

# Investigation on Chaotic Characteristic of PD Magnitude Series during Propagation of Electrical tree in XLPE Power Cables

**Abstract.** The electrical tree propagation experiments were performed under high voltages with a frequency of 50Hz and amplitudes ranging from 12kV to 21kV at room temperature by utilizing an actual XLPE cable as the test sample, where the concentration of electrical field was simulated by a metal needle tip. The chaotic theory was introduced to analyze the PD magnitude series during the propagation process of electrical tree. Experimental results show that deterministic chaos exists in the propagation of electrical tree in XLPE cables, and the largest Lyapunov exponent and correlation of the strange attractors increase with the decrease of electrical tree fractal dimension. The results may provide a new approach for online diagnosis of electrical tree morphology.

**Streszczenie.** Przedstawiono wyniki badania wyładowania w kablu XPLE przy częstotliwości 50 Hz i napięciu 12 – 21 kV. Stwierdzono chaotyczny charakter drzewienia elektrycznego. Zaproponowano opis teoretyczny umożliwiający łatwe rozpoznanie typu wyładowania. (Badania i opis teoretyczny chaotycznego charakteru wyładowania w kablach XLPE)

**Keywords:** XLPE cables; electrical tree; partial discharge magnitude series; the largest Lyapunov exponent; correlation dimension

**Słowa kluczowe:** wyładowanie niepełne, drzewienie elektryczne, kable XLPE.

## Introduction

Cross-linked polyethylene (XLPE) cable has been widely used in high-voltage power systems due to its superior insulation performance. The insulation system gradually degrades subjected to long-term electrical, thermal, mechanical and environmental stresses [1]. With the formation of dendritic discharge channel, electrical tree is often regarded as the main route of local breakdown occurred in XLPE insulation system. Therefore, electrical tree is always a key issue to the safe operating of XLPE cables.

It has been proved that partial discharges were closely related with the propagation process of electrical tree and useful to the insulation diagnosis [2]. A large number of studies have been focused on partial discharge of electrical trees by several signal processing methods such as phase resolved distribution (PRPD) [3], single pulse waveform features [4], trends analysis [5] and inter time distribution (ITD) [6], aiming in search of suitable partial discharge features to facilitate the diagnosis of electrical trees. However, it is quite difficult to establish explicit relationships between these features and physical mechanism of electrical trees. In recent years, chaos theory has been introduced to the analysis of electrical tree propagation. It has been identified [7-9] that deterministic chaos existed in the propagation of electrical trees in polymeric solid insulation, which was supported by analysis of the trend of PD numbers developing in unit time. Besides, three normalized parameters, i.e. PD magnitude, applied voltages and differences of PD occurrence time were also used for chaotic analysis of partial discharge in XLPE cable [10]. Despite these efforts, it is needed to carry out deeper research in chaotic characteristic of electrical trees to achieve a full understanding about the physical mechanism of the electrical tree propagation, and provide effective PD features for electrical tree diagnosis.

In this paper, an actual 15kV XLPE cable was utilized as the test sample stressed under a metal needle electrode, and four electrical trees samples were obtained under various voltages at room temperature. By utilizing the maximum and overall PD magnitude to construct the time series, deterministic chaotic characteristic during the propagation of electrical tree were confirmed. Finally, the relationships between the chaotic features with tree morphology were discussed.

## Chaos theory

In real systems, the necessary number of parameters to describe the state of a system is often unknown, and only a limited number of observations related to the dynamics of system are available. It has been confirmed that if  $m > 2d_A + 1$ , the topological properties of the original attractor can be restored in the reconstructed phase space according to Takens theorem [11], where  $m$  represents the embedding dimension of the reconstructed space,  $d_A$  is considered as the fractal dimension of the original attractor. Phase space reconstruction from scalar time observations can be implemented based on the delay coordinates method [12].

The essence of chaos is the nonlinear interaction within systems. The usual methods to determine chaos in systems mainly include Lyapunov exponent method, power spectrum method, Poincare section method and principal component analysis, etc. Poincare mapping can well depict the reciprocating nonperiodic characteristic of chaos. If there are exactly one or a few discrete points in Poincare section, the motion is periodical. Otherwise, if there is a closed curve instead, the motion is quasi-periodical. If there is neither a finite points set nor a closed curve, the corresponding motion is quite likely to be chaotic. Furthermore, Poincare mapping section of the system with a certain damping without external noise disturbance would be a point set with meticulous structure.

