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Application of Stochastic Differential Equations in Second-Order Electrical Circuits Analysis

Abstract. *The paper deals with an unconventional approach to the analysis of electrical circuits with randomly varying parameters based on stochastic differential equations (SDE). A response of the electrical circuit is computed in the form of a sample mean with a particular confidence interval to provide credible estimate of the result. The method is applied to get voltage and current responses of the second-order RLGC network excited from a voltage source with a noise term. The results are compared with the classical deterministic state-variable approach.*

Streszczenie. *W artykule przedstawiono niekonwencjonalna metodę analizy obwodu elektrycznego o przypadkowo zmienianych parametrach. Metoda bazuje na stochastycznych równani ach różnicowych SDE. Metodę sprawdzono na przykładzie określania napięcia i prądu w sieci drugiego rzędu RLGC zasilanej napięciem z szumami. (**Zastosowanie stochastycznych równań różnicowych w obwodach elektrycznych drugiego rzędu**)*

Keywords: stochastic differential equation, Itô formula, second-order electrical circuit, state variable.
Słowa kluczowe: równania stochastyczne, obwody elektryczne drugiego rzędu, analiza obwodów

Introduction

The study of stochastic phenomena has been stimulated by the need of taking into account random effects while modelling physical systems. An interesting approach of that is to replace parameters in the deterministic model by random processes. In this context the stochastic differential equations (SDE) can be utilized to characterize a response of such a model [1-3].

The most attention has still been paid to the first-order RL or RC circuits, see e.g. [4-7], where main SDE concepts and suitable numerical techniques were widely studied and verified. From practical point of view, however, just second-order RLC and RLGC networks are of major importance as they may often serve as building blocks of more complex physical models. Namely, our intention in the paper is to get ready a concept for describing responses of transmission lines (TL) modeled by the lumped-parameter circuits, RLGC networks [8], when random changes of their parameters or sources excitations can occur in physical implementations, e.g. [9]. The stochastic solutions derived for the basic RLGC network have a potential to be further generalized towards higher-order electrical models used just for transmission structures simulation in high-speed electronic systems, multiconductor transmission lines (MTL) included [10-12].

Mathematical issues

Vector stochastic differential equations. A general N-dimensional stochastic differential equation (vector SDE), can be written in a vector form as

$$d\mathbf{X}(t)=\mathbf{A}\big(t,\mathbf{X}(t)\big)dt+\sum_{j=1}^{M}\mathbf{B}_j\big(t,\mathbf{X}(t)\big)dW_j(t)\ , \tag{1}$$

where $\mathbf{A}$: $\langle t_0,T\rangle\times\mathbb{R}^N\to\mathbb{R}^N$ is a vector function, $\mathbf{B}_j$ represents the j-th column of the matrix function $\mathbf{B}$: $\langle t_0,T\rangle\times\mathbb{R}^N\to\mathbb{R}^{N\times M}$ and $d\mathbf{W}(t)=(dW_1(t),...,dW_M(t))$ is a column vector, where $dW_1(t),...,dW_M(t)$ are independent Wiener processes representing the noise. (A continuous stochastic process $W(t)$ is called the Wiener process if it has independent increments, $W(0)=0$ and $W(t)-W(s)$ distributed $N(0,t-s)$, $0\le s<t$).

The solution of a vector stochastic differential equation is a stochastic vector process $\mathbf{X}(t)=(X_1(t),...,X_N(t))$. By an SDE we understand in fact an integral equation

$$\mathbf{X}(t)=\mathbf{X}_0+\int_{t_0}^{t}\mathbf{A}\big(s,\mathbf{X}(s)\big)ds+\sum_{j=1}^{M}\int_{t_0}^{t}\mathbf{B}_j\big(s,\mathbf{X}(s)\big)dW_j(s)\ , \tag{2}$$

where the integral with respect to ds is the Lebesgue integral and the integrals with respect to $dW_j(s)$ are stochastic integrals, called the Itô integrals, see [3].

Although the Itô integral has some very convenient properties, the usual chain rule of classical calculus doesn't hold. Instead, the appropriate stochastic chain rule, known as Itô formula, contains an additional term, which, roughly speaking, is due to the fact that the stochastic differential of the Wiener process $(dW(t))^2$ is equal to dt in the mean square sense, i.e. $E[(dW(t))^2]=dt$, so the second order term in $dW(t)$ should really appear as a first order term in dt.

