Abderrahmane KACIMI[1], Abdellah MOKHTARI[2], Benatman KOUADRI[2]

University of Oran, Algeria (1), University of Science and Technology MB, Oran, Algeria (2)

Sliding Mode Control Based on Adaptive Backstepping Approch for a Quadrotor Unmanned Aerial Vehicle

Abstract. *A sliding mode control based on adaptive backstepping approch, is developped respectively in order to synthesise tracking errors and to ensure Lyapunov stability, handling of all system nonlinearities and desired tracking trajectories. Under ground effects and wind disturbances, the body inertia becomes badly known , and non parametric uncertainties are considered in the system model. Dynamic modelling of quadrotor takes into account the high-order nonholonomic constraints, that are considered here in order to test this new control scheme on a model that takes into account the various physical phenomenas, which can influence the dynamics of a flying structure. Finally simulation results are provided to illustrate the performances of the proposed controller.*

Streszczenie. *Przedstawiono system sterowania ślizgowego z modulem typu backstepping w zastosowaniu do bezzalogowego pojazdu powietrznego (helikoptera). Zaproponowany system sterowania pozwala na zmniejszenie błędu trasy i zapewnia stabilność z uwzględnieniem nieliniowości, zmian wiatru i inercji pojazdu. Model dynamiczny uwzglednia różne możliwe fizyczne zjawiska. (Sterowanie slizgowe typu adaptacyjny backstepping w zastosowaniu do bezzalogowego pojazdu powietrznego).*

Keywords: Tuning function, Nonholonomic constraints, Adaptive backstepping, Sliding mode.
Słowa kluczowe: Slowa kluczowe: sterowanie śliozgowe, helicopter bezzalogowy, backstepping.

Introduction

Unmanned aerial vehicles are characterized by high level of complexity, high dimension of the state space, multiple inputs and outputs, parametric uncertainty, unmodeled dynamics. Its inertia will slowly vary as a result of wind disturbances and ground effects. We need a control law that adapts itself to such changing conditions. Affected by aerodynamic forces, the Quadrotor dynamics is nonlinear, multivariable, and is subject to parameter uncertainties and external disturbances. In turn, controlling of the Quadrotor is required i) to meet the stability, robustness and desired dynamic properties. ii) to be able to handle nonlinearity. iii) to be adaptive to changing parameters and environmental disturbances. The adaptive control deals with system which the parameters are slowly time-varying or uncertain. The adaptive backstepping control has been widely studied in the two last decade [3], [13], [9], [11], [10], [12], and [14]. In the latter newly developed strategies are presented. The designed controllers are shown to guarantee all signals bounded in the system and yield good transient and tracking performance.Based on backstepping approch,this methode use the tunning functions for updating the unknown parameter to be estimated and overcome the overparametrisation problem. An adaptive controller is designed by combining a parameter estimator, which provides the unknown parameter estimation, with a control law. The parameters of the controller are adjusted during the operation of the plant. Affected by wind aerofynamic force and wind aerodynamic torque, the control scheme is reinforced by sliding mode control, based on adaptive backstepping approch. This control is developped by taking error of virtual controls as sliding surface. The same strategy used into [15] is followed, except the backstepping approch is remplaced by adaptive backstepping approch, while taking into account the high-order nonholonomic constraints. This method presents the following major advantages: i)It ensures Lyapunov stability. ii)It ensures the robustness and all properties of the desired dynamics. iii)It ensures the handling of all system nonlinearities.

In this paper, the dynamic modelling of system [4] with the nonholonomic constraints taked into account[15] and neglecting a Coriolis torque and a gyroscopic torque [1] are presented in section II. Theorical background of adaptive backstepping approch and his application to quadrotor system are developped in section III. In section IV, sliding mode control, based on adaptive backstepping approch of the quadrotor system is developped. Simulation results are presented in Section V and a conclusion is drawn in Section VI.

Dynamical Model of X4-Flyer

Dynamics of X4-Flyer

The coordinate system and X4-flyer configuration are shown in (Fig. 1).

Let $E = \{E_x\ E_y\ E_z\}$ denote a right-hand inertial frame such that E_z denotes the vertical direction downwards into the earth. Let the vector $\zeta = [x\ y\ z]^T$ denote the position of the centre of mass of the airframe in the frame E relative to a fixed origin $O \in E$.

Let c be a (right-hand) body fixed frame for the airframe. When defining the rotational angles $\eta = [\phi\ \theta\ \psi]^T$ around X-, Y -, and Z-axis in the frame c, the orientation of the rigid body is given by a rotation $R : c \rightarrow E$, where $R \in \Re^{3\times 3}$ is an orthogonal rotation matrix given by [1]

$$(1)\quad R = \begin{pmatrix} c\theta c\psi & s\phi s\theta c\psi - c\phi s\psi & c\phi s\theta c\psi + s\phi s\psi \\ c\theta s\psi & s\phi s\theta s\psi + c\phi c\psi & c\phi s\theta s\psi - s\phi c\psi \\ -s\theta & s\phi c\theta & c\phi c\theta \end{pmatrix}$$

Let J denote the inertia matrix around the center of mass (expressed in the body fixed frame c), where the moment of inertia around each axis is given by I_x, I_y and I_z. Moreover, let f_i denote the thrust generated by the rotor i in free air (expressed in c), m is the mass of the airframe, g denotes the acceleration due to gravity, and l denotes the distance from the rotors to the center of gravity of the airframe. We present a dynamical model of the X4-flyer using a Lagrangian approach. The translational kinetic energy of the X4-flyer will be

$$(2)\qquad T_{trans} = \frac{1}{2} m\, \dot{\zeta}^T \dot{\zeta}$$

The rotational kinetic energy is

$$(3)\qquad T_{rot} = \frac{1}{2}\, \dot{\eta}^T J\, \dot{\eta}$$

with $J = diag\{I_x, I_y, I_z\}$ And the only potential energy which needs to be considered is the standard gravitational