The most common parameters to describe the properties of strange attractors were the largest Lyapunov exponent and correlation dimension. The theoretical backgrounds related to the definition of these two parameters were presented in Reference [12]. Lyapunov exponents reflect the sensitivity of chaotic motion to the initial value. The largest Lyapunov exponent which measures the divergence of nearby trajectories in the phase space is also one of the most efficient methods to determine the presence of chaos. At present, many methods were used to calculate the largest Lyapunov exponent among which Small Data Sets method [13] was introduced in this paper. Small Data Sets method has short calculation time, and is quite appropriate for calculating the largest Lyapunov exponent of PD magnitude series since it's more reliable for small data mixed with noises. Correlation dimension is one of fractal dimension, which is non-integer for chaotic systems. It characterizes the complexity of geometric structure of the strange attractor. Grassberger-Procaccia algorithm, G-P for short, is a method which was primarily used to calculate the correlation

dimension through the correlation integral  $C_m(r)$  and the distance  $r$  in the reconstructed phase space [14].

### Experimental Arrangement

The type of YJV<sub>22</sub>-3×95, 8.7/15kV XLPE cable was used as test sample, and the structure is shown as figure 1. The wire core diameter is 11.5mm, internal shielding layer thickness is 0.5mm, XLPE insulation layer thickness is 4.8mm, and external shielding layer thickness is 0.55mm. The shielding copper strip is close to external shielding layer.

The test sample is made by the following steps. Firstly, the armor of 15kV XLPE cable with the length of 0.9m is removed. Then, make both the terminals expose the wire core with 25mm length, and remove 190mm external shielding layer. Unfold the middle cooper strip, and insert a metal needle with a curvature radius of 5μm. Finally, make the cooper strip been restored. The distance between needle tip and internal shielding layer is 2.8mm.

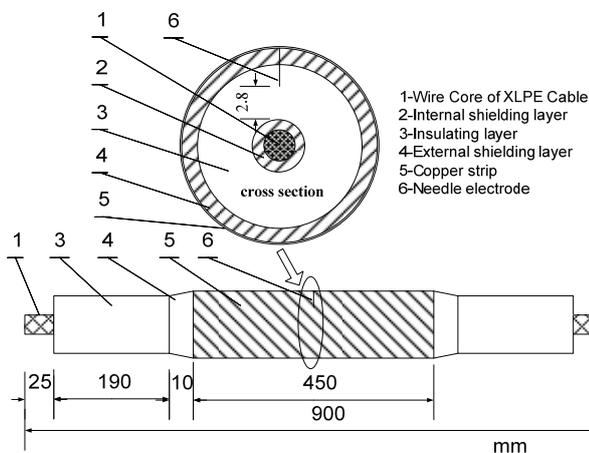


Fig.1. Sample structure

The test samples with electrode system were all placed in an oil-filled tank to prevent PD occurring in the sample terminals, which would influence the experimental results. The structure of the system is shown in Figure 2. The oil filled in the tank was 25# transformer oil.

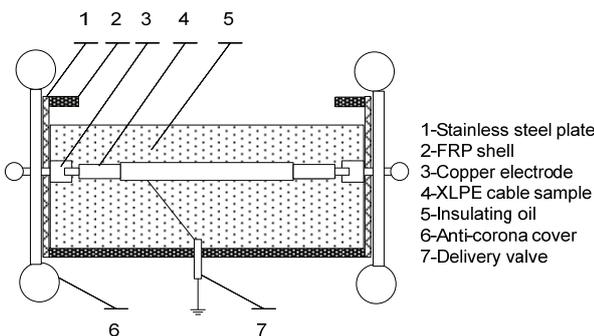


Fig. 2. The structure of the electrode system

In the current study, the electrical tree propagation was investigated by means of partial discharge measurement. PD signals were acquired by a digital measurement system, shown in Figure 3. A discharge-free AC voltage source (50 kV/50 kVA) with amplitude of 0–50 kV and frequency of 50 Hz consists of a voltage regulator, a transformer, a protection resistor of 10kΩ and a coupling capacitance of 1000pF. The PD detector was a Rogovski coil with the bandwidth of 50kHz~15MHz. PD signals were observed and

collected by Wavepro7100 oscilloscope after being amplified by 10~100 times and filtered by a high-pass filter of 100kHz. The oscilloscope's bandwidth is 1GHz and highest sampling rate is 20GS/s.

