Stefan F. FILIPOWICZ[1,2], Tomasz RYMARCZYK

Warsaw University of Technology, Institute of the Theory of Electrical Engineering, Measurement and Information Systems (1), Electrotechnical Institute, Warszawa (2)

Measurement Methods and Image Reconstruction in Electrical Impedance Tomography

***Abstract**. In this work was shown a method to examine the non-destructive analysis defects of the objects by solving the inverse problem in the electrical impedance tomography. The measurements use point-like electrodes at the boundary of the object. By two of these current are injected. The representation of the shape of the boundary and its evolution during an iterative reconstruction process is achieved by the level set function and Chan-Vese model. The forward problem was solved by the finite element method.*

***Streszczenie**. W pracy zaprezentowano dwie metody pomiarów tomograficznych wybranych obiektów oraz rekonstrukcje obrazów ich nieznanych właściwości w wyniku rozwiązania zagadnienia odwrotnego. Algorytm rekonstrukcji oparty został na idei zbiorów poziomicowych a do rozwiązania zagadnienia prostego wykorzystano metodę elementów skończonych. Reprezentację wyglądu obrazu badanego obiektu wykonano rozwiązując wielokrotnie zadanie proste tak, aby w procesie iteracyjnym otrzymać rozkład napięć maksymalnie zbliżony do uzyskanego z pomiarów.* (**Metody pomiaru i konstrukcja obrazu metodą zbiorów poziomicowych w tomografii impedancyjnej**).

Keywords: Electrical Impedance Tomography, Finite Element Method, Inverse Problem, Level Set Method
Słowa kluczowe: tomografia impedancyjna, metoda elementów skończonych, zagadnienie odwrotne, metoda zbiorów poziomicowych

Introduction

Electrical impedance tomography is a widely investigated problem with many applications in physical and biological sciences. It is well known that the inverse problem is nonlinear and highly ill-posed. The objection function is minimized (the difference between the potential due to the applied current and the measured potential). The conductivity values in different regions are determined by the finite element method [1]. The representation of the boundary shape and its evolution during an iterative reconstruction process is achieved by the level set method [2,9,11]. Given the boundary, the potential is obtained by solving the Laplace equation in the entire domain. The advection diffusion equation is then solved with the given boundary conditions. The extension methodology discussed earlier is used to build a velocity field through the narrow band, which is then used to update the level set function advancing the void boundary. The idea was also used successfully in the context of the inverse problem. The pioneering work of Osher and Santosa [7] uses the level set method for an inverse problem associated with the shape optimization. This algorithm is the numerical technique which can follow the evolution of interfaces. These interfaces can develop sharp corners, break apart, and merge together. The level set method tracks the motion of an interface by embedding the interface as the zero level set of the signed distance function [5,6]. The motion of the interface is matched with the zero level set of the level set function, and the resulting initial value partial differential equation for the evolution of the level set function resembles a Hamilton-Jacobi equation. Shapes may be easily evaluated, topological changes occur in a natural manner.

Measurements

The current is fed and the voltage is measured through different pairs of electrodes to avoid the error due to the contact impedance. In the following there was described some of the measurement methods that are used. The measurement system was presented in figure 1.

Neighbouring method

In this method the current is applied through neighboring electrodes and the voltage is measured successively from all other adjacent electrode pairs. The application of this method consists N equally spaced electrodes (e.g. 16). The current is first applied through electrodes 1 and 2. The current density is, of course, highest between these electrodes, decreasing rapidly as a function of distance. The voltage is measured successively with electrode pairs 3-4, 4-5, . . . 15-16. All these 13 measurements are independent. Each of them is assumed to represent the impedance between the equipotential lines intersecting the measurement electrodes.

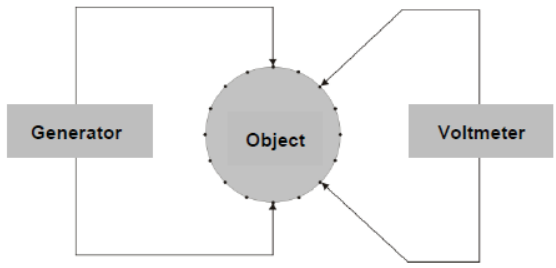


Fig. 1. The measurement system

Split-drive pair

The voltage is measured successively between electrodes relate to one electrode. This method gives only $N(N–4)/2$ independent measurements.