The multidimensional Itô formula. Let the stochastic process **X**(*t*) be a solution of the stochastic differential equation (1) for some suitable matrix functions $\mathbf{A}$, $\mathbf{B}$, (see [3], p. 48). Let $\mathbf{g}(t,\mathbf{x})$: $(0,\infty)\times\mathbb{R}^N\to\mathbb{R}^P$ is a twice continuously differentiable function. Then

$$\mathbf{Y}(t)=\mathbf{g}\big(t,\mathbf{X}(t)\big)=\big(g_1(t,\mathbf{X}),\dots,g_P(t,\mathbf{X})\big) \tag{3}$$

is a stochastic process, whose k−th component is given by

$$\begin{aligned}dY_k&=\frac{\partial g_k}{\partial t}(t,\mathbf{X})dt+\sum_i\frac{\partial g_k}{\partial X_i}(t,\mathbf{X})dX_i\\&+\frac{1}{2}\sum_{i,j}\frac{\partial^2 g_k}{\partial X_i\partial X_j}(t,\mathbf{X})(dX_i)(dX_j)\end{aligned} \tag{4}$$

where $dX_i\cdot dX_j$ is computed according to the rules $dt\cdot dt=dt\cdot dW_i=dW_i\cdot dt=0$ and $dW_i\cdot dW_j=\pm\,\delta_{i,j}\,dt$.

Linear stochastic differential equations. A general vector linear Itô stochastic differential equation has a form

$$d\mathbf{X}(t)=\big(\mathbf{A}(t)\mathbf{X}(t)+\mathbf{a}(t)\big)dt+\sum_{j=1}^{M}\big(\mathbf{B}_j(t)\mathbf{X}(t)+\mathbf{b}_j(t)\big)dW_j(t)\ , \tag{5}$$

where $\mathbf{A}(t)$ and $\mathbf{B}_i(t)$ are measurable and bounded $N\times N$ matrix functions of time or constants and similarly $\mathbf{a}(t)$ and $\mathbf{b}_i(t)$ are measurable and bounded M−dimensional vector functions of time or constants. In the case, when $\mathbf{B}_i(t)\equiv\mathbf{0}$, $i=1,...,M$, we say that the equation (5) is a general linear stochastic differential equation with an additive noise.

Basic RLGC network analysis

Herein, a basic 2nd-order RLGC network which is a part of the whole TL model will be considered, see Fig. 1.

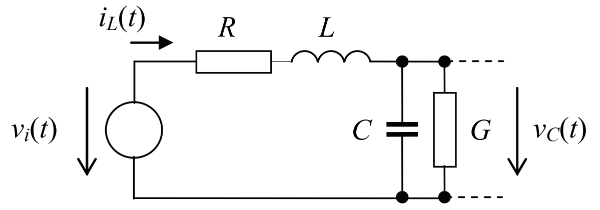

Fig.1. Basic 2nd-order RLGC network

The responses $v_C(t)$ and $i_L(t)$, being simultaneously the state variables of the circuit, will be analyzed, while feeding the circuit from a step voltage source, $v_i(t) = V_i \cdot \underline{1}(t)$. Because of their analytical solutions are known this will enable to judge a correctness of characteristics obtained by means of the SDE technique.

First, a classical deterministic approach will be resumed. If the state-variable method is applied the 1st-order matrix ordinary differential equation is to be solved, namely

$$\begin{bmatrix} C & 0 \\ 0 & L \end{bmatrix} \cdot \frac{d}{dt} \begin{bmatrix} v_C(t) \\ i_L(t) \end{bmatrix} + \begin{bmatrix} G & -1 \\ 1 & R \end{bmatrix} \cdot \begin{bmatrix} v_C(t) \\ i_L(t) \end{bmatrix} = \begin{bmatrix} 0 \\ v_i(t) \end{bmatrix} . \tag{6}$$