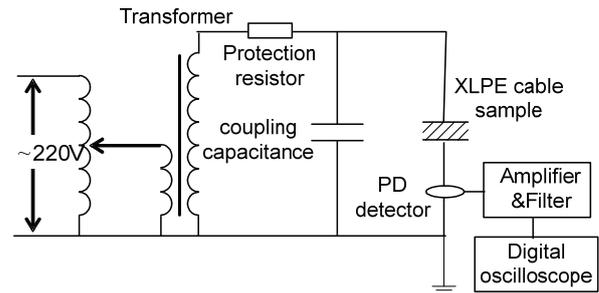


Fig.3. Experimental Circuit

After the treeing test, the XLPE cable samples were cut into slices with the approximate thickness of 2mm, which were observed by the digital microscope with a camera whose maximum amplification factor is 300 times.

In this paper, four groups of samples were made for the test. Samples a, b, c and d were stepped up to 12kV, 15kV, 18kV and 21kV with a rise rate of 1kV/s respectively at room temperature. Then, the PD signals were collected by the digital detection system mentioned above. For comparative analysis of the experimental results, the voltage applied on all samples was stopped when the maximum PD magnitude reached 1500 pC.

### Experiment Results

Typical captured photographs of tree structure grown in XLPE cable insulation are shown in table 1 and figure 4, where  $D_1$  is the observed tree width of cross section slices and  $D_2$  is the observed tree width of vertical section slices. It can be found that the tree structure changed from branch-type to bush-type while the stressed voltage increased.

Table 1. Results of test of electrical tree

Test samples	applied voltage / kV	Width of tree /mm		Propagation time of tree / s
		$D_1$	$D_2$	
a	12	0.58	0.63	720
b	15	0.75	0.93	600
c	18	0.76	0.85	600
d	21	0.87	0.95	1500

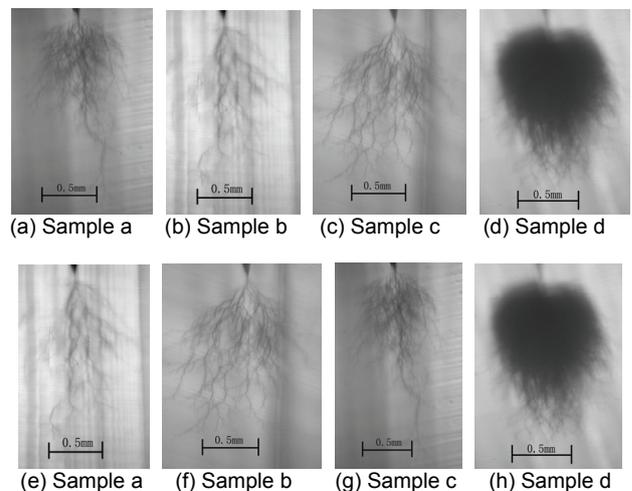


Fig. 4. Photographs of electrical tree under various applied voltages (a)~(d) are observed on cross section, (e)~(h) are observed on vertical section

In order to study the chaotic characteristic of the electrical tree propagation, one observable variable which can reflect the stochastic fluctuation of electrical tree propagation need to be determined first. Results in literature [15] showed that the stochastic fluctuation of electrical tree propagation could be measured by partial discharge, mainly including discharge numbers and magnitude. Dissado [7] chose the time series of PD numbers per unit time for chaotic analysis of electrical tree. However, the statistic value of PD numbers was relatively sensitive to PD measurement system, which means PD pulses with small magnitude might be ignored. Therefore, the time series of PD magnitude was chosen for chaotic analysis in this paper.

Similar to the time series of PD numbers, an appropriate time interval is quite essential for establishment of PD magnitude series. The PD magnitudes were recorded in every 0.3s. The time series, i.e. the maximum PD magnitude series  $q_{max}$  and overall PD magnitude series  $q_n$ , of samples a, b, c, d were shown in figure 5 and 6. The total points  $N$  for each sample were 2400, 2000, 2000 and 5000, respectively.

