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Control of a Uniform Step Asymmetrical Multilevel Inverter using Particle Swarm Optimization

***Abstract**. Harmonic Elimination Strategy (HES) has been a widely researched alternative to traditional PWM techniques. This paper presents the harmonic elimination strategy of a Uniform Step Asymmetrical Multilevel Inverter (USAMI) using Particle Swarm Optimization (PSO) which eliminates specified higher order harmonics while maintaining the required fundamental voltage. This method can be applied to USAMI with any number of levels. As an example, in this paper a seven-level USAMI is considered and the optimum switching angles are calculated to eliminate the fifth and seventh harmonics. The HES-PSO approach is compared to the well-known Sinusoidal Pulse-Width Modulation (SPWM) strategy. Simulation results demonstrate the better performances and technical advantages of the HES-PSO controller in feeding an asynchronous machine. Indeed, the harmonic distortions are efficiently cancelled providing thus an optimized control signal for the asynchronous machine. Moreover, the technique presented here substantially reduces the torque undulations.*

***Streszczenie.** W artykule zaprezentowano strategię eliminacji harmonicznych HES w przekształtniku wielopoziomowym asymetrycznym USAMI. Wykorzystano metodę algorytmów rojowych i porównano tę metodę z klasyczną metoda PWM. (**Sterowanie wielopoziomowym symetrycznym przekształtnikiem USAMI przy wykorzystaniu algorytmów rojowych**)*

Keywords: Uniform step asymmetrical multilevel inverter (USAMI), Harmonic Elimination Strategy (HES), Particle Swarm Optimization (PSO), Sinusoidal Pulse-Width Modulation (SPWM).
Słowa kluczowe: przekształtniki USAMI, sterowanie, eliminacja harmonicznych, algorytm rojowy.

1. Introduction

Multilevel inverters have been widely used in last years for high-power applications [1]. Variable-speed drives have reached a wide range of standard applications such as pumps, fans and others. Many of these applications use medium-voltage motors (2300, 3300, 4160 or 6600V), due to their lower current ratings in higher power levels [2]. Static Var compensators and active filters are other applications that use multilevel converters [3].

Several topologies of multilevel inverters have been studied and presented. Among them, neutral point clamped inverters [4], flying capacitors inverters also called imbricated cells [5], and series connected cells inverters also called cascaded inverters [6]. The industry often has used the neutral-point-clamped inverter