

Comparative studies of control systems with fractional controllers

Abstract. The paper presents the realization of fractional order controller implemented in National Instruments sbRIO-9631 controller programmed in LabView. In order to digitally realize the fractional order controller transfer function, operator-based continuous fraction expansion (CFE) scheme is applied. DC motor – generator plant model is used as the controlled system. The controlled variable is the speed of the rotor.

Streszczenie. W pracy przedstawiono praktyczną realizację regulatora niecałkowitego rzędu w sterowniku National Instruments sbRIO-9631 programowanym w środowisku LabView. W celu wyznaczenia dyskretnej transmitancji aproksymującej transmitancję ciągłą regulatora niecałkowitego rzędu wykorzystano rozwinięcie transmitancji niewymiernej w ułamek łańcuchowy i przyjęcie skończonej liczby elementów tego rozwinięcia. Obiektem regulacji jest model zespołu silnik - generator z silnikiem prądu stałego. Wielkością regulowaną jest prędkość obrotowa wału silnika. (Badania porównawcze układów regulacji z regulatorami niecałkowitego rzędu).

Keywords: controller, fractional order, speed control, DC motor.

Słowa kluczowe: regulator, niecałkowity rząd, regulacja prędkości, silnik prądu stałego.

Introduction

In recent years considerable attention has been paid to fractional calculus and its application in many areas in science and engineering (see, e.g. [1, 2, 3, 4]).

In control system fractional order controllers are used to improve the performance of the feedback control loop. One of the most developed approaches to design robust and fractional order controllers is CRONE control methodology, French acronym of "Commande Robuste d'Ordre Non Entier" (non-integer order robust control) [5, 6].

The fractional PID controllers, namely $PI^\lambda D^\mu$ controllers, including an integrator of λ order and a differentiator of μ order were proposed in [7, 8]. Many design methods of tuning the $PI^\lambda D^\mu$ controllers have been presented in the literature (see, e.g. [9, 10, 11, 12, 13, 14]). These methods are based on the mathematical description of the process. The first order plant with time delay is the most frequently used model for tuning fractional and integral controllers [15].

The asymptotic stability is the basic requirement of a closed-loop system. Some methods for determining the asymptotic stability regions in the fractional controller parameter space were proposed in [16, 17]. Gain and phase margins are measures of relative stability for a feedback system, therefore the synthesis of control systems is very often based on them. In typical control systems the phase margin is from 30° to 60° whereas the gain margin is from 5dB to 10dB. In paper [18] a simple method of determining the stability region (satisfying the conditions of gain and phase margins) in the parameter space of a fractional order inertial plant with time delay and a fractional order PID controller was given.

In order to implement (digital realization) of the fractional order controller the approximate time rational transfer function should be determined. Some methods of obtaining the discrete transfer function approximating the continuous fractional order transfer function have been proposed in the literature [4, 19, 20, 21]. Generally, the digital transfer function is based on the canonical form of the infinite impulse response filter (IIR filter). Such an algorithm can be directly implemented into any processor based devices as for instance programmable logic controller (PLC) [19].

In this paper the realization of fractional-order controller implemented in National Instruments sbRIO-9631 controller programmed in LabView is given. The DC motor-generator plant model is used as the controlled system. The controlled variable is the speed of the rotor. The comparative studies of these control systems are presented.

Controlled system: DC motor – generator model

The object to be controlled is a DC drive system. A general layout is shown in Figure 1. The unit consists of two mechanically coupled motor generator system built of two identical mini DC motors supplied by a voltage of up to 12V and a rated current of 0.35A. An analog signal proportional to motor speed is available at the analog output n . The change in the speed of 1000 rpm corresponds to a change of 1V. The rotational speed varies from 0 to 6000 rpm.

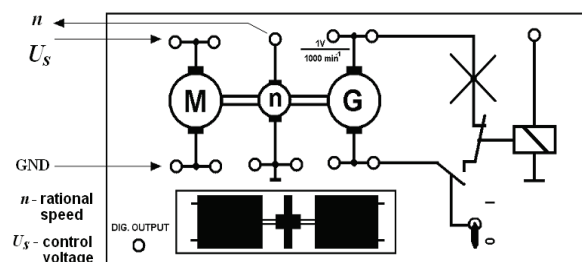


Fig.1. General layout of the DC motor control system

In order to identify the parameters of the plant model a step change of control voltage U_s from the initial value of 5V to the value of 8V is made. A corresponding change of output voltage n is recorded at sampling time 0.005s. The step response of the object is shown in Figure 2 (dashed line). The figure shows that the real object can be described by the model of the transfer function

$$(1) \quad G(s) = \frac{Ke^{-sh}}{1+s\tau}, \quad K > 0, \quad \tau > 0, \quad h \geq 0,$$

where K is the gain, τ is the time constant, and h is the time delay.