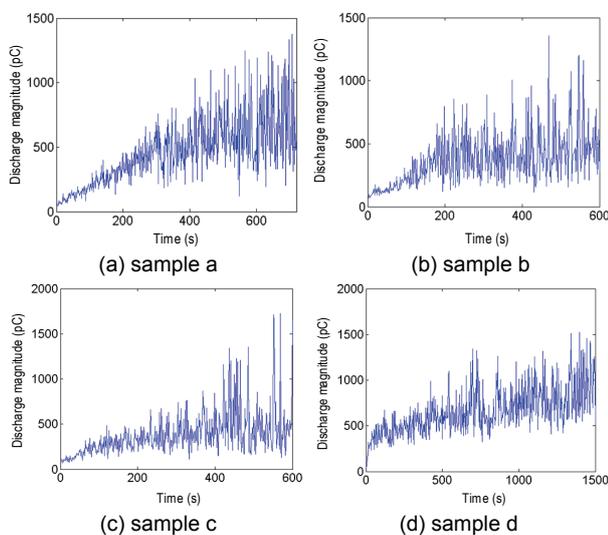


Fig. 5. The maximum PD magnitude series  $q_{max}$

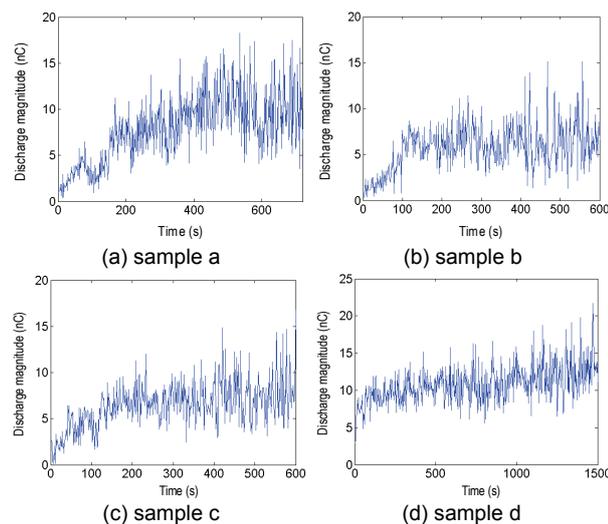


Fig. 6. The overall PD magnitude series  $q_n$

For the purpose of eliminating the influence of both pseudo-data and external interferences, a normalization pre-processing method for PD series was introduced before data analysis. Denote  $\{y_k : k=1, 2, \dots, N\}$  as the PD magnitude series observed by the same time interval, the normalization can be expressed as follows:

$$(1) \quad x_i = \frac{y_i - y_{\min}}{y_{\max} - y_{\min}}, i = 1, 2, \dots, N$$

where:  $y_{\min} = \min\{y_j, j=1, 2, \dots, N\}$ ,  $y_{\max} = \max\{y_j, j=1, 2, \dots, N\}$ .

### Preliminary chaotic test and chaotic features extraction of PD magnitude series

Poincare section method was applied for the preliminary chaotic test. The Poincare sections of PD series of sample a were shown in figure 7. Results showed both sections of the series were some crowded spots with fractal structure, which demonstrated that the maximum PD magnitude series  $q_{max}$  and overall PD magnitude series  $q_n$  both had chaotic characteristic. The similar structure was also confirmed in other samples.

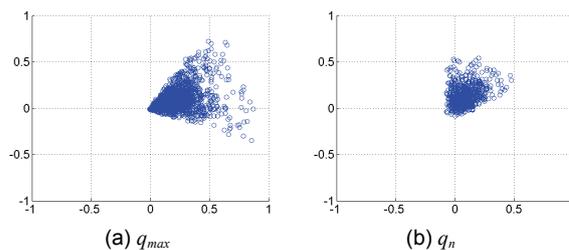


Fig. 7. Poincare section of the time series of sample a (a) The maximum PD magnitude series; (b) The overall PD magnitude series

The phase space was then reconstructed to restore the topological structure of the attractors. In this paper, Mutual information method [16] was applied to obtain the best delay time, and Cao's method [17] was used to calculate the optimal embedded dimension. Take sample b for example, the relationship of the mutual information  $I(\tau)$  between the original and the delay vector and the delay time  $\tau$  is shown as figure 8. It can be found that the mutual information reach the first local minimum while  $\tau$  were 2 and 3 respectively. The delay time of all the samples were obtained in table 2. After the best time delay  $\tau$  was obtained, the calculated embedded dimension based on Cao's method showed that the embedded dimensions of four samples are all 10.