[1]where c denote cos and s denote sin

potential given by

$$U = -mgz \tag{4}$$

The Lagrangian is given by

$$\begin{aligned} L &= T_{trans} + T_{rot} - U \\ &= \tfrac{1}{2} m\, \dot{\zeta}^T \dot{\zeta} + \tfrac{1}{2}\, \dot{\eta}^T J\, \dot{\eta} + mgz \end{aligned} \tag{5}$$

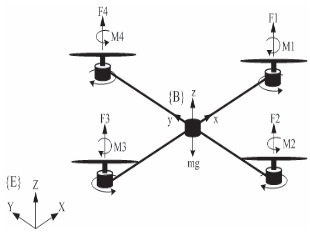

Fig. 1. Coordinate system of X4-Flyer

Defining the generalized coordinate as $q = [\zeta\ \eta]^T$, the model for the full X4-flyer dynamics is obtained from the Euler-Lagrange equations with external generalized force F

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = F \tag{6}$$

$F = [F_\zeta + F_t\ \ \tau]^T$, Where F_ζ is the translational force applied to the X4-flyer due to the control inputs, F_t is the resultant of the drag forces along (X, Y, Z) axis and $\tau = [\tau_\phi\ \tau_\theta\ \tau_\psi]^T$ is the generalized moments around the airframe. Letting the control input as the translational force be defined as

$$u_1 = f_1 + f_2 + f_3 + f_4$$

And the unit vector of Z-direction in the airframe be $e_3 = [0\ 0\ 1]^T$,it yields that

$$\begin{aligned} F_\zeta &= -Re_3 u_1 \\ &= -u_1 \begin{bmatrix} c\phi s\theta c\psi + s\phi s\psi \\ c\phi s\theta s\psi - s\phi c\psi \\ c\phi c\theta \end{bmatrix} \end{aligned} \tag{7}$$

$$F_t = \begin{pmatrix} -K_{ftx} & 0 & 0 \\ 0 & -K_{fty} & 0 \\ 0 & 0 & -K_{ftz} \end{pmatrix} \dot{\zeta} \tag{8}$$

Such as K_{ftx}, K_{fty} and K_{ftz} are the translation drag coefficients. and τ becomes

$$\tau_\phi = (f_2 - f_4)\, l$$

$$\tau_\theta = (f_1 - f_3)\, l$$

$$\tau_\psi = \sum_{i=1}^{4} \tau_{M_i}$$

$$J\dot{\omega} = \omega \times J\omega + \sum_{i=1}^{4} J_r (\omega \times e_3)\, \omega_i + \tau \tag{9}$$

Where $\omega = \begin{bmatrix} \dot{\phi} & \dot{\theta} & \dot{} \end{bmatrix}^T$, J_r denotes the moment of inertia for the rotor, ω_i is the rotational speed of the rotor i.
Neglecting a term consisting of a Coriolis torque and a gyroscopic torque, the final dynamics of X4 flyer can be reduced to

$$m\ddot{x} = (\cos\phi \sin\theta \cos\psi + \sin\phi \sin\psi)\, u_1 - K_{ftx}\, \dot{x} \tag{10}$$

$$m\ddot{y} = (\cos\phi \sin\theta \sin\psi - \sin\phi \cos\psi)\, u_1 - K_{fty}\, \dot{y} \tag{11}$$

$$m\ddot{z} = (\cos\phi \cos\theta)\, u_1 - mg - K_{ftz}\, \dot{z} \tag{12}$$

$$I_x \ddot{\phi} = \dot{\theta}\ \dot{}\ (I_y - I_z) + l u_2 \tag{13}$$

$$I_y \ddot{\theta} = \dot{\phi}\ \dot{}\ (I_z - I_x) + l u_3 \tag{14}$$

$$I_z\ \ddot{}\ = \dot{\phi}\,\dot{\theta}\,(I_x - I_y) + u_4 \tag{15}$$

Where $f_i = -b\omega_i^2$ (b is a thrust factor),$\tau_{M_i} = d\omega_i^2$ (d is a drag factor), and Ω,u_1,u_2,u_3 and u_4 are respectively given by

$$\Omega = (\omega_2 + \omega_4 - \omega_1 - \omega_3) \tag{16}$$

$$u_1 = f_1 + f_2 + f_3 + f_4 = b\left(\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2\right) \tag{17}$$

$$u_2 = f_2 - f_4 = b\left(\omega_2^2 - \omega_4^2\right) \tag{18}$$

$$u_3 = f_1 - f_3 = b\left(\omega_1^2 - \omega_3^2\right) \tag{19}$$

$$u_4 = \sum_{i=1}^{4} \tau_{M_i} = d\left(-\omega_2^2 - \omega_4^2 + \omega_1^2 + \omega_3^2\right) \tag{20}$$

Assuming that the electric rotors are velocity controlled then (u_1,u_2,u_3 and u_4) may be considered directly as control inputs.

