\,,$$

where $K^*_0(\alpha,r_{2k})$ is the spectral form of the potential kernel, while the first terms in both integrands represent spectral reflection coefficients (SRCs):

$$\text{(4a)}\qquad \widetilde{R}_{\eta10}(\alpha)=\frac{u_0-u_1}{u_0+u_1},\ u_i=\sqrt{\alpha^2+\underline{\gamma}_i^2}\ ,\ i=0,1\,,$$

$$\text{(4b)}\qquad \widetilde{R}_{z10}(\alpha)=\frac{\underline{n}^2u_0-u_1}{\underline{n}^2u_0+u_1},\ u_i=\sqrt{\alpha^2+\underline{\gamma}_i^2}\ ,\ i=0,1\,,$$

$r_{1k}=(\rho`_k{}^2+(z-h)^2)^{1/2}$, $r_{2k}=(\rho`_k{}^2+(z+h)^2)^{1/2}$, $\rho`_k{}^2=(x-x`_k)^2+(y-y`_k)^2$, $k=1, 2$.

Based on (1a, b) and (2), one can see that for determining the structure of the Hertz's vector and electric scalar potential it is necessary to numerically solve two types of SIKs (3a and b). Based on [11] and [13], the solution of the SIK $S^v_{00}(r_{2k})$ will be considered known, so the attention will be devoted only to modeling of integrals of the form (3a).

SIK modeling

The authors propose an approach to this problem whose first step considers assuming the SRC in a certain approximate form. The goal is to create a model with a simple form, whose implementation will lead to a satisfyingly accurate closed-form solution of the SIK. Also, the model can be considered a rather good approximation if its application is not limited by the type of soil, i.e. the accuracy of the calculation remains in a satisfactory range for all types of the LHS above which the HDA is located.

Let's assume the SRC in the form that follows:

$$(5)\quad \widetilde{R}_{\eta 10}(u_0) \cong B + A\, e^{-(u_0-\underline{\gamma}_0)\underline{d}_0} .$$

Matching expression (5) at points $u_0\to\infty$ and $u_0\to\underline{\gamma}_0$, and its first derivative at $u_0\to\underline{\gamma}_0$, all three unknown constants A, B and $\underline{d}_0$ are determined, i.e. $B=0$, $A=-R_0$, $\underline{D}_0=\underline{\gamma}_0\underline{d}_0=2/\underline{n}$, $R_0=(\underline{n}-1)/(\underline{n}+1)$, and $\underline{n}$ - refractive index. Substituting (5) into (3a), the following SIK model is obtained:

$$(6)\quad S^h_{00}(r_{2k}) \cong -R_0\, e^{\underline{D}_0} K_0(r_{3k}) ,$$

where $r_{3k}=(\rho`_k{}^2+(z+h+\underline{d}_0)^2)^{1/2}$.

A similar SRC model, i.e. the well-known Bannister-Wait approximation, [1] and [3], is derived in a different manner, and its application leads to the following SIK model:

$$(7)\quad S^h_{00}(r_{2k}) \cong -K_0(r_{3k}),\ r_{3k}=\sqrt{\rho_k'^2+(z+h+\underline{d}_0)^2} .$$

It should be emphasized that the model (7) can be obtained only if one considers that $n>>1$.

SIE-H and the point-matching method

In order to determine the unknown current distribution (UCD), we will assume it in the following polynomial form:

$$(8)\quad I_k(u_k'=x_k'/l_k) = \sum_{m=0}^{\mathrm{M}_k} I_{mk}\, u_k'^{m} ,\ 0\le u_k'\le 1,\ k=1,2 ,$$

where I_{mk} - unknown complex current coefficients, and M_k - the polynomial degree. Unknown current coefficients will be determined solving the system of integral equations of Hallén`s type (SIE-H). This system will be formed satisfying the boundary condition for the tangential component of the electric field on the antenna surface. Then, using the point-matching method, a corresponding system of linear equations will be formed. Solving the established system of equations, the unknown current distribution along the antenna conductors will be determined, and based on it, the input impedance/admittance of the HDA:

$$(9)\quad Z_{in} = R_{in} + \mathrm{j}X_{in} = \frac{1}{Y_{in}} = \frac{1}{G_{in}+\mathrm{j}B_{in}} = \frac{U}{I_1(u_1'=0)} .$$