The experimentally obtained step responses of the system are approximated to the characteristics of a step model (1) taking into account $K = 0.67$, $\tau = 0.082$, $h = 0.02$. The characteristic of the model (1) for given values of model parameters is shown in Figure 2 (solid line).

Fractional PID Controller

The fractional order $PI^\lambda D^\mu$ controller was proposed as a generalization of the PID controller with integrator of real order λ and differentiator of real order μ [7, 8]. The transfer function of such a type controller in the Laplace domain takes the form

$$(2) \quad C(s) = k_p + k_i s^{-\lambda} + k_d s^{\mu}, \quad \lambda > 0, \quad \mu > 0,$$

where k_p , k_i and k_d denote the proportional, integral and differential gains of the controller respectively.

Taking $\lambda=1$ and $\mu=1$ we obtain a classical PID controller. If $\mu=0$ and $k_d=0$ we obtain a PI^{λ} controller etc. All these types of controllers are particular cases of the $PI^{\lambda}D^{\mu}$ controller.

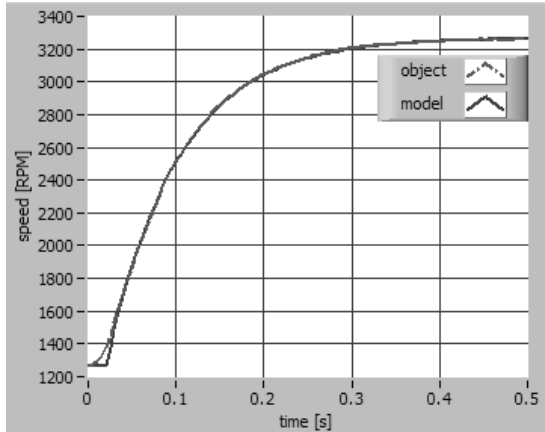


Fig.2. Step responses of object and its model

Parametric synthesis of the controller

In paper [18] the method of determining the stability region in the parameter space of $PI^{\lambda}D^{\mu}$ controller was given. The inertial plant of a fractional order with time delay was considered. The transfer function (1) is the particular case of that model (integral order of inertia). In [18] on the basis of the D-partition method analytical forms expressing the boundaries of stability regions for specified gain and phase margin requirements were determined. Stability regions are located between the real zero boundary and the complex zero boundary. When the stability regions are known, the tuning of the fractional controller can be carried out. On choosing any point from the stability region we can obtain the controller parameter values provided that the conditions of gain and phase margins of control systems are fulfilled.

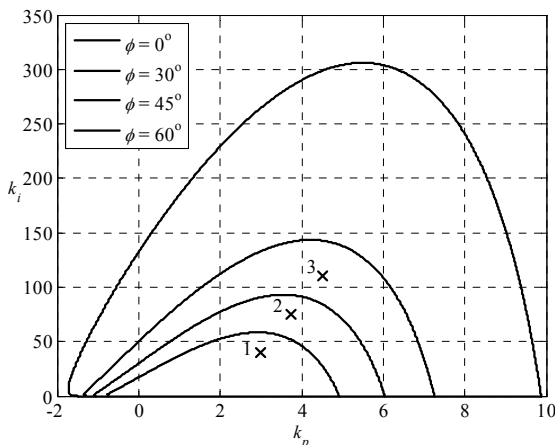


Fig.3. Stability regions for $k_d=0.19$, $\mu=0.5$, $\lambda=1$ and different values of ϕ (the controller $PI^{0.5}$)

On computing complex and real zero boundaries by the proposed method [18], we obtain the stability regions in the controller parameter plane (k_p, k_i). The stability regions for $k_d=0.19$, $\mu=0.5$, $\lambda=1$ and a some values of phase margin are shown in Figure 3. Boundaries of stability regions are calculated for transfer function (1) with $K=0.67$, $\tau=0.082$, $h=0.02$. The stability regions lie between line $k_i=0$ (the real

zero boundary) and the curve assigned to specified phase margin ϕ (the complex zero boundary). On choosing any point from the stability region we obtain the controller parameter values provided the phase margin of this system is greater than that specified for drawing the complex boundary. For example, any point from the region limited by the line $k_i=0$ and the curve corresponding to $\phi=60^\circ$ provides a phase margin of this system greater than 60° . The controller parameters and stability margins of the control system for all points marked in Figure 3 are listed in Table 1. It is shown that the stability margin values are greater than that specified for drawing the complex boundaries of the stability regions.