Nonholonomic Constraints

Taking into account nonholonomic constraints which define the coupling between various states of the system. From the equations (10), (11)and (12) we can extract the expressions of the high-order nonholonomic constraints:

(21)

$$\begin{cases} \tan\theta = \dfrac{\left(\ddot{x} + \frac{K_{ftx}}{m}\dot{x}\right)\cos\psi + \left(\ddot{y} + \frac{K_{fty}}{m}\dot{y}\right)\sin\psi}{\ddot{z} + g + \frac{K_{ftz}}{m}\dot{z}} \\ \sin\phi = \dfrac{\left(\ddot{x} + \frac{K_{ftx}}{m}\dot{x}\right)\sin\psi + \left(\ddot{y} + \frac{K_{fty}}{m}\dot{y}\right)\cos\psi}{\sqrt{\left(\ddot{x} + \frac{K_{ftx}}{m}\dot{x}\right)^2 + \left(\ddot{y} + \frac{K_{fty}}{m}\dot{y}\right)^2 + \left(\ddot{z} + g + \frac{K_{ftz}}{m}\dot{z}\right)^2}} \end{cases}$$

Adaptive backstepping

Theorical background

In this section, we will consider unknown parameters which appear linearly in system equations[14]. An adaptive controller is designed by combining a parameter estimator, which provides estimates of unknown parameters, with a control law. The parameters of the controller are adjusted during the operation of the plant. In the presence of such parametric uncertainties, the adaptive controller is able to ensure the boundedness of the closed-loop states and asymptotic tracking.
The adaptive backstepping control with tuning function is considered here. Let the following sytem:

(22)

$$\begin{aligned} \dot{x_{n-1}} &= x_n + \phi_{n-1}^T (x_1, ...x_{n-1})\, \theta + \psi_n (x_1, ...x_{n-1}) \\ \dot{x_n} &= bu + \phi_n^T (x)\, \theta + \psi_n (x) \end{aligned}$$

Where $x = \left[\begin{array}{cccc} x_1, & . & . & . \;\; , x_{n-1} \end{array}\right]^T \in R^n$, the vector $\theta \in R^r$ is constant and unknown, $\phi_i \in R^r$, $\psi_i \in R$, $i = 1, \ldots n$ are known nonlinear functions and b is unknown constant gain.

The control objective is to force the output x_1 to asymptotically track the reference signal x_r with the following assumptions.

Assumption 1 : The sign of b is known.

Assumption 2 : The reference signal x_r and its n order derivatives are piecewise continuous and bounded.

For system (22), the number of design steps required is equal to n. At each step, an error variable z_i, a stabilizing function α_i and a tuning function τ_i are designed.

Step1 :

$$\begin{aligned} & z_1 = x_1 - x_r \\ & \dot{z_1} = \dot{x_1} - \dot{x_r} = z_2 + \alpha_1 + \phi_1^T \widehat{\theta} + \psi_1 \\ & \alpha_1 = -c_1 z_1 - \phi_1^T \widehat{\theta} - \psi_1 \\ & \tau_1 = \phi_1 z_1 \\ & V_1 = \tfrac{1}{2} z_1^2 \end{aligned}$$

Step $i : i = 2, \ldots n$

$$\begin{aligned} & \dot{z_i} = z_{i+1} + \alpha_i + \psi_i - \sum_{j=1}^{i-1} \tfrac{\partial \alpha_{i-1}}{\partial x_j} \left(x_{j+1} + \psi_j\right) \\ & +\theta^T \left(\phi_i - \sum_{j=1}^{i-1} \tfrac{\partial \alpha_{i-1}}{\partial x_j} \phi_j \right) - \tfrac{\partial \alpha_{i-1}}{\partial \widehat{\theta}} \Gamma \tau_i \\ & + \left(\sum_{j=2}^{i-1} z_j \tfrac{\partial \alpha_{j-1}}{\partial \widehat{\theta}} \right) \Gamma \left(\phi_i - \sum_{j=1}^{i-1} \tfrac{\partial \alpha_{i-1}}{\partial x_j} \phi_j \right) + \\ & \sum_{j=1}^{i-1} \tfrac{\partial \alpha_{i-1}}{\partial x_r^{(j-1)}} x_r^{(j)} \end{aligned} \tag{23}$$

We select the stabilizing function α_i

$$\begin{aligned} & \alpha_i = -c_i z_i - z_{i-1} - \psi_i + \sum_{j=1}^{i-1} \tfrac{\partial \alpha_{i-1}}{\partial x_j} \left(x_{j+1} + \psi_j\right) \\ & -\theta^T \left(\phi_i - \sum_{j=1}^{i-1} \tfrac{\partial \alpha_{i-1}}{\partial x_j} \phi_j \right) + \tfrac{\partial \alpha_{i-1}}{\partial \widehat{\theta}} \Gamma \tau_i \\ & + \left(\sum_{j=2}^{i-1} z_j \tfrac{\partial \alpha_{j-1}}{\partial \widehat{\theta}} \right) \Gamma \left(\phi_i - \sum_{j=1}^{i-1} \tfrac{\partial \alpha_{i-1}}{\partial x_j} \phi_j \right) \\ & + \sum_{j=1}^{i-1} \tfrac{\partial \alpha_{i-1}}{\partial x_r^{(j-1)}} x_r^{(j)} \end{aligned} \tag{24}$$

And tuning function

$$\tau_i = \tau_{i-1} + \left(\phi_i - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j} \phi_j \right) z_i \tag{25}$$

In the last step n, the actual control input u appears and is at our disposal. We derive the z_n dynamics

$$\begin{aligned} & \dot{z_n} = bu + \psi_n - \sum_{j=1}^{n-1} \tfrac{\partial \alpha_{i-1}}{\partial x_j} \left(x_{j+1} + \psi_j\right) \\ & +\theta^T \left(\phi_n - \sum_{j=1}^{n-1} \tfrac{\partial \alpha_{i-1}}{\partial x_j} \phi_j \right) - \tfrac{\partial \alpha_{n-1}}{\partial \widehat{\theta}} \dot{\widehat{\theta}} \\ & - \sum_{j=1}^{n-1} \tfrac{\partial \alpha_{i-1}}{\partial x_r^{(j-1)}} x_r^{(j)} - x_r^{(n)} \end{aligned} \tag{26}$$