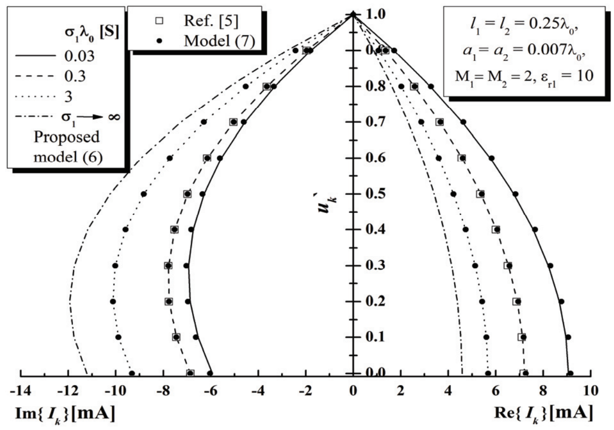

Fig.2. Real and imaginary part of current distributions for the half-wave HDA placed above LHS

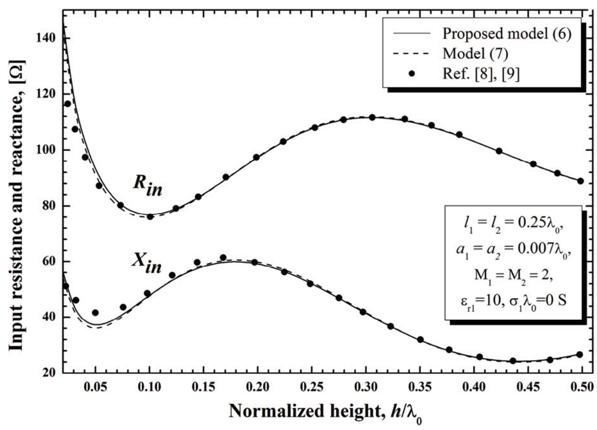

Fig.3. Input resistance and reactance of a half-wave HDA placed above LHS versus normalized height

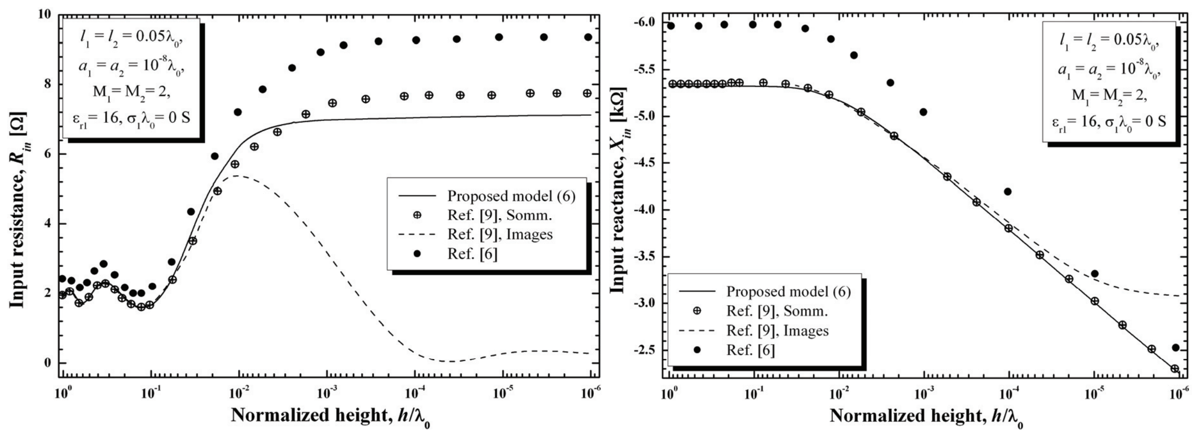

Fig.4. Input resistance and reactance of a HDA placed above a perfect dielectric versus normalized height