Cross method

A more uniform current distribution is obtained when the current is injected between a pair of more distant electrodes. In the cross method, adjacent electrodes. The voltage is measured successively for all other 13 electrodes with the aforementioned electrode 1 as a reference. The measurement sequence is then repeated using electrodes as current and voltage reference electrodes. From these 182 measurements only 104 are independent. The cross method does not have as good a sensitivity in the periphery as does the neighboring method, but has better sensitivity over the entire region.

Opposite method

Another alternative for the impedance measurement is the opposite method. Another alternative for the impedance measurement is the opposite method. The electrode adjacent to the current-injecting electrode is used as the voltage reference. Voltage is measured from all other electrodes except from the current electrodes, yielding 13 voltage measurements. The current distribution in this method is more uniform and, therefore, has a good sensitivity.

Linear array

There can use the liner array when the area doesn't around by the measurement system. When 16 electrodes are used, the opposite method yields 8×13 = 104 data points. The linear array and equipotential lines by supply current shows figure 2.

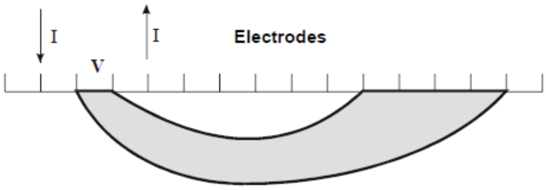


Fig. 2. The linear array and equipotential lines by supply current

Image reconstruction

From the collected data, the image of the distribution of the electric impedance may be constructed by use of certain reconstruction algorithms.

The level set method is known to be a powerful and versatile tool to model the evolution of interfaces. The idea is merely to define a smooth function ϕ, that represents the interface and has the following properties

$$\begin{aligned} \phi(x,t) > 0 \text{ for } x \in \Omega \\ \phi(x,t) < 0 \text{ for } x \notin \Omega \\ \phi(x,t) = 0 \text{ for } x \in \partial\Omega = \Gamma(t) \end{aligned} \tag{1}$$

The interface information such as the tangential and normal the derivatives and the curvature at projections are obtained from the values of the level set function at grid point plus a bilinear interpolations.

Thus, the interface is captured for all later time, by localization of the set Γ(t) for which ϕ vanishes. This deceptively trivial statement is of great significance for numerical computation primarily, because topological changes such as breaking and merging are well defined and performed. The motion is analyzed by the convection the ϕ values (levels) with the velocity field ν. The Hamilton-Jacobi equation of the form:

$$\frac{\partial\phi}{\partial t} + \nu \cdot \nabla\phi = 0 \tag{2}$$

is describing this process.

Here ν is the desired velocity on the interface, and is arbitrary elsewhere. Actually, only the normal component of ν is needed $\nu_N = \nu \cdot \frac{\nabla\phi}{|\nabla\phi|}$, so (2) becomes

$$\frac{\partial\phi}{\partial t} + \nu_N \cdot |\nabla\phi| = 0 \tag{3}$$

In the level set representation, the interface which is the set of points (*x,y*) satisfying $\phi(x,t) = 0$ is not explicitly given. There is only information $\phi(x_i, y_i)$ at each grid point.

The expression for the curvature of the zero level set assigned to the interface itself is given by:

$$\kappa = \nabla \cdot \frac{\nabla\phi}{|\nabla\phi|} = \frac{\phi_{xx}\phi_y^2 - 2\phi_y\phi_x\phi_{xy} + \phi_{yy}\phi_x^2}{\left(\phi_x^2 + \phi_y^2\right)^{3/2}} \tag{4}$$

Update the level set function ϕ (*x,t*) by solving the Hamilton-Jacobi equation:

$$\frac{\phi^{k+1} - \phi^k}{\Delta t} + \nu_k \left|\nabla\phi^k\right| = 0 \tag{5}$$

The objection function is minimized (the difference between the potential due to the applied current and the measured potential).