Table 1. Controller parameters, gain and phase margins

Point	Controller parameters	Phase margin [°]	Gain margin [dB]
1	$k_p = 3, k_i = 40, k_d = 0.19, \lambda = 1, \mu = 0.5$	70.16	8.25
2	$k_p = 3.75, k_i = 75, k_d = 0.19, \lambda = 1, \mu = 0.5$	51.29	6.83
3	$k_p = 4.5, k_i = 110, k_d = 0.19, \lambda = 1, \mu = 0.5$	38.22	5.27

Table 1 confirms that points from the stability regions satisfy the phase margin requirements.

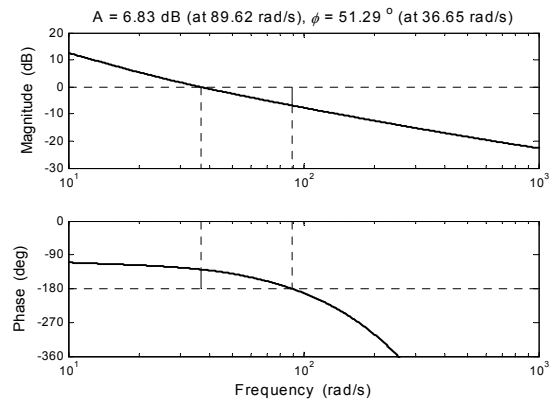


Fig.4. Bode plot with gain and phase margins for $k_p = 3.75$, $k_d = 0.19$, $\mu = 0.5$, $\lambda = 1$ (point 2)

The Bode plot with gain and phase margins for point 2 is illustrated in Figure 4. The phase margin for this system is greater than 45° and equal to 51.29° .

Digital realization of the fractional order controller

Transfer function (2) of the controller is the irrational function in the variable s . Therefore, accurate physical realization of the differentiation and integration of a fractional order is not possible. In order to implement the fractional order controller the approximate time rational transfer function should be determined. The approximate time rational transfer function may be in the form of a discrete transfer function with integral orders. Such a transfer function can be directly implemented into any processor based devices. Some methods of obtaining the discrete transfer function approximating the continuous fractional order transfer function have been proposed in the literature [4, 19, 20, 21].

In general, the discretization of the fractional order differentiator/integrator $s^{\pm r}$ ($r \in R$) is expressed by the so-called generating function $s \approx w(z^{-1})$. In this paper as a generating function we use [9, 19]

$$(3) \quad \left(\omega(z^{-1}) \right)^{\pm r} = \left(\frac{1+a}{T} \frac{1-z^{-1}}{1+az^{-1}} \right)^{\pm r},$$

where T is sampling time, a is ratio term and r is fractional order.

Depending on the value of $a = \{0, 1/7, 1\}$ we obtain Euler rule, Al-Alaoui rule and Tustin rule, respectively. In this paper we use a different value of a . In expanding the above in rational functions we use the continued fraction expansion (CFE). By using this method the discrete transfer function approximating fractional order operators can be expressed as

$$(4) \quad \begin{aligned} (\omega(z^{-1}))^{\pm r} &\approx \left(\frac{1+a}{T}\right)^{\pm r} CFE \left\{ \left(\frac{1-z^{-1}}{1+az^{-1}} \right)^{\pm r} \right\}_{p,q} \\ &= \left(\frac{1+a}{T}\right)^{\pm r} \frac{P(z^{-1})}{Q(z^{-1})} \\ &= \left(\frac{1+a}{T}\right)^{\pm r} \frac{p_0 + p_1z^{-1} + p_2z^{-2} + \dots + p_mz^{-p}}{q_0 + q_1z^{-1} + q_2z^{-2} + \dots + q_mz^{-q}} \end{aligned}$$

where P and Q are polynomials of degree p and q , respectively, in the variable z^{-1} . Normally we can set $p=q$. In this paper we use the order of the approximate model $p=q=1$, then we can calculate coefficients of P and Q polynomials from the following forms

$$(5) \quad \begin{aligned} p_0 = q_0 &= \frac{2}{a+r+ra-1}, \\ p_1 &= \frac{a-r-ra-1}{a+r+ra-1}, \quad q_1 = 1. \end{aligned}$$