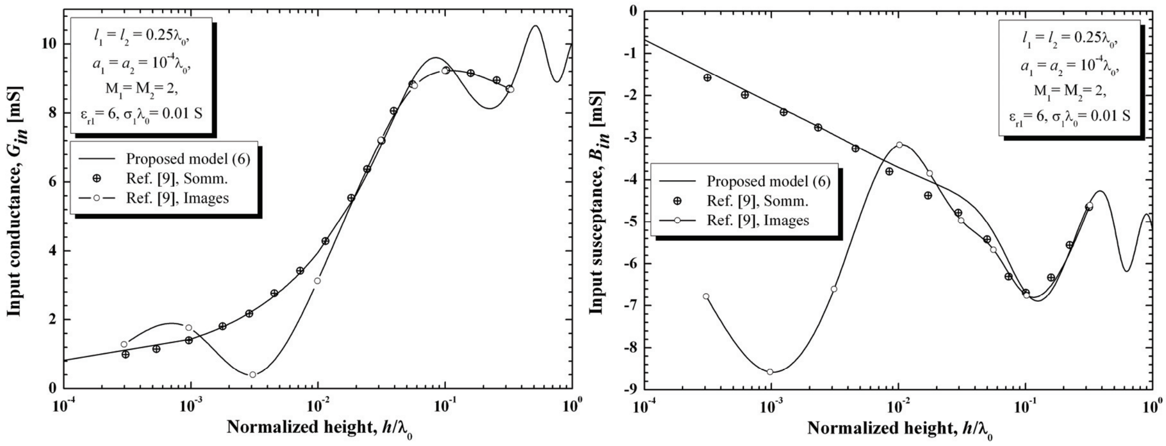

Fig.5. Input conductance and susceptance of a half-wave HDA placed above dry ground versus normalized height

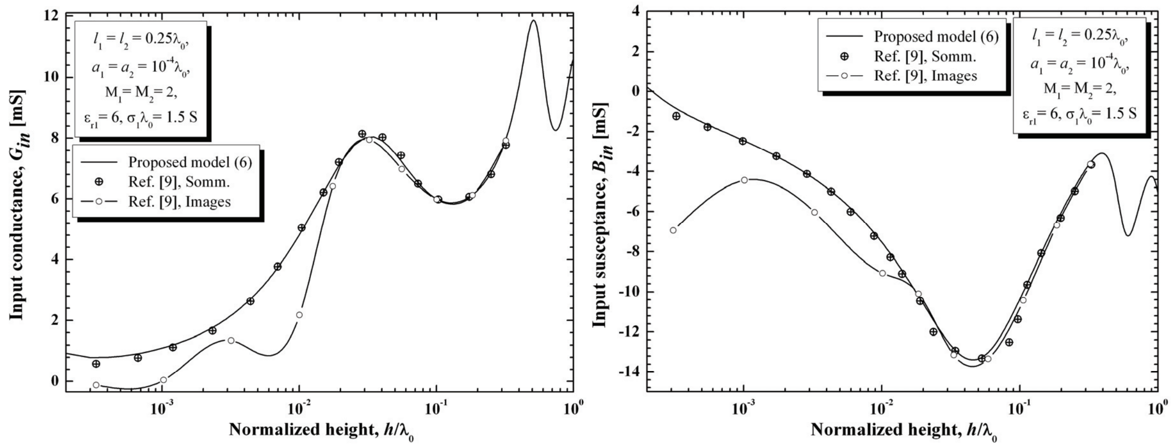

Fig.6. Input conductance and susceptance of a half-wave HDA placed above moist ground versus normalized height

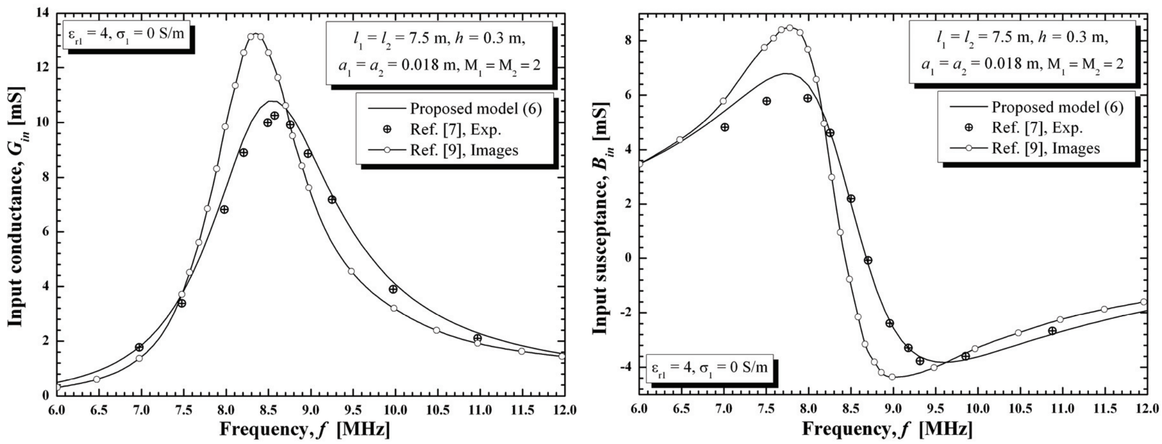

Fig.7. Input conductance and susceptance of a HDA 15m long and 0.3m above a very poorly conducting ground plane versus frequency