The following steps are used in numerical algorithm [3,4,8,12]:

- Initialization the zero level set function $\phi(x,y)$ covering the unknown shape,
- Calculate the electric potential (solving the Laplace equation by using Finite Element Method)
- Compute the difference of the computed solution with the observed data
- Solve the adjoint equation
- Find the component of the normal velocity of the surface due to the electric potential
- Find the normal velocity of the level set function
- Calculate the velocity:

$$\nu_k = \nabla\lambda_k \cdot \nabla\varphi \tag{6}$$

where: ν_k – velocity, λ_k – potential of the adjoint equation, $\boldsymbol{\varphi}$ – potential

- Update the level set function:

$$\phi_t^{k+1} = \phi_t^k - \Delta t\left(\nabla\lambda_k \cdot \nabla\varphi\right)\left|\nabla\phi^k\right| \tag{7}$$

where: ϕ - level set function, *t* - time

- Reinitialize the level set function.

Results

Applied the measurement systems and the image reconstruction algorithm to the practical examples such as the tree trunks, the copper-mine ceiling, the moisture wall with different objects.

The copper-mine ceiling

In this model were used 16 electrodes, the opposite method yields 8×13 = 104 data points. The linear array was applied. The idea of the measurements copper-mine ceiling and the image reconstruction by the level set method were shown in fig 3. Assumed: the conductivity of copper mine is 30 [S/m], the air gap is 0,03 [S/m].

The moisture wall

The effectiveness of the image reconstruction by using level set methods was examined for the moisture wall. Real model of the wall with electrodes fixed to both sides of the wall is presented in the fig. 4 and 5. Electrodes are fastened to the wall by special conducting glue, which minimizes contact impedance. Different fastening materials with electrically conductive ingredients (copper, brass, and graphite powders) were tested in the laboratory. Non-repeating contact impedance values of the order of kΩ were obtained with standard masonry mortars and glazing adhesives. Then some adhesives mixtures were used to fasten the electrodes to the wall. The best results were obtained when mixture of epoxy adhesive and graphite powder was used. In this case 30 Ω contact impedance was obtained. This impedance could be neglected in moisture content evaluation as wall impedance is much higher [10].

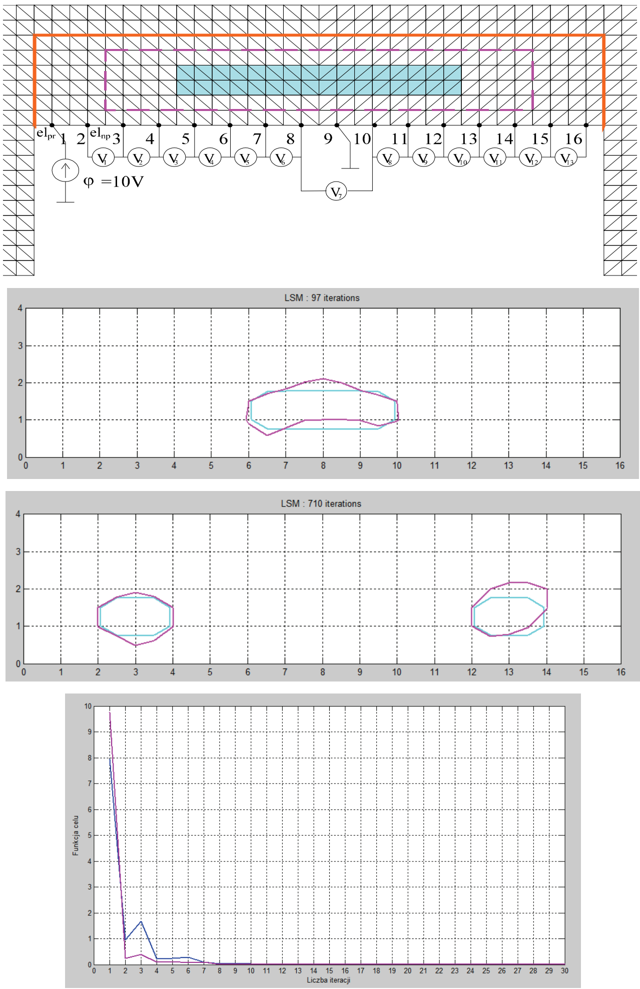


Fig. 3. The idea of the measurements copper-mine ceiling, the image reconstruction by the level set method and the objective function

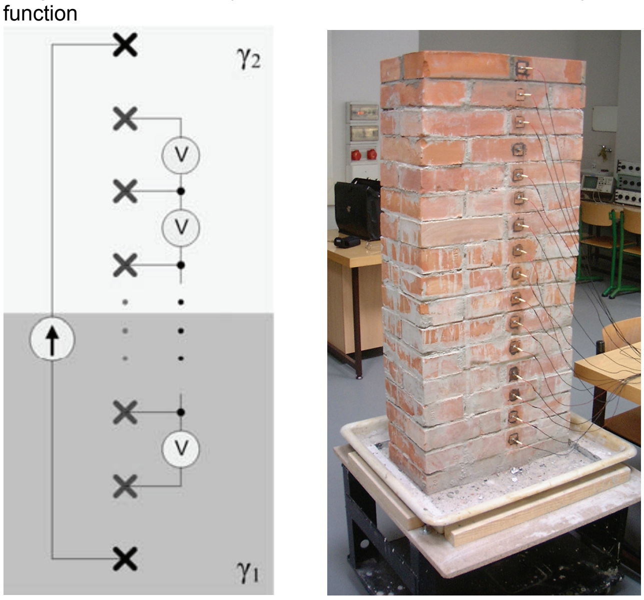


Fig. 4. The measurement system and the real model of the moisture wall

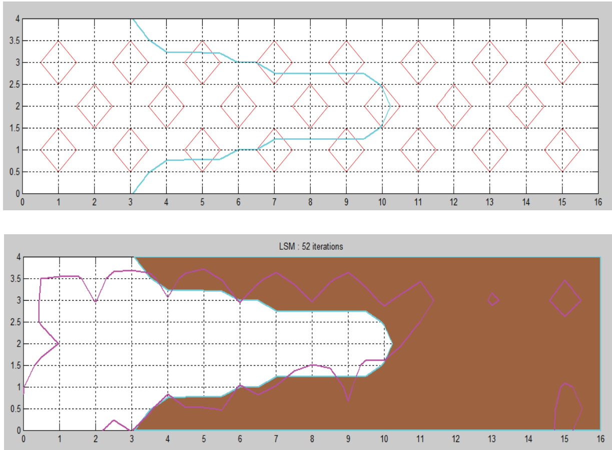


Fig. 5. The simulation of the image reconstruction

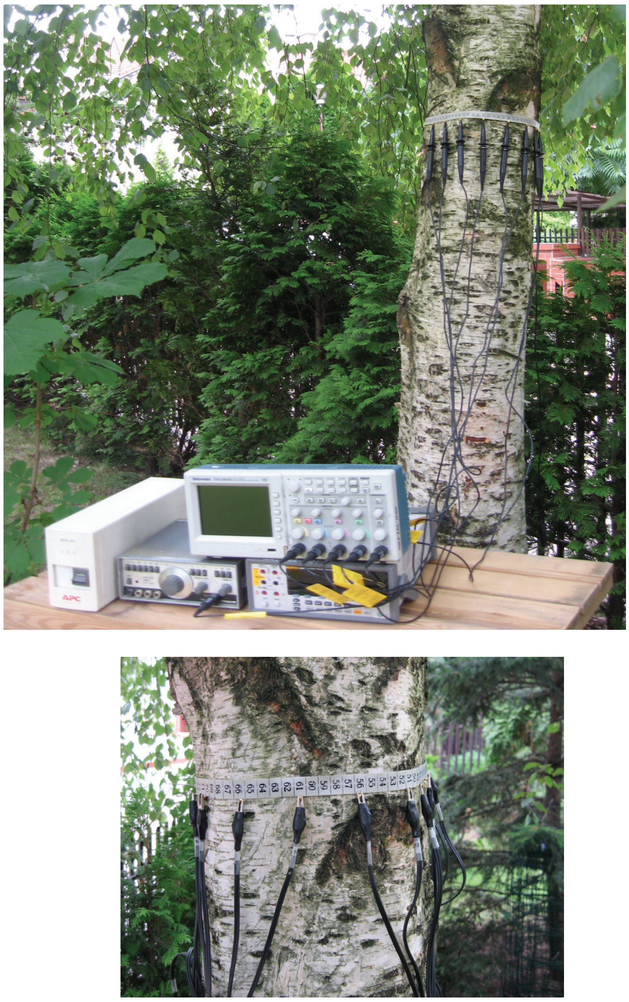


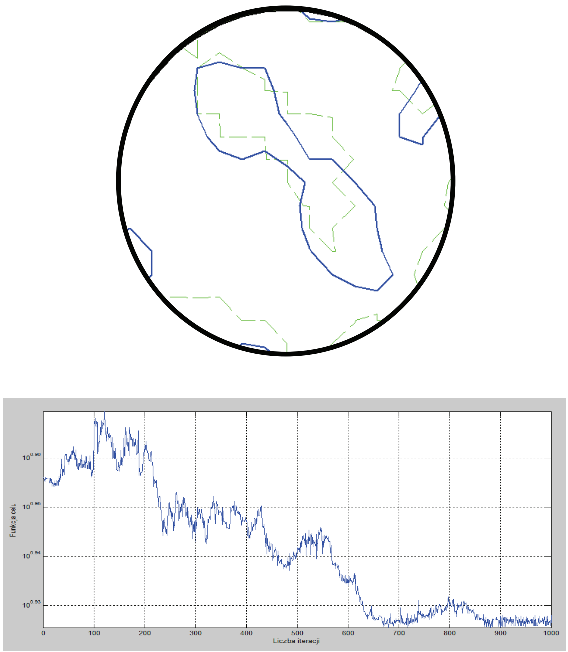
Fig. 6. The measurement stand during the tomography testing of the birch

The tree trunks

The tree trunk was measured (diameter 78 cm on high 130 cm above earth). There were used 16 electrodes, power supply 3,900V, frequency 1kHz. Figure 6 illustrates the application of this method for the tree with equally spaced electrodes.

The figure 7 presents the results of the image reconstruction and the objective function. The pictures show the practical examples of the process reconstruction. The images show the reconstructed shape after the process iterations.

a)



b)

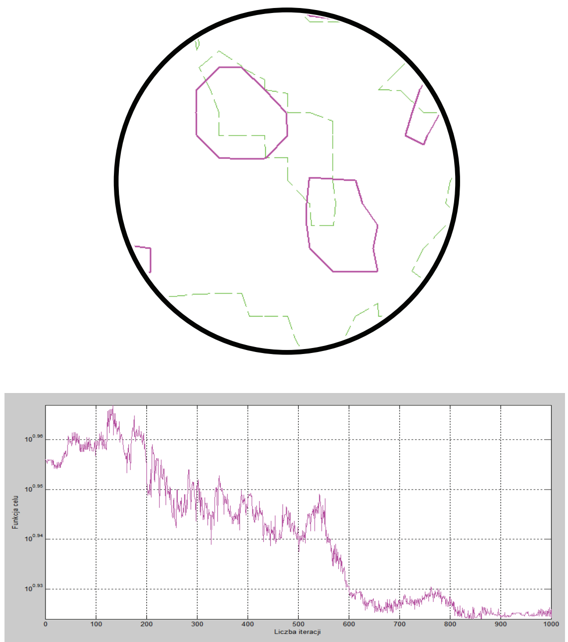


Fig. 7. The result of the image reconstruction and the objective function by the level set method: a) without regularization, b) with regularization

Summary

The image reconstruction was based on the level set methods and the finite element method. The pictures show a few of the practical examples such as the tree trunks, the copper-mine ceiling, the moisture wall with different objects and their process reconstruction. It is sometimes more advantageous to recover the shape of the domains containing different materials than to recover the parameters of the materials. The level set methods can produce good results in identifying the sharp interfaces. This approach is, therefore, useful for solution of the practical problems. The presented techniques were shown to be successful to identify unknown boundary shapes.

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***Authors**: Stefan F. FILIPOWICZ, Warsaw University of Technology, Institute of the Theory of Electrical Engineering, Measurement and Information Systems, Pl. Politechniki 1, 00-661 Warszawa, e-mail: 2xf@nov.iem.pw.edu.pl*
Tomasz RYMARCZYK, Net-art,
ul. Lotnicza 3,20-322 Lublin, e-mail: tomasz@rymarczyk.com