We are finally in this position to design control u and update laws $\dot{\widehat{\theta}}$ and $\dot{\widehat{p}}$ as

$$u = \widehat{p} \overline{u} \tag{27}$$

$$\overline{u} = \alpha_n + x_r^{(n)} \tag{28}$$

$$\dot{\widehat{\theta}} = \Gamma \tau_n \tag{29}$$

$$\dot{\widehat{p}} = -\gamma sign\left(b\right) \overline{u} z_n \tag{30}$$

Where γ is a positive constant and $\widehat{p}$ is an estimate of $p = 1/b$. Note that

$$bu = b \widehat{p} \overline{u} = \overline{u} - b \widetilde{p} \overline{u} \tag{31}$$

Where $\widetilde{p} = p - \widehat{p}$. We choose the Lyapunov function

$$V_n = V_{n-1} + \frac{|b|}{2\gamma} \widetilde{p}^2 = \sum_{i=1}^{n} \frac{1}{2} z_i^2 + \frac{1}{2} \widetilde{\theta}^T \Gamma^{-1} \widetilde{\theta} + \frac{|b|}{2\gamma} \widetilde{p}^2 \tag{32}$$

Where γ is a positive design parameter. Then its derivative is given by

$$\begin{aligned} & \dot{V_n} = -\sum_{i=1}^{n} c_i z_i^2 + \left(\sum_{j=2}^{n} z_j \tfrac{\partial \alpha_{j-1}}{\partial \widehat{\theta}} \right) \left(\Gamma \tau_n - \dot{\widehat{\theta}} \right) \\ & +\widetilde{\theta}^T \left(\tau_n - \Gamma^{-1} \dot{\widehat{\theta}} \right) - \tfrac{|b|}{\gamma} \widetilde{p} \left(\dot{\widehat{p}} + \gamma sign\left(b\right) \overline{u} z_n \right) \\ & = -\sum_{i=1}^{n} c_i z_i^2 \leq 0 \end{aligned} \tag{33}$$

The controller designed in this section achieves the goals of stabilization and tracking. The proof of these properties is a direct consequence of the recursive procedure, because a Lyapunov function is constructed for the entire system including the parameter estimates. The overparametrization problem is overcomed by using tuning functions. The number of parameter estimates are equal to the number of unknown parameters.

Adaptive backstepping control of the quadrotor

The model (10) to (15) developed in the previous section can be rewritten in the state-space form (22) with six subsystems and wind disturbances are considered. Let

$$\begin{aligned} & X = \left[\begin{array}{cccccccccccc} x & \dot{x} & y & \dot{y} & z & \dot{z} & \phi & \dot{\phi} & \theta & \dot{\theta} & \psi & \dot{\psi} \end{array} \right]^T \\ & X = \left[x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}\right]^T \\ & u = \left[u_1, u_2, u_3, u_4\right]^T \end{aligned}$$

$$\begin{aligned} & \dot{x_1} = x_2 \\ & \dot{x_2} = U_x \tfrac{u_1}{m} - a_1 x_2 + \tfrac{A_x}{m} \\ & \dot{x_3} = x_4 \\ & \dot{x_4} = U_y \tfrac{u_1}{m} - a_2 x_4 + \tfrac{A_y}{m} \\ & \dot{x_5} = x_6 \\ & \dot{x_6} = \cos\left(x_7 x_9\right) \tfrac{U_1}{m} - g - a_3 x_6 + \tfrac{A_z}{m} \\ & \dot{x_7} = x_8 \\ & \dot{x_8} = \theta_1 x_{10} x_{12} + b_1 u_2 + \tfrac{A_p}{I_x} \\ & \dot{x_9} = x_{10} \\ & \dot{x_{10}} = \theta_2 x_8 x_{12} + b_2 u_3 + \tfrac{A_q}{I_y} \\ & \dot{x_{11}} = x_{12} \\ & \dot{x_{12}} = \theta_3 x_8 x_{10} + b_3 u_4 + \tfrac{A_r}{I_z} \end{aligned} \tag{34}$$

With

$$\begin{array}{l} a_1 = \frac{K_{ftx}}{m} \quad a_2 = \frac{K_{fty}}{m} \quad a_3 = \frac{K_{ftz}}{m} \\ b_1 = \frac{l}{I_x} \quad b_2 = \frac{l}{I_y} \quad b_3 = \frac{1}{I_y} \\ \theta_1 = \frac{(I_y - I_z)}{I_x} \quad \theta_2 = \frac{(I_x - I_z)}{I_y} \quad \theta_3 = \frac{(I_x - I_y)}{I_z} \\ U_x = (\cos x_7 \sin x_9 \cos x_{11} + \sin x_7 \sin x_{11}) \\ U_y = (\cos x_7 \sin x_9 \sin x_{11} - \sin x_7 \cos x_{11}) \end{array}$$