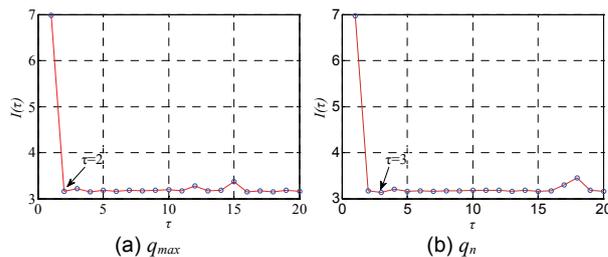


Fig. 8. Varying of  $I(\tau)$  with  $\tau$  of sample b

Table 2. Results of the delay time

Samples	a	b	c	d
$q_{max}$	2	2	2	2
$q_n$	2	3	3	3

By utilizing Small Data Set method, the largest Lyapunov exponent  $\lambda_1$  of PD magnitude series were shown in Table 3. It can be seen that the largest Lyapunov exponent  $\lambda_1$  of the time series of  $q_{max}$  and  $q_n$  were all positive number, which confirmed that PD magnitude series of the electrical tree has the determinate chaotic characteristic.

Table 3. Results of the largest Lyapunov exponent

Samples	a	b	c	d
$q_{max}$	0.157	0.123	0.101	0.083
$q_n$	0.343	0.131	0.142	0.013

According to G-P algorithm, the  $\ln(C(r)) \sim \ln(r)$  curves of  $q_{max}$  and  $q_n$  series of electrical tree with the increase of embedded dimension  $m$  were shown in figure 9. While  $m$  was greater than 9, the slope of the linear segment of the curve had no change at all. Therefore, the embedded dimension  $m=10$  was chosen to calculate the correlation dimension, which accorded with the results by Cao's method. The correlation dimensions of four samples were shown in Table 3, which were all non-integral confirming the deterministic chaotic characteristic in PD magnitude series during the propagation of electrical tree.

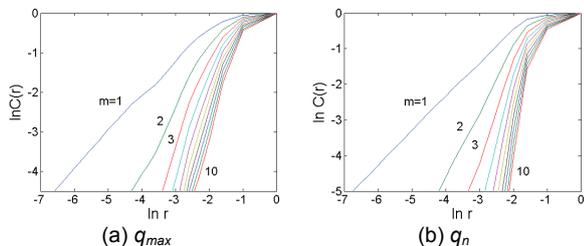


Fig. 9. Varying of  $\ln C(r)$  with  $\ln r$  of sample b

Table 4. Results of the correlation dimension

Samples	a	b	c	d
$q_{max}$	3.31	3.03	2.95	2.89
$q_n$	3.72	3.44	3.53	3.13

### Correlation analysis between chaotic characteristics of PD and the electrical tree shape

It had been identified that there were three kinds of electrical tree structures which are branch, bush and mix configurations developed in XLPE cable insulation. Electrical tree with branch structure grows rapidly and continuously. On the contrary, bush-like electrical tree grows very slowly and has a long lag phase. Therefore, the damage to the insulating system of the branch structure is greater than the bush structure. Fractal dimension is an important parameter to represent the morphological characteristic of the electrical tree shape. Fractal dimension of branch-like tree is small while of bush-like tree is large. In this paper, fractal dimension of the 2-D electrical tree photographs was calculated by the algorithm of box dimension presented in [18].

As mentioned above, the chaotic degree can be characterized quantitatively by the largest Lyapunov exponents and correlation dimension of strange attractor. Specifically, the system instability is characterized by the largest Lyapunov exponents, and correlation dimension reflect the complexity of system's change. Figure 10 and figure 11 gave the relationship between the largest Lyapunov exponent and correlation dimension and electrical tree fractal dimension in cross section, respectively. It can be found that the largest Lyapunov exponents and correlation dimension both showed a decreasing trend with the increase of electrical tree fractal dimension. Moreover, the dispersion of overall PD magnitude series was obviously greater. It showed us that the propagation of bush-like trees is more stable than branch-like trees. The same results can also be obtained by utilizing the electrical trees in the vertical section.