From the above it follows that transfer function (2) corresponds in the discrete domain to the discrete transfer function in the following expression

$$(6) \quad C(z^{-1}) = k_p + k_i (\omega(z^{-1}))^{-\lambda} + k_d (\omega(z^{-1}))^{\mu}.$$

Practical fractional controller realization is implemented in National Instruments (NI) sbRIO-9631 programmable controller equipped with a real-time processor with FPGA interface, so it is possible to perform control algorithms with precisely determined timing operations. The NI sbRIO-9631 main board includes:

- integrated real-time controller, reconfigurable FPGA, and I/O on a single board,
- up to 128 MB DRAM, 256 MB non-volatile storage,
- analog and digital I/O,
- RS232 serial port for peripheral devices, Ethernet 10/100 Mb/s.

The sbRIO-9631 devices are programmed using the NI LabView graphical programming language. The real-time processor runs the LabView Real-Time Module on the Wind River VxWorks real-time operating system (RTOS) for extreme reliability.

It is possible to program the onboard reconfigurable FPGA on sbRIO-9631 devices using the LabView FPGA Module for high-speed control, custom I/O timing, and inline signal processing. The programmed device performs measurement of voltage level on object output n as well controlling analog outputs to motor U_s .

One of the application feature is a set of controller parameters like $k_p, k_i, k_d, \lambda, \mu$ and approximation parameters T, a . All those data are used to determine coefficients of approximating discrete transfer function (Fig. 5). For example, if controller parameters are $k_p = 3.75, k_i = 75, k_d = 0.1875, \lambda = 1, \mu = 0.5$ and approximating coefficients are $T = 0.005, a = 0.5$ than discrete transfer function

approximating the continuous fractional order transfer function (2) has the form

$$(7) \quad \begin{aligned} C(z^{-1}) &= 3.75 + 75 \frac{0.003333 + 0.001667z^{-1}}{1-z^{-1}} + \\ &+ 0.1875 \frac{138.6 - 86.6z^{-1}}{8+z^{-1}} = \\ &= \frac{-57.9808 + 67.2187z^{-1} - 12.613z^{-2}}{-8 + 7z^{-1} + z^{-2}}. \end{aligned}$$

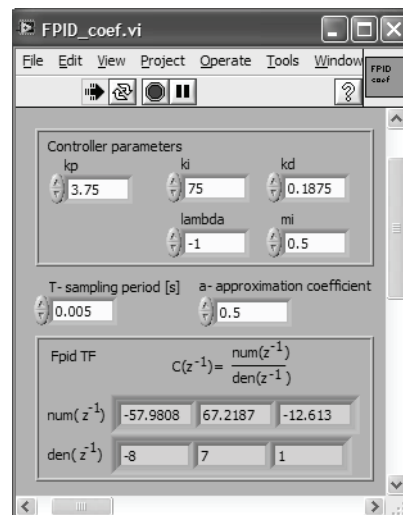


Fig.5. Program window for setting up fractional control coefficients

Experiments

Experiments are performed on laboratory test bench shown in Figure 6. The object is controlled by the sbRIO-9631 device. The software prepared is sent to the sbRIO environment where operations of measure, analysis, calculations and set up outputs take place.

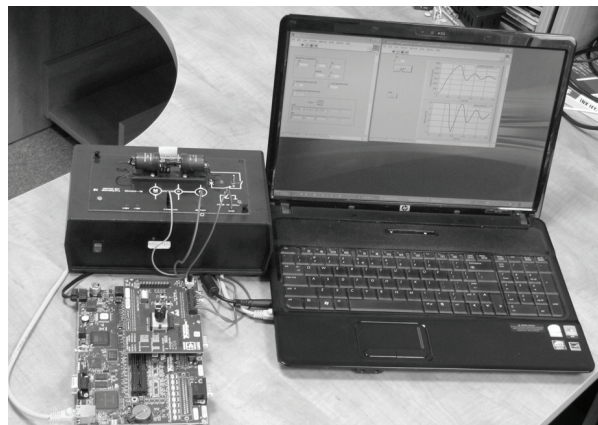


Fig.6. Laboratory test bench

The experiments made it possible to observe how the speed control loop operates as a result of the set point changing from 0 to 2000 rpm. All the experiments were carried out for sampling time $T=0.005s$. The speed characteristics obtained are shown in Figures 7, 8 and 9. The controller parameter values were chosen according to Table 1 which presents gain and phase margin specifications (points marked in Figure3). In these Figures there are two step responses. The first is obtained for the fractional PID controller (dashed line) and the second is obtained for the classical (integer) PID controller for the same values of k_p, k_i, k_d . From these Figures we can see

that a fractional order of μ causes faster time responses for points 1 and 2 versus using the classical PID controller.