Denoting the matrices by

$$\begin{gathered} \mathbf{x}(t) = \begin{bmatrix} v_C(t) \\ i_L(t) \end{bmatrix}, \ \mathbf{M} = \begin{bmatrix} C & 0 \\ 0 & L \end{bmatrix}, \\ \mathbf{H} = \begin{bmatrix} G & -1 \\ 1 & R \end{bmatrix}, \ \mathbf{u}(t) = \begin{bmatrix} 0 \\ v_i(t) \end{bmatrix}, \end{gathered} \tag{7}$$

and consequently considering equalities

$$\begin{gathered} -\mathbf{M}^{-1}\mathbf{H} = \begin{bmatrix} -G/C & 1/C \\ -1/L & -R/L \end{bmatrix} = \mathbf{A}, \\ \mathbf{M}^{-1}\mathbf{u}(t) = \begin{bmatrix} 0 \\ v_i(t)/L \end{bmatrix} = \frac{1}{L}\mathbf{u}(t), \end{gathered} \tag{8}$$

the last equation can be written in its compact form

$$\frac{d\mathbf{x}(t)}{dt} = \mathbf{A}\mathbf{x}(t) + \frac{1}{L}\mathbf{u}(t) \ , \tag{9}$$

with a solution

$$\mathbf{x}(t) = e^{\mathbf{A}t}\mathbf{x}(t_0) + \frac{1}{L}\int_{t_0}^{t} e^{\mathbf{A}\cdot(t-\tau)}\mathbf{u}(\tau)d\tau \ . \tag{10}$$

For our purposes the initial time is chosen as $t_0 = 0$, and zero initial conditions are only considered, i.e. $\mathbf{x}(0) = \mathbf{0}$. Then the response to the external feeding source is given by

$$\mathbf{x}(t) = \frac{1}{L}\int_{0}^{t} e^{\mathbf{A}\cdot(t-\tau)}\mathbf{u}(\tau)d\tau = \frac{e^{\mathbf{A}t}}{L}\int_{0}^{t} e^{-\mathbf{A}\tau}\mathbf{u}(\tau)d\tau \ . \tag{11}$$

Evaluating the matrix exponential function $e^{\mathbf{A}t}$, and making necessary integrations, the analytical results are classified depending on the type of roots of the characteristic equation $|\mathbf{A} - \lambda\mathbf{I}| = 0$, see Tab. 1.

Table 1. Possible solutions of (11)

roots	complex: $\lambda_{1,2} = -\alpha \pm j\omega$ (underdamped case)
$v_C(t)$	$\frac{V_i}{RG+1}\left[1 - e^{-\alpha t}\left(\frac{\alpha}{\omega}\sin\omega t + \cos\omega t\right)\right]$
$i_L(t)$	$\frac{V_i}{RG+1}\left[G - e^{-\alpha t}\left(\frac{(\alpha L - R)G - 1}{\omega L}\sin\omega t + G\cos\omega t\right)\right]$
roots	double: $\lambda_{1,2} = -\alpha$ (critically damped case)
$v_C(t)$	$\frac{V_i}{RG+1}\left[1 - e^{-\alpha t}(\alpha t + 1)\right]$
$i_L(t)$	$\frac{V_i}{RG+1}\left[G - e^{-\alpha t}\left(\frac{(\alpha L - R)G - 1}{L}t + G\right)\right]$
roots	real: $\lambda_{1,2} = -\alpha \pm \omega$ (overdamped case)
$v_C(t)$	$\frac{V_i}{RG+1}\left[1 - e^{-\alpha t}\left(\frac{\alpha}{\omega}\sinh\omega t + \cosh\omega t\right)\right]$
$i_L(t)$	$\frac{V_i}{RG+1}\left[G - e^{-\alpha t}\left(\frac{(\alpha L - R)G - 1}{\omega L}\sinh\omega t + G\cosh\omega t\right)\right]$

In Tab.1, designations were applied as follows

$$\alpha = \frac{1}{2}\left(\frac{R}{L} + \frac{G}{C}\right) \ , \ \omega = \sqrt{\left|\alpha^2 - \frac{RG+1}{LC}\right|} \ . \tag{12}$$

RLGC network with stochastic source

In this section the Itô stochastic calculus will be used to get responses of the RLGC network to a source influenced by random effects. Instead of $v_i(t)$ a non-deterministic version of this function is considered as

$$v_i^*(t) = v_i(t) + \text{"noise"} \ . \tag{13}$$

To be able to substitute this into (9) we have to describe mathematically the “noise”. It is reasonable to look at it as a stochastic process called the “white noise process”, denote it by $\xi(t)$. We get the following equation (α is a constant)

$$\frac{d\mathbf{x}(t)}{dt} = \mathbf{A}\mathbf{x}(t) + \frac{1}{L}\mathbf{u}(t) + \frac{\alpha}{L}\begin{bmatrix} 0 \\ \xi(t) \end{bmatrix} \ , \tag{14}$$

The last equation is multiplied by dt and then $\xi(t)dt$ replaced by $dW(t)$; where $W(t)$ as the Wiener process. Formally the “white noise” is the time derivative of the Wiener process $W(t)$ (that is really just a formal statment, because the sample paths of a Wiener process $W(t)$ are nowhere differentiable). We get a stochastic differential equation

$$d\mathbf{x}(t) = \left(\mathbf{A}\mathbf{x}(t) + \frac{1}{L}\mathbf{u}(t)\right)dt + \frac{\alpha}{L}\begin{bmatrix} 0 \\ dW(t) \end{bmatrix} \ , \tag{15}$$

or componentwise

$$dv_C(t) = \left(-\frac{G}{C}v_C(t) + \frac{1}{C}i_L(t)\right)dt \tag{16}$$

$$di_L(t) = \left(-\frac{1}{L}v_C(t) - \frac{R}{L}i_L(t)\right)dt + \frac{1}{L}\left(v_i(t) + \alpha\, dW(t)\right). \tag{17}$$

The solution of a stochastic differential equation is a stochastic process, which is usually denoted by a capital letter. To distinguish the deterministic model from the stochastic one, we denote the stochastic solution by $\mathbf{X}(t)$. So we have the linear vector stochastic differential equation

(18) $$d\mathbf{X}(t)=\left(\mathbf{A}\mathbf{X}(t)+\frac{1}{L}\mathbf{u}(t)\right)dt+\mathbf{B}dW(t) \ ,$$

where $\mathbf{B}=[0,\alpha/L]^{\mathrm{T}}$. To find the analytic solution we compute, using the multidimensional Itô formula, the derivative of the function $g(t,\mathbf{X}(t))=e^{-\mathbf{A}t}\cdot\mathbf{X}(t)$:

(19) $$\begin{aligned} &dg\big(t,\mathbf{X}(t)\big)=d\big(e^{-\mathbf{A}t}\mathbf{X}(t)\big)=\\ &-\mathbf{A}e^{-\mathbf{A}t}\mathbf{X}(t)\,dt+e^{-\mathbf{A}t}d\mathbf{X}(t)-0\big(d\mathbf{X}(t)\big)^2=\\ &=-\mathbf{A}e^{-\mathbf{A}t}\mathbf{X}(t)\,dt+e^{-\mathbf{A}t}\mathbf{A}\mathbf{X}(t)\,dt+e^{-\mathbf{A}t}\left(\frac{1}{L}\mathbf{u}(t)dt+\mathbf{B}dW(t)\right). \end{aligned}$$

We have

(20) $$d\big(e^{-\mathbf{A}t}\mathbf{X}(t)\big)=e^{-\mathbf{A}t}\left(\frac{1}{L}\mathbf{u}(t)dt+\mathbf{B}dW(t)\right) \ .$$

Integrating the last equation we get

(21) $$e^{-\mathbf{A}t}\mathbf{X}(t)-\mathbf{X}(0)=\frac{1}{L}\int_0^t e^{-\mathbf{A}\tau}\mathbf{u}(\tau)\,d\tau+\int_0^t e^{-\mathbf{A}\tau}\mathbf{B}\,dW(\tau)$$

From this we can easily get the solution

(22) $$\mathbf{X}(t)=e^{\mathbf{A}t}\mathbf{X}(0)+\frac{1}{L}\int_0^t e^{\mathbf{A}(t-\tau)}\mathbf{u}(\tau)\,d\tau+\int_0^t e^{\mathbf{A}(t-\tau)}\mathbf{B}\,dW(\tau)$$