Where assuming, values of inertias, unknown and constant or slowly varing in time, then:
θ_1, θ_2 and θ_3 are the unknown parameters but constant.
b_1, b_2 and b_3 are constant gains unknown with a known sign.
$\begin{pmatrix} A_x & A_y & A_z \end{pmatrix}^T$ and $\begin{pmatrix} A_p & A_q & A_r \end{pmatrix}^T$ are resulting aerodynamic forces and moments acting on the UAV, and are computed from the aerodynamic cefficients C_i as
$A_i = (1/2)\,\rho_{air} C_i W^2$
Where ρ_{air} is the air density and W is the velocity of the UAV with respect to air.
Using the adaptive backstepping approach as a recursive algorithm for the control-laws synthesis, the following control and adaptation laws are obtained.
(35)

$$\begin{cases} U_x = \frac{m}{u_1}\left(-z_1 - c_2 z_2 + a_1 x_2 - c_1\left(x_2 - \dot{x_{1d}}\right) + \ddot{x_{1d}}\right) \\ U_y = \frac{m}{u_1}\left(-z_3 - c_4 z_4 + a_2 x_4 - c_3\left(x_4 - \dot{x_{3d}}\right) + \ddot{x_{3d}}\right) \\ With\ \ u_1 \neq 0 \\ u_1 = \frac{m}{D}\begin{pmatrix} -z_5 - c_6 z_6 + a_3 x_6 + g - c_5\left(x_6 - \dot{x_{5d}}\right) \\ +\ddot{x_{5d}} \end{pmatrix} \\ \overline{u_2} = \left(-z_7 - c_8 z_8 - \widehat{\theta_1} x_{10} x_{12} - c_7\left(x_8 - \dot{x_{7d}}\right) + \ddot{x_{7d}}\right) \\ u_2 = \widehat{p_1}\,\overline{u_2} \\ \overline{u_3} = \begin{pmatrix} -z_9 - c_{10} z_{10} - \widehat{\theta_2} x_8 x_{12} - c_9\left(x_{10} - \dot{x_{9d}}\right) \\ +\ddot{x_{9d}} \end{pmatrix} \\ u_3 = \widehat{p_2}\,\overline{u_3} \\ \overline{u_4} = \begin{pmatrix} -z_{11} - c_{12} z_{12} - c_{11}\left(x_{12} - \dot{x_{11d}}\right) \\ -\widehat{\theta_3} x_8 x_{10} + \ddot{x_{11d}} \end{pmatrix} \\ u_4 = \widehat{p_3}\,\overline{u_4} \end{cases}$$

With

$$D = cx_7 cx_9 \ and\ c_i > 0 \quad for \quad i = 1, ..., 12.$$

U_x and U_y are the effort of the control input u_1 along the x-axis and along the y-axis respectively.
The desired trajectories x_{7d} of ϕ angle and x_{9d} of θ angle are extracted from the equations (21) of nonholonomic constraints.
Adaptation laws :

(36) $$\dot{\widehat{\theta_1}} = \Gamma_1 \tau_8, \quad \dot{\widehat{\theta_2}} = \Gamma_2 \tau_{10}, \quad \dot{\widehat{\theta_3}} = \Gamma_3 \tau_{12},$$

Where Γ_1, Γ_2 and Γ_3 are the adaptation gain and with the tunning functions :

$$\tau_i = 0 \quad i = 7, 9, 11$$

(37) $$\begin{array}{l} \tau_8 = \phi_8 z_8 = x_{10} x_{12} z_8 \\ \tau_{10} = \phi_{10} z_{10} = x_8 x_{12} z_{10} \\ \tau_{12} = \phi_{12} z_{12} = x_8 x_{10} z_{12} \end{array}$$

(38) $$\begin{array}{l} \dot{\widehat{p_1}} = -\gamma_1 sign\,(b_1)\,\overline{u_2} z_8 \\ \dot{\widehat{p_2}} = -\gamma_2 sign\,(b_2)\,\overline{u_3} z_{10} \\ \dot{\widehat{p_3}} = -\gamma_3 sign\,(b_3)\,\overline{u_4} z_{12} \end{array}$$

For robustness oposite the non parametric uncertainties (wind disturbances) the sliding mode control is developed in next section.

Sliding mode control of the quadrotor

The choice of the sliding surfaces is based upon the synthesized tracking errors which permitted the synthesis of stabilizing control laws [15], so from (23) we define :

(39) $$\begin{cases} S_x = z_2 = x_2 - \dot{x_{1d}} - c_1 z_1 \\ S_y = z_4 = x_4 - \dot{x_{3d}} - c_3 z_3 \\ S_z = z_6 = x_6 - \dot{x_{5d}} - c_5 z_5 \\ S_\phi = z_8 = x_8 - \dot{x_{7d}} - c_7 z_7 \\ S_\theta = z_{10} = x_{10} - \dot{x_{9d}} - c_9 z_9 \\ S_\psi = z_{12} = x_{12} - \dot{x_{11d}} - c_{11} z_{11} \end{cases}$$

Such as S_x, S_y, S_z, S_ϕ, S_θ and S_ψ are the dynamic sliding surfaces. To synthesize a stabilizing control law by sliding mode, the necessary sliding condition $(S\,\dot{S} < 0)$ must be verified, so the synthesized stabilizing control laws are as follows:

(40)

$$\begin{cases} U_x = \frac{m}{u_1}\begin{pmatrix} -k_1 sign\,(S_x) - c_2 S_x - a_1 x_2 \\ -c_1\left(x_2 - \dot{x_{1d}}\right) + \ddot{x_{1d}} \end{pmatrix} \\ U_y = \frac{m}{u_1}\begin{pmatrix} -k_2 sign\,(S_y) - c_4 S_y - a_2 x_4 \\ -c_3\left(x_4 - \dot{x_{3d}}\right) + \ddot{x_{3d}} \end{pmatrix} \\ With \quad u_1 \neq 0 \\ u_1 = \frac{m}{D}\begin{pmatrix} -k_3 sign\,(S_z) - c_6 S_z - a_3 x_6 + g \\ -c_5\left(x_6 - \dot{x_{5d}}\right) + \ddot{x_{5d}} \end{pmatrix} \\ \overline{u_2} = \begin{pmatrix} -k_4 sign\,(S_\phi) - c_8 S_\phi - \widehat{\theta_1} x_{10} x_{12} \\ -c_7\left(x_8 - \dot{x_{7d}}\right) + \ddot{x_{7d}} \end{pmatrix} \\ u_2 = \widehat{p_1}\,\overline{u_2} \\ \overline{u_3} = \begin{pmatrix} -k_5 sign\,(S_\theta) - c_{10} S_\theta - \widehat{\theta_2} x_8 x_{12} \\ -c_9\left(x_{10} - \dot{x_{9d}}\right) + \ddot{x_{9d}} \end{pmatrix} \\ u_3 = \widehat{p_2}\,\overline{u_3} \\ \overline{u_4} = \begin{pmatrix} -k_6 sign\,(S_\psi) - c_{12} S_\psi - \widehat{\theta_3} x_8 x_{10} \\ -c_{11}\left(x_{12} - \dot{x_{11d}}\right) + \ddot{x_{11d}} \end{pmatrix} \\ u_4 = \widehat{p_3}\,\overline{u_4} \end{cases}$$

With $k_i > 0$ for $\quad i = 1, ..., 6.$

Proof Apply to first subsystem (34), we know a priori from (32) and (39)

(41) $$V_2 = \tfrac{1}{2} z_1^2 + \tfrac{1}{2} S_x^2$$

and

(42) $$S_x = z_2 = x_2 - \dot{x_{1d}} - c_1 z_1$$

With equation (26) apply to $\dot{z_2}$ we have
(43)

$$\begin{cases} \dot{V_2} = z_1\,\dot{z_1} + S_x\,\dot{S_x} \\ \dot{V_2} = z_1\,\dot{z_1} + S_x\left(a_1x_2 + \frac{U_1}{m}U_x + c_1\left(x_2 - \dot{x_{1d}}\right) - \ddot{x_{1d}}\right) \end{cases}$$

The chosen law for the attractive surface is the time derivative of (41) satisfying $(S\,\dot{S} < 0)$:

$$\text{(44)}\qquad \begin{aligned} \dot{S_x} &= -k_1 sign\left(S_x\right) - c_2S_x \\ &= \dot{x_2} - \ddot{x_{1d}} + c_1\,\dot{z_1} \\ &= a_1x_2 + \frac{U_1}{m}U_x - \ddot{x_{1d}} + c_1\left(x_2 - \dot{x_{1d}}\right) \end{aligned}$$

From (44), the control input U_x is extracted:
(45)

$$U_x = \frac{m}{u_1}\begin{pmatrix} -k_1 sign\left(S_x\right) - c_2S_x - c_1\left(x_2 - \dot{x_{1d}}\right) \\ -a_1x_2 + \ddot{x_{1d}} \end{pmatrix}$$

The same steps are followed to extract U_y and u_1.
For u_2, we know a priori from (32) and (39)

$$\text{(46)}\qquad \begin{cases} V_8 = \frac{1}{2}z_7^2 + \frac{1}{2}z_8^2 + \frac{1}{2}\widetilde{\theta_1}^T\Gamma_1^{-1}\widetilde{\theta_1} + \frac{|b_1|}{2\gamma_1}\widetilde{p_1}^2 \\ S_\phi = z_8 = x_8 - \dot{x_{4d}} - c_7z_7 \end{cases}$$

And from (33) after applying (36) and (38), with equation (26) apply to $\dot{z_8}$ we have
(47)

$$\begin{cases} \dot{V_8} = z_7\,\dot{z_7} + S_\phi\,\dot{S_\phi} + \left.\sum_{j=8}^{8} z_j\frac{\partial\alpha_{j-1}}{\partial\hat{\theta}_1}\right)\left(\Gamma_1\tau_8 - \dot{\widehat{\theta}_1}\right) \\ \quad +\widetilde{\theta}_1\left(\tau_8 - \Gamma_1^{-1}\dot{\widehat{\theta}_1}\right) - \frac{|b_1|}{\gamma_1}\widetilde{p_1}\left(\dot{\widehat{p_1}} + \gamma_1 sign\left(b_1\right)\overline{u}_2z_8\right) \\ \dot{V_8} = z_7\,\dot{z_7} + S_\phi\left(\widehat{\theta}_1x_{10}x_{12} + \frac{u_2}{\widehat{p_1}} + c_7\left(x_8 - \dot{x_{7d}}\right) - \ddot{x_{7d}}\right) \end{cases}$$

The chosen law for the attractive surface is the time derivative of second equation of (46) satisfying $(S\,\dot{S} < 0)$:

$$\text{(48)}\qquad \begin{aligned} \dot{S_\phi} &= -k_4 sign\left(S_\phi\right) - c_8S_\phi \\ &= \dot{x_8} - \ddot{x_{7d}} + c_7\,\dot{z_7} \\ &= \widehat{\theta}_1x_{10}x_{12} + \frac{u_2}{\widehat{p_1}} - \ddot{x_{7d}} + c_7\left(x_8 - \dot{x_{7d}}\right) \end{aligned}$$

From (48), the control input u_2 is extracted:
(49)

$$\overline{u_2} = \begin{pmatrix} -k_4 sign\left(S_\phi\right) - \widehat{\theta}_1x_{10}x_{12} - c_7\left(x_8 - \dot{x_{7d}}\right) \\ -c_8S_\phi + \ddot{x_{7d}} \end{pmatrix}$$

$$u_2 = \widehat{p_1}\,\overline{u_2}$$

The same steps are followed to extract u_3 and u_4.