The results also demonstrate that bush-like trees which grew at a higher voltage had more weakened chaotic characteristic. Bush-like trees have a denser discharge channel whose conductance is higher [19]. Then, space charge is easier to transfer through the channel, which results in that the fluctuation of the local electrical field nearby the channel is harmonious. It would be difficult to accumulate charges in the channel, and PD pulses with high

magnitude hardly occur. However, PD pulses with small magnitude are difficult to extend to the tree end. It is the key reason that bush-like trees grow slower than branch-like trees, which can be seen from Table 1. Therefore, chaotic characteristic of PD magnitude series and the growth mechanism of electrical tree were supported by each other very well.

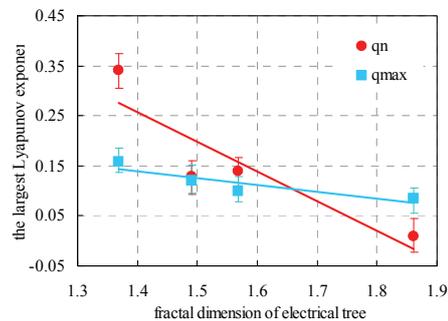


Fig. 10. Relationship between fractal dimension and the largest Lyapunov exponent of electrical tree

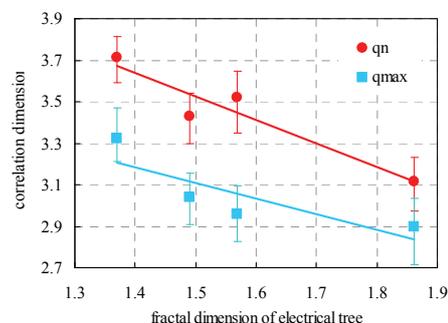


Fig. 11. Relationship between fractal dimension and correlation dimension of electrical tree

Meanwhile, branch-like tree whose largest Lyapunov exponent was larger was more sensitive to initial condition can also be confirmed by the influence of voltage rising rate on electrical tree. Figure 12 showed the electrical tree of samples (cross section) under 12kV and 21kV with the voltage rising rate of 0.1kV/s and 1kV/s respectively. Under 12kV, electrical tree fractal dimension changed from 1.22 to 1.37. While under 21kV, the change was smaller, from 1.83 to 1.86. It can be obtained that the branch-like tree under 12kV is more sensitive to initial condition than the bush-like tree under 21kV.

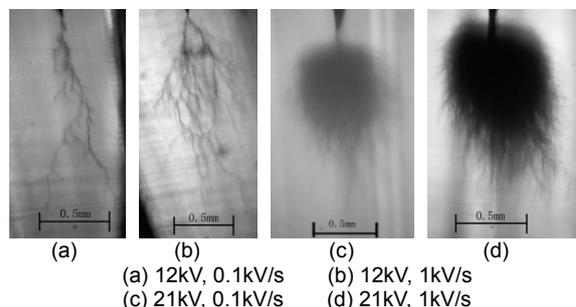


Fig. 12. Photographs of electrical tree under the voltage rising rate of 0.1kV/s and 1kV/s

Chaotic analysis of PD magnitude series showed that deterministic chaotic characteristic existed in the process of electrical tree propagation, and its chaotic degree was closely related to electrical tree shape. The largest Lyapunov exponent and correlation dimension decreased with the increase of fractal dimension of electrical tree, which may provide a new technology for the diagnosis of

tree morphology. However, on-site applications need to be confirmed by further researches.

### Conclusion

The important conclusions arrived at based on the present investigations are for the followings:

(1) Results of Poincare section, the largest Lyapunov exponent and correlation dimension of PD magnitude series show that deterministic chaotic characteristic exists in the process of electrical tree propagation. Bush-like trees have weaker chaotic characteristic than branch-like trees.

(2) Both the largest Lyapunov exponent and correlation dimension of PD magnitude series show a decreasing tendency with the increase of fractal dimension, which coincide with the sensitivity of electrical tree with branch and bush structure to the step-stress rate.

(3) Chaotic features of PD magnitude series can be regarded as a new tool to identify the morphology of electrical tree. Moreover, the maximum magnitude series are more effective than the overall discharge magnitude series.

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