The solution $\mathbf{X}(t)$ is a random process and for it's expectation we have for every $t>0$

(23) $$E\big[\mathbf{X}(t)\big]=e^{\mathbf{A}t}\cdot E\big[\mathbf{X}(0)\big]+\frac{1}{L}\int_0^t e^{\mathbf{A}(t-\tau)}\mathbf{u}(\tau)\,d\tau \ ,$$

while the expectation of the Itô integral is zero. We can see, that for the initial value $\mathbf{X}(0)$ constant, which corresponds to the deterministic exciting voltage source, the expectation of the stochastic solution coincides with the deterministic solution (10), or (11) in case of $\mathbf{X}(0)=\mathbf{0}$.

SDE numerical solution technique

To simulate the solution of a stochastic differential equation numerical techniques have to be used. One of the approaches to deriving numerical solution of SDEs is based on numerical methods for ordinary differential equations. Here some care is required, because according to the definition of the Itô integral, the stochastic integral must always be evaluated at lower endpoint of the discretization subinterval. The stochastic version of the Euler scheme for a vector SDE is like follows.

Let the stochastic process $\mathbf{X}_t$, $t_0\le t\le T$, be the solution of the multidimensional SDE

(24) $$d\mathbf{X}(t)=\mathbf{A}\big(t,\mathbf{X}(t)\big)dt+\sum_{j=1}^{M}\mathbf{B}_j\big(t,\mathbf{X}(t)\big)dW_j(t) \ ,$$

on $t_0\le t\le T$ with the initial value $\mathbf{X}(t_0)=\mathbf{X}_0$. Let us consider an equidistant discretisation of the time interval $t_n=t_0+nh$, where $h=(T-t_0)/N=t_{n+1}-t_n$, for $n=0,\dots,N\text{-}1$, and the corresponding discretization of the M independent Wiener processes $\Delta W_j^n=W_j(t_{n+1})-W_j(t_n)=\int_{t_n}^{t_{n+1}}dW_j(\tau)$, $j=1,\dots,M$.

To be able to apply any stochastic numerical scheme, first we have to generate the random increments of the Wiener processes as independent Gauss random variables with $E[\Delta W_j^n]=0$ and $E[(\Delta W_j^n)^2]=h$.

The Euler scheme for the i-th component of the multidimensional SDE has the form

(25) $$X_i^{n+1}=X_i^n+A_i\big(t_n,\mathbf{X}_n\big)h+\sum_{j=1}^{M}B_{i,j}\big(t_n,\mathbf{X}_n\big)\Delta W_j^n \ .$$

It is consistent with Itô stochastic calculus because the noise term in (25) approximates the stochastic integral over the subinterval $\langle t_n,t_{n+1}\rangle$ by evaluating its integrand at the lower end point:

(26) $$\begin{aligned} &\int_{t_n}^{t_{n+1}}B_{i,j}\big(\tau,\mathbf{X}(\tau)\big)dW_j(\tau)\approx\int_{t_n}^{t_{n+1}}B_{i,j}\big(t_n,\mathbf{X}(t_n)\big)dW_j(\tau)=\\ &=B_{i,j}\big(t_n,\mathbf{X}(t_n)\big)\int_{t_n}^{t_{n+1}}dW_j(\tau)=B_{i,j}\big(t_n,\mathbf{X}(t_n)\big)\Delta W_j^n \ . \end{aligned}$$

For measuring the accuracy of a numerical solution to an SDE we use the strong order of convergence. We say that a stochastic numerical scheme converges with strong order γ if there exist real constants $K>0$ and $\delta>0$, so that

(27) $$E\Big[\big|\mathbf{X}(T)-\mathbf{X}^N(T)\big|\Big]\le Kh^{\gamma} \ , \quad h\in(0,\delta) \ ,$$

where the numerical solution is denoted by $\mathbf{X}_T^h$. The Euler scheme converges with strong order $\gamma=½$.

Statistical estimates and confidence intervals

An important goal of our simulation is to approximate an expectation $\mu=E[X]$ of the solution of the SDE in view. In practice it can be only estimated by the arithmetic mean of a finite number of realisations of the numerical scheme. To obtain statistically reliable results, confidence intervals should also be used. Denoting $X^n=X(t_n)$, the sample mean and the sample standard deviation can be calculated by

(28) $$\bar{X}_n=\frac{1}{K}\sum_{k=1}^{K}X_k^n \ ,$$

(29) $$s_n=\sqrt{\frac{1}{K-1}\sum_{k=1}^{K}\big(X_k^n-\bar{X}_n\big)^2} \ ,$$

respectively, K as the number of realizations, $n=0,\dots,N\text{-}1$.

There are good reasons to suppose a „noise" to have a normal distribution $N(\mu,\sigma^2)$, μ and σ as the population mean and standard deviation, respectively. Then, using the sample statistics (28) and (29), a two-sided $100\cdot(1-\alpha)\%$ confidence interval can be determined as

(30) $$\left(\bar{X}_n-t_{1-\alpha/2,K-1}\frac{s_n}{\sqrt{K}}\ ,\ \bar{X}_n+t_{1-\alpha/2,K-1}\frac{s_n}{\sqrt{K}}\right)$$

based on the student-t distribution with $K-1$ degrees of freedom.

RLGC network simulation results

Examples of deterministic solutions for the RLGC circuit in Fig. 1 are presented in Fig. 2, see respective formulae in Tab. 1, with parameters given by Tab. 2, and $V_i=1\mathrm{V}$.

Table 2. The RLGC network parameters

Case \ Parameter	R [Ω]	L [H]	G [S]	C [F]
underdamped	0.3	1	0.3	1
critically damped	3	1	1	1
overdamped	6	1	1	1

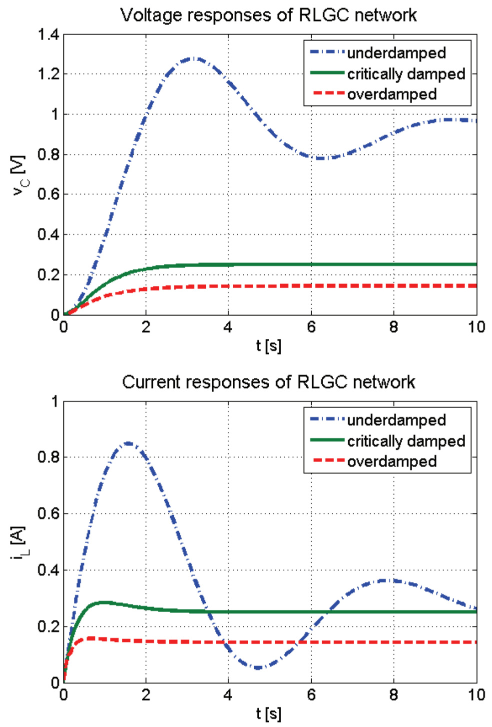

Fig.2. Voltage and current responses of RLGC network

To verify the SDE approach the exciting voltage (13) will now be applied to the RLGC network, while choosing an intensity of the noise $\alpha = 0.15$. The SDE (18) was solved numerically, based on the Euler scheme described above, by utilizing a Matlab language environment.

The stochastic responses are shown in Fig. 3 (voltages), and Fig. 4 (currents), with the sample means and relevant 99% confidence intervals highlighted, compare to the deterministic solutions in Fig. 2. We can clearly see that for a sufficient number of realizations the sample mean curves are approaching theoretical waveforms and respective confidence intervals are becoming narrower, as expected. It is interesting to notice that the impact of the noise on the voltage $v_C(t)$ is weaker than on the current $i_L(t)$. It is seen in (16) - (17) that $v_C(t)$ is affected implicitly only by the Wiener process, just via $i_L(t)$. Beside this network forms a low-pass filter suppresing high-frequency components on its output.

Conclusions

The second-order RLGC network analyzed in this paper plays an important role in building higher-order lumped-parameter models for transmission structures simulation in high-speed electronic systems. Namely, when connecting a sufficient number of such RLGC blocks in cascade one can get the transmission line model required. Especially, in case of transmission paths on semiconductor substrates small random changes of their parameters occur due to a number of various physical factors [8, 9]. That is why the SDE approach has its real foundation and is an interesting alternative to other probabilistic methods introduced to the electronic circuits simulation, see e.g. [13-15].

Taking into account the above perspectives, our further investigations will be focused on a few directions. First, it is necessary to develop SDE solutions for the 2nd-order RLGC network with randomly changed individual parameters R, L, G and C, and under their simultaneous actions. Finaly, the higher-order TL models should be developed and studied in this respect, considering MTLs models as well [10-12].

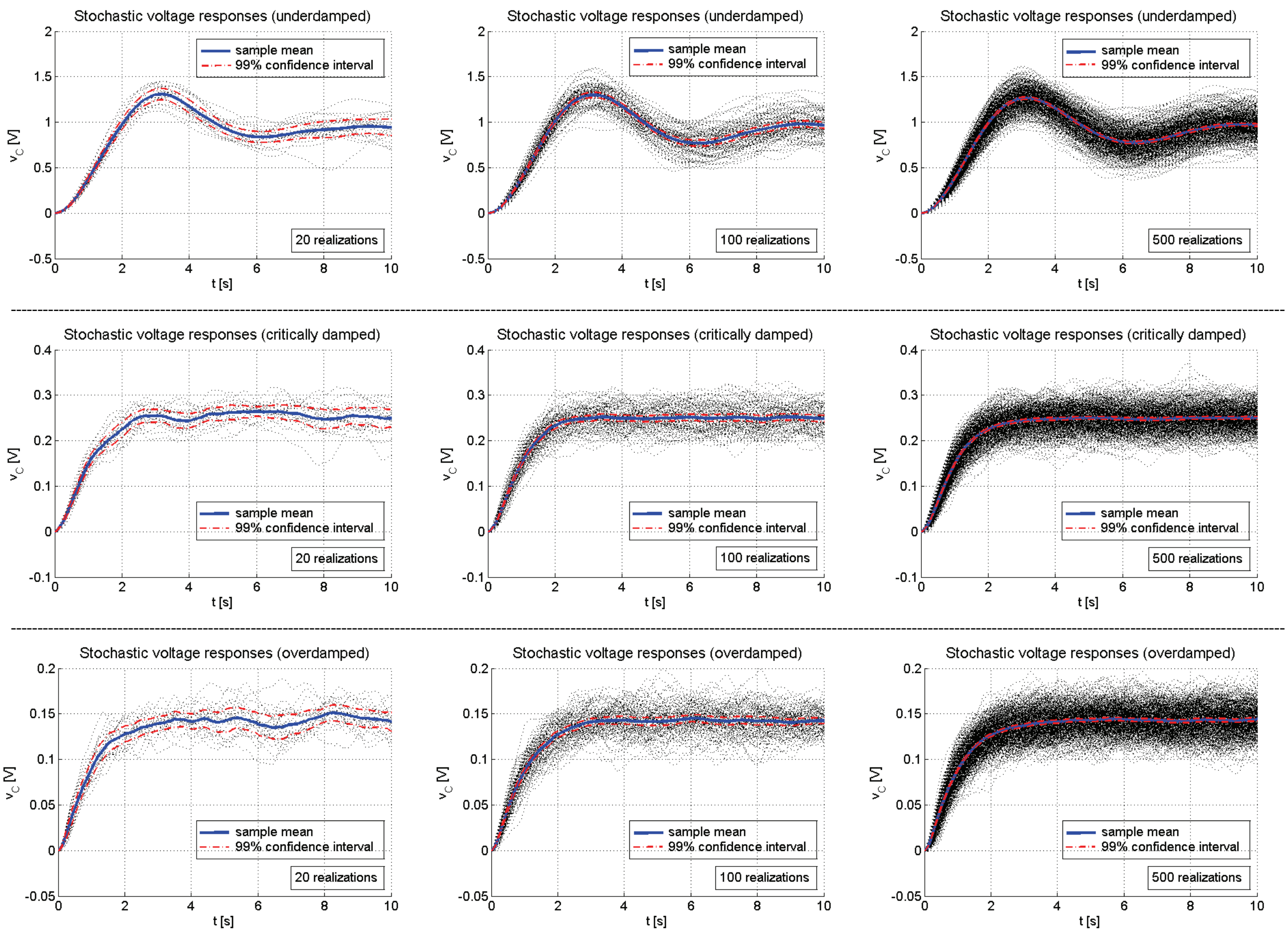

Fig.3. Stochastic voltage responses of RLGC network

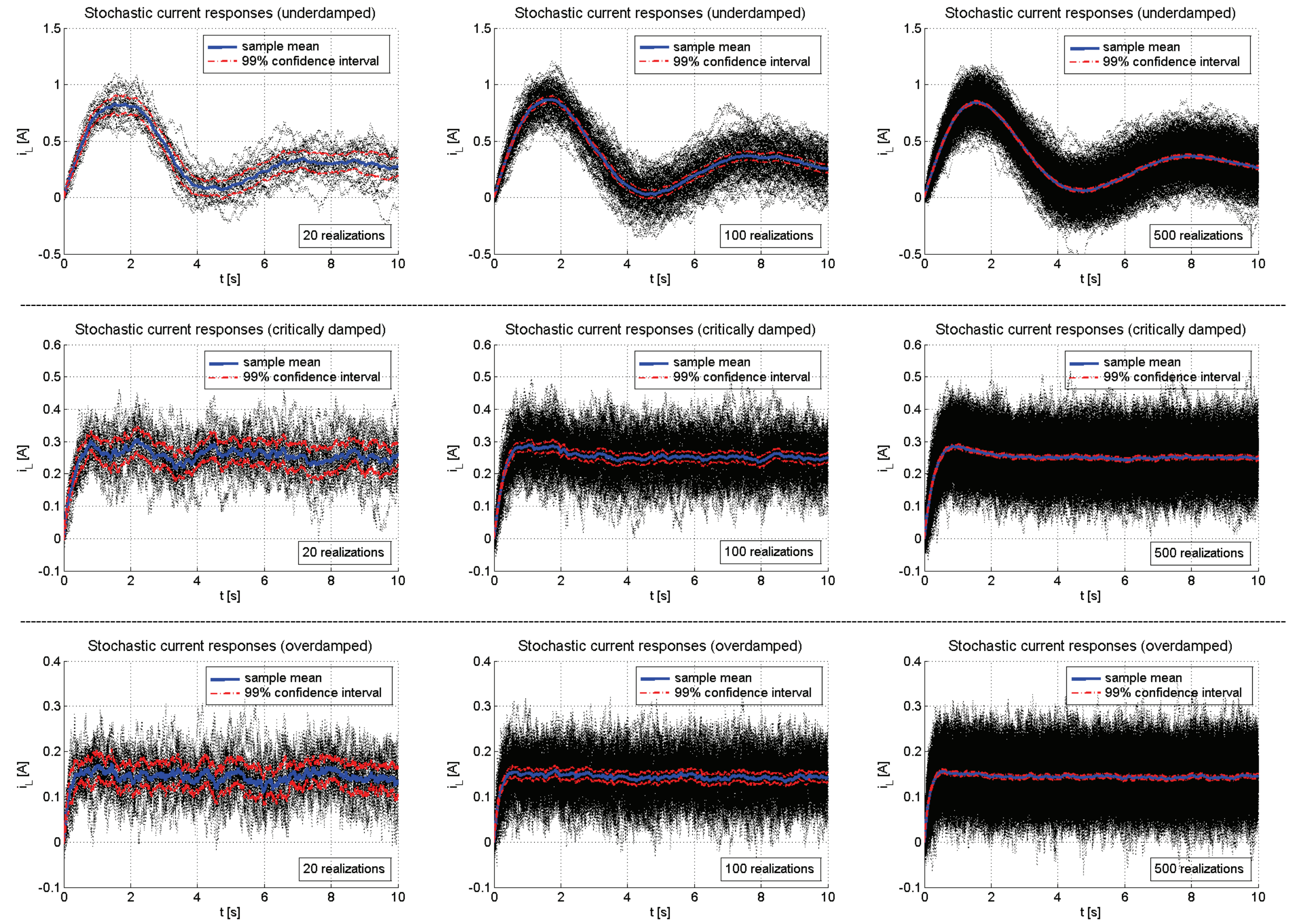

Fig.4. Stochastic current responses of RLGC network

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