The settling time and overshoot of time responses of the rotor speed for all the points marked in Figure 3 are listed in Table 2. It is shown that for the fractional controller the decreasing value of ϕ results in greater oscillations.

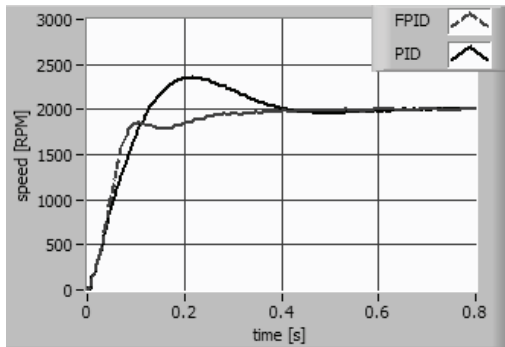


Fig.7. Transient responses of the closed-loop system with the fractional PID controller for point 1 (Table 1) for $a = 0.5$ and with the classical PID controller for the same values of k_p, k_i, k_d

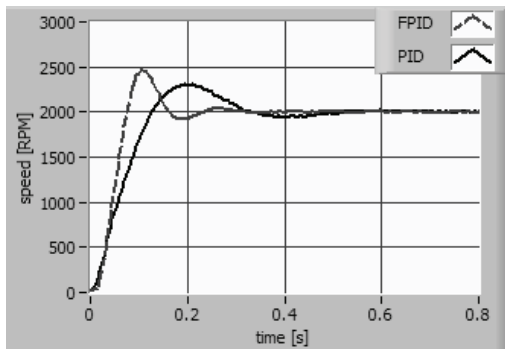


Fig.8. Transient responses of the closed-loop system with the fractional PID controller for point 2 (Table 1) for $a = 0.5$ and with the classical PID controller for the same values of k_p, k_i, k_d

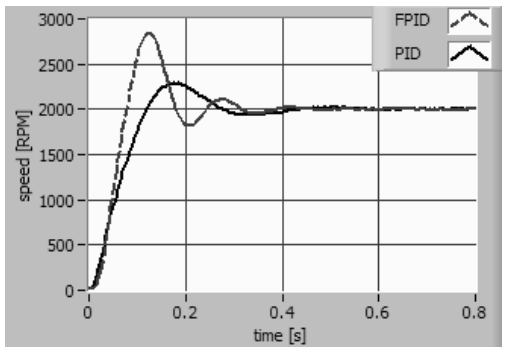


Fig.9. Transient responses of the closed-loop system with the fractional PID controller for point 3 (Table 1) for $a = 0.5$ and with the classical PID controller for the same values of k_p, k_i, k_d

Table 2. Overshoot and settling time

Point	Fractional order controller		Integral order controller	
	Settling time [s]	Overshoot [%]	Settling time [s]	Overshoot [%]
1	0.25	0.2	0.35	18.0
2	0.15	23.6	0.30	15.3
3	0.29	42.4	0.25	14.5

Now, we choose controller values for point 2 and check the influence of approximating parameter a for time

responses. The settling time and overshoot of time responses of the rotor speed for specified values of a are listed in Table 3. It is shown that for smaller value of a we obtain a shorter settling time and lower overshoot. Examples of step responses for $a = 0.005$ and $a = 1$ are shown in Figures 10 and 11 respectively.

Table 3. Overshoot and settling time for different values of a

Parameter a	Fractional order controller	
	Settling time [s]	Overshoot [%]
0.005	0.14	5.9
0.1	0.16	9.8
0.2	0.18	15.4
0.3	0.19	18.0
0.4	0.19	20.7
0.5	0.20	23.0
0.6	0.21	26.0
0.7	0.21	25.9
0.8	0.22	29.4
0.9	0.22	29.0
1	0.19	33.6

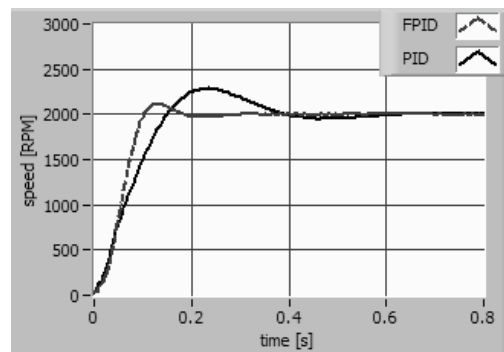


Fig.10. Transient responses of the closed-loop system with the fractional PID controller for point 2 (Table 1) for $a = 0.005$ and with the classical PID controller for the same values of k_p, k_i, k_d

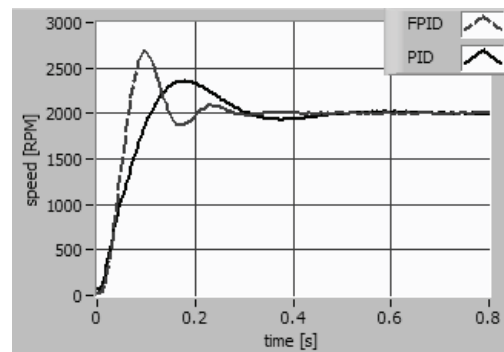


Fig.11. Transient responses of the closed-loop system with the fractional PID controller for point 2 (Table 1) for $a = 1$ and with the classical PID controller for the same values of k_p, k_i, k_d

Now, we choose controller values for point 2 and check the influence of the fractional order μ for time responses. The settling time and overshoot of time responses of the rotor speed for specified values of μ are listed in Table 4. It is shown that for lower value of μ we obtain a shorter settling time and lower overshoot. Examples of step responses for $\mu = 0.1$ and $\mu = 0.9$ are shown in Figures 12 and 13 respectively.

Table 4. Overshoot and settling time for different values of μ

Parameter μ	Fractional order controller	
	Settling time [s]	Overshoot [%]
0.1	0.15	5.1
0.2	0.15	6.0
0.3	0.15	7.6
0.4	0.16	11.8
0.5	0.16	12.5
0.6	0.17	12.6
0.7	0.19	14.0
0.8	0.23	16.0
0.9	0.27	18.6
1	0.33	22.7

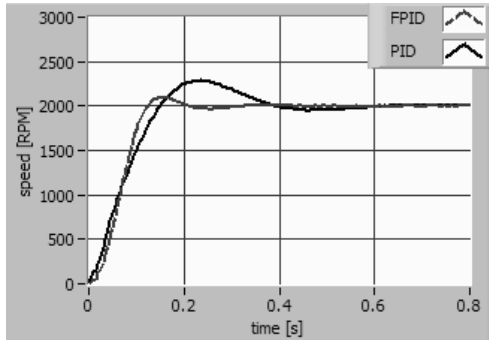


Fig.12. Transient responses of the closed-loop system with the fractional PID controller for $k_p = 3.75$, $k_i = 75$, $k_d = 0.19$, $\lambda = 1$, $\mu = 0.1$, $\alpha = 0.2$ and with the classical PID controller for the same values of k_p , k_i , k_d

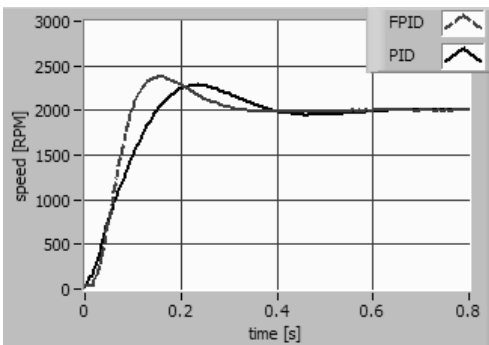


Fig.13. Transient responses of the closed-loop system with the fractional PID controller for $k_p = 3.75$, $k_i = 75$, $k_d = 0.19$, $\lambda = 1$, $\mu = 0.9$, $\alpha = 0.2$ and with the classical PID controller for the same values of k_p , k_i , k_d

Conclusion

In this paper the practical realization of the fractional order controller implemented into National Instruments sbRIO-9631 controller programmed in LabView is given. Such a controller is tested and verified in the real rotor speed control system. Comparative studies of this control systems are presented. It was shown that if we have the same values of k_p , k_i , k_d and two controllers (fractional PID and classical PID) than the fractional order of differentiation μ causes faster time responses. However the value of approximating parameter α is very important. The studies have shown that for lower values of α shorter time responses with lower overshoot were obtained.

Parametric synthesis of the fractional order PID controller was made using the method of determining the stability region in the parameter space of the controller for specified gain and phase margin requirements [18].

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