SIMULATION RESULTS

The simulation results are obtained with the following parameters:

$$\begin{aligned} &m = 1.5\,kg;\ l = 0.35\,m;\ I_x = I_y = 1.8\ \ 10^{-1}\ N/rad/s^2 \\ &I_z = 9.1\ \ 10^{-2}\ N/rad/s^2;\ g = 9.81\,m/s^2 \\ &K_{ftx} = K_{fty} = K_{ftz} = 5.567\ \ 10^{-4}\ N.rad/s \\ &A_x = A_y = 3\,N.m \quad A_z = 5\,N.m \\ &A_p = 1.5\,N.m \quad A_q = A_r = 0.8\,N.m \end{aligned}$$

The vector of parameters θ^T is initiated to zero, unknown constant gains b_1, b_2 and b_3 are initiated to any values except at zero.

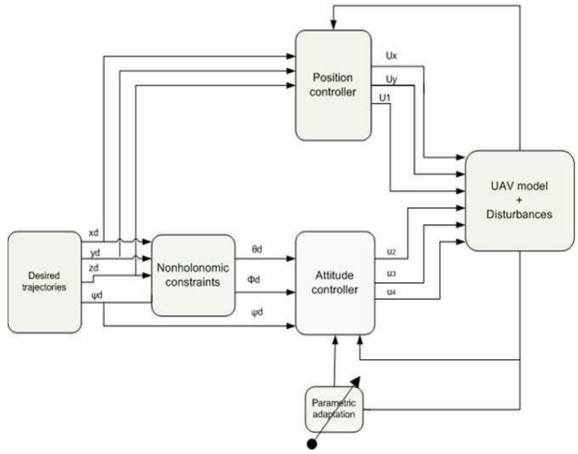

Fig. 2. The overall closed-loop system

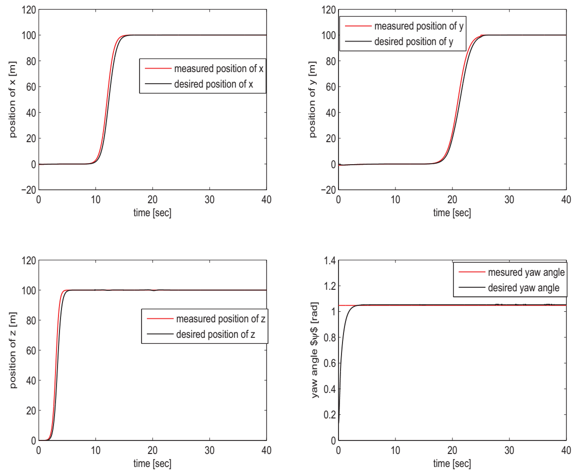

Fig. 3. x,y,z and yaw angle ψ tracking desired trajectories

Quality of the results

(Fig. 3) shows the good tracking of desired trajectory along the axis x, y, z and along yaw angle ψ. (Fig. 4) shows the evolution of the quadrotor in space and its stabilization. The response time of the system is about 4s. (Fig. 5) shows the good tracking of desired trajectory along the angle ϕ and θ, except, in the begining a little discrepancy for the angle θ is present. (Fig. 6) represents the errors made on the desired trajectory tracking. (Fig. 7) shows the convergence of estimated parameters to their true valeus and (Fig. 8) shows the convergence of estimated unknown constant gains, their convergence is slower then the estimated parameters.

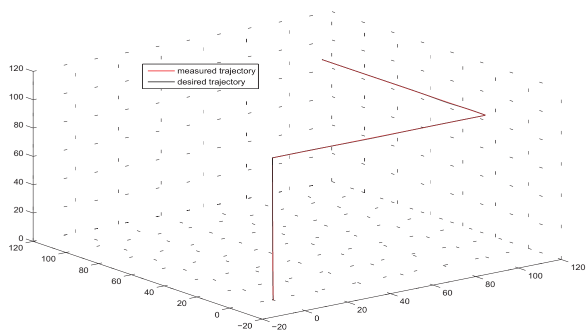

Fig. 4. x,y,z tracking desired trajectories in 3D

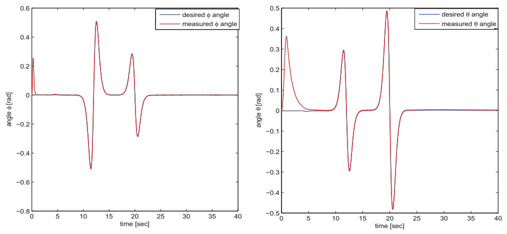

Fig. 5. θ and ϕ angle tracking desired trajectories

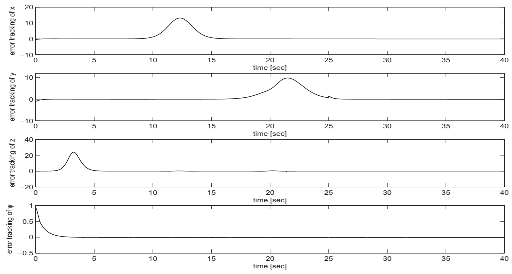

Fig. 6. Tracking errors according (x, y, z) and yaw angle ψ respectively

Conclusion

In this paper, stabilizing control law synthesis by sliding mode based on adaptive backstepping approach is presented. The dynamic model of the quadrotor taking into account the different physical phenomena which can influence the evolution of the system in the space is considered, with neglecting the phenomena less pertinant, as a Coriolis torque, a gyroscopic torque and the development of the high order nonholonomic constraints imposed to the system motions. The adaptive backstepping control is developed to overcome the parametric uncertainties caused by changes in the values of inertia under ground effects or sudden change of climatic conditions. The rapid convergence settings to their real value makes a good trajectory tracking and ensures the stability of the closed loop. The sliding mode control enhances the stability of the global system against disturbances experienced by the external system such as the wind and keeps the system performance obtained by The adaptive backstepping control. For all outputs, the tracking error reaches zero after a few seconds.

REFERENCES

[1] A. Mokhtari, N. K. Msirdi, K. Meghriche, A. Belaidi. Feedback linearization and linear observer for a quadrotor unmanned. *Advanced Robotics*, 20:71–91, 2006.

[2] I. Kanellakopoulos, P. V. Kokotovic, A. S. Morse. Systematic

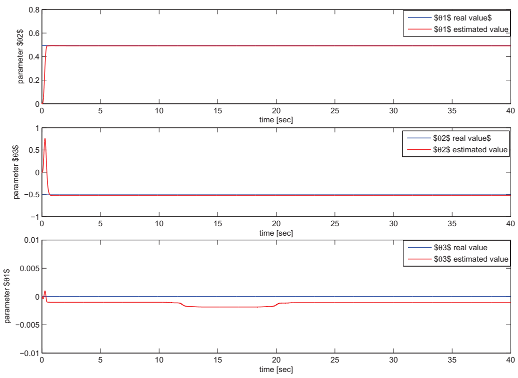

Fig. 7. parameters estimated $\widehat{\theta_1}$, $\widehat{\theta_2}$ and $\widehat{\theta_3}$.

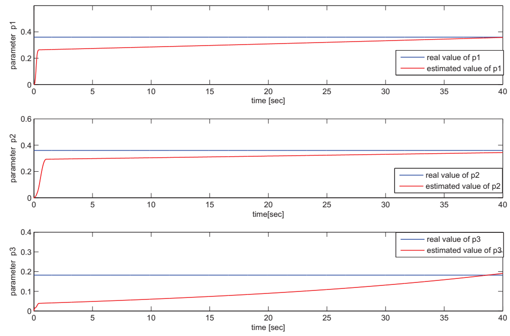

Fig. 8. constant gains estimated $\widehat{p_1}$, $\widehat{p_2}$ and $\widehat{p_3}$.

Design of Adaptive Controllers for Feedback Linearizable Systems. *IEEE Trans. Automat. Contr*, 36, 1241–1253, 1991.

[3] M. Krstic, I. Kanellakopoulos, P.V. Kokotovic. Adaptive nonlinear control without overparametrization. *Systems & Control Letters*, volume 19, pages 177–185. North-Holland, 1992.

[4] K. Watanabe, K. Tanaka, K. Izumi,K. Okamura, R. Syam. Discontinuous Control and Backstepping Method for the Underactuated Controlof VTOL Aerial Robots with Four Rotors. *Unmanned Systems, SCI 192*, pages 83-100. Berlin Heidelberg, Springer-Verlag, 2009.

[5] H. Xu, M. mirmirani, P. A. Ioannou, H. R. Boussalis. Robust adaptive sliding control of linearizable systems. *Proceeding of the American Control Conference*, pages 4351–4356. Arlington, VA June 25-27, 2001.

[6] J. E. Slotine, W. Li Applied nonlinear control. Printice Hall, 1991.

[7] L. Beji, A. Abichou. Streamlined rotors mini rotorcraft: Trajectory generation and tracking. *International journal of automation and systems*, 03, 01, pages 87–99, Mars 2005.

[8] A. Isidori. Nonlinear Control Systems. London, Springer-Verlag, 1995.

[9] C. Wen, Y.C. Soh. Decentralized adaptive control using integrator backstepping. *Automatica*, vol.33, pages 1719-1724, 1997.

[10] Y. Zhang, C. Wen, Y.C. Soh. Adaptive backstepping control design for systems with unknown high-frequency gain. *IEEE Trans. on Automat. Contr*,vol.45, pages 23502354, 2000.

[11] C. Wen, Y. Zhang, Y.C. Soh. Robustness of an adaptive backstepping controller without modification. *Systems & Control Letters*,vol.36, pages 87-100, 1999.

[12] Costa. R.R, al. Adaptive backstepping control design for mimo plants using factorization. *Proceedings of the American Control Conference*, pages 4601-4606, 2002.

[13] Y. Ling, G. Tao. Adaptive backstepping control design for linear multivariable plants. *Proceedings of the IEEE Conference on Decision and Control*, pages 2438-2443, 1996.

[14] J. Zhou, C. Wen, Y. Zhang. Adaptive backstepping control of a class of uncertain nonlinear systems with unknown backlash-like hysteresis. *IEEE Transactions on Automatic Control*, vol.49, pages 1751-1757, 2004.

[15] H. Bouadi, M. Bouchoucha, and M. Tadjine. Sliding Mode Control based on Backstepping Approach for an UAV Type-Quadrotor. World Academy of Science, Engineering and Technology,vol. 26, 2007.

***Authors**: Mr. Abderrahmane Kacimi, Institute of Industrial Safety and Maintenance Measurement and Instrumentation Systems, University of Oran, Algeria, email:abderrahm2001@yahoo.fr, Prof. Abdellah Mokhtari, Laboratory LAHN marine departement, Marine Institute, University of Science and Technology MB, Oran, Algeria, email:amok_usto@yahoo.co.uk, Ph.D. Benatman Kouadri, Institute of Electrotechnic, Faculty of Electrical Engineering, University of Science and Technology MB, Oran, Algeria, email:benatmank@yahoo.com,*