Numerical results

Although the applied method was described on a general example of an asymmetric HDA above LHS, the results presented in this section will correspond to a symmetric HDA with equal conductor lengths, l_1=l_2, and equal cross-section radii, a_1=a_2, since this is the HDA geometry that is most often analyzed in the literature, [4-9], [13, 14]. All the results are obtained using the polynomial current approximation with the polynomial degree of M_1=M_2=2.

Current distributions along the half-wave HDA determined for different values of the normalized conductivity of the ground are given in Fig. 2. Continual lines correspond to the proposed SIK model (6), while scattered values correspond to results obtained using the SIK model (7) from [3] (solid circles) and results taken from [5] (open squares).

Following set of results, Fig. 3, corresponds to the input impedance (resistance and reactance) of the analyzed HDA, as one of the crucial integral characteristics of an antenna that clearly reflects on the accuracy of the near field calculations. Displayed curves are obtained for the case of a perfect dielectric (ε_{r1}=10). The results were compared with the corresponding ones from [8, 9], where the authors applied the method of images (with six images), and the ones obtained using the SIK model (7). The later ones are also in very good agreement with the ones from [8, 9], although the model is derived under a condition that the refractive index is much greater than 1, n>>1. This was expected since the value of the refractive index in this example is n=$\sqrt{10}$, which practically satisfies the set limitation.

Previous results confirmed that the application of the described methodology provides accurate analysis of the HDA placed at heights higher than h/λ_0=0.025 above the LHS. However, it was of importance to evaluate the applicability of the described method and proposed model (6) to cases of the HDA positioned at heights lower than h/λ_0= =0.025 since the standard method of images rapidly losses on the accuracy for these HDA positions.

The real and imaginary part of the impedance of a very thin and relatively short HDA located above a perfect dielectric are illustrated in Fig. 4. Solid lines correspond to the results obtained using the method described here, and are compared to the ones that gives rigorous Sommerfeld`s analysis, [9]. Also, in order to illustrate the accuracy that this method offers, the results from [6] (where the authors applied sinusoidal current approximation and solved the Pocklington`s integral equation applying the MoM method) and the ones from [9] obtained applying the method of images, are also shown. It is obvious that even for very low-placed antennas proposed method gives the most accurate results.

A similar numerical experiment has been performed with different values of the electrical parameters of the ground and geometry. This time we considered the HDA with a radius a_1=a_2=0.0001λ_0, and two types of the LHS below it: dry ground (ε_{r1}=6, $\sigma_1\lambda_0$=0.01S, see Fig. 5), and moist ground (ε_{r1}=6, $\sigma_1\lambda_0$=1.5S, see Fig. 6). Both figures illustrate real and imaginary parts of the HDA input admittance. Comparison is done with two sets of values: the ones that are a result of rigorous Sommerfeld`s analysis, and the results of employed method of images. The same conclusions as previously can be also made here. The accordance of the results is better for higher placed antennas, i.e. for antenna heights $h/\lambda_{0>}$0.02, which is the lower boundary of the validity range of the referenced method of images. It should be mentioned that our results correspond to the second order of the polynomial current approximation, whereas the ones obtained using the method of images correspond to six pairs of images and the third order of the polynomial.

Figure 7 gives comparison between theoretical calculations performed using the SIK model (6) proposed in this paper and the results of conductance and susceptance measurements for a frequency range of 6-12 MHz, [7]. The observed HDA is 15m long suspended at height of 0.3m above the LHS, a large flat granodiorite outcrop on which there had been no rain for several months ensuring a dry ground plane. For the sake of comparing, corresponding results obtained using the method of images, [8, 9], are also given in figures. Obviously, a much better accordance with experimental data is obtained applying the method described in this paper. This was expected since the observed antenna is very close to the ground (for the frequency of 10 MHz, the 0.3m corresponds to 0.01λ_0). As stated in previous examples, in the case of the method of images accuracy rapidly decreases with the height of antenna less than h/λ_0=0.025, [8, 9]. This is of importance since it is suggested in the literature that input impedance measurements using an antenna located at about h/λ_0=0.01 above ground provide valuable data for geological mapping surveys, [7].

Conclusion

Clearly, application of the proposed SIK model and described numerical procedure gives highly accurate results for the near field integral characteristics in all ranges of the electrical parameters of the ground and geometry parameters. Simplicity of the SIK model does not depend on the ground characteristics, which is a great advantage in regard to the method of images, [8, 9], which in order to achieve higher accuracy of calculations calls for a greater number of images. It should also be emphasized that the accuracy of calculations is stable even for low-placed antennas.

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REFERENCES

[1] Wait J.R., Spies K.P., On the Image Representation of the Quasi-static Fields of a Line Current Source above the Ground, *Can.J.Phys.*, 47 (1969), 2731-2733.
[2] Sarkar T.K., Analysis of Radiation from Arbitrarily Oriented Wire Antennas over Imperfect Ground (Sommerfeld Formulation), Computer Program Description, *Electronics letters*, 12 (1976), No. 18, 462.
[3] Bannister P.R., Extension of Quasi-static Range of Finitely Conducting Earth Image Theory Technique to Other Ranges, *IEEE Trans. on AP*, 26 (1978), No. 3, 507-508.
[4] Parhami P., Rahmat-Samii Y., Mittra R., An Efficient Approach for Evaluating Sommerfeld Integrals Encountered in the Problem of a Current Element Radiating over Lossy Ground, *IEEE Trans. on AP*, 28 (1980), No. 1, 100-104.
[5] Surutka J.V., Mitić D.N., Horizontal Dipole Antenna above an Imperfectly Conducting Ground Fed by a Two-wire Line, *Extrait du BULLETIN T. LXXVIII de l`Acedemie serb des sciences et des arts, Class des sciences techniques*, 19 (1981), 1-14.
[6] Burke G.J., Johnson W.A., Miller E.K., Modelling of Simple Antennas Near to and Penetrating an Interface, *IEEE Proc.*, 71 (1983), No. 1, 174-175.
[7] Nicol J.L., Ridd P.V., Antenna Input Impedance: Experimental Confirmation and Geological Application, *Can.J.Phys*, 66 (1988), 818-823.
[8] Popović B.D., Petrović V.V., Horizontal Wire Antenna Above Lossy Half-Space: Simple Accurate Image Solution, *International journal of numerical modelling: Electronic networks, devices and fields*, 9 (1996), 194-199.
[9] Petrović V.V., Analysis of Wire Antennas in the Presence of Real Ground Using the Method of Images, PhD Thesis, University of Belgrade, Electrotechnical faculty, Serbia, 1996. (in Serbian)
[10] Arand B.A., Hakkak M., Horizontal Wire Antenna above Lossy Half-space, Proc. of IEEE Antennas and Propagation Society International Symposium, 4 (2003), 970-973.
[11] Rančić M., Rančić P., Vertical Dipole Antenna above a Lossy Half-Space: Efficient and Accurate Two-Image Approximation for the Sommerfeld's Integral, CD Proc. of EuCAP`06, paper N^{o}121, Nice, France, 2006.
[12] Makki S.V.A., Ershadi T.Z., Abrishamian M.S., Determining the Specific Ground Conductivity Aided by the Horizontal Electric Dipole Antenna near the Ground Surface, *Progress in Electromagnetics Research B*, 1 (2008), 43–65.
[13] Rančić M., In the Addition to Modelling of the Real Ground Influence on Characteristics on Vertical Wire Structures in the Air, M.Sc. thesis, University of Niš, Faculty of Electronic Engineering, Niš, Serbia, 2008. (in Serbian)
[14] Rančić M., Rančić P., Horizontal Dipole Antenna Above Real Ground, CD Proc. of 14th International IGTE Symposium on Numerical Field Calculation in Electrical Engineering - IGTE'10, Graz, Austria, 2010, 87-92.
[15] Rančić M., Rančić P., Horizontal Linear Antennas Above a Lossy Half-Space: A New Model for the Sommerfeld`s Integral Kernel, *International Journal of Electronics and Communications (AEUE)*, 65 (2011), No. 10, 879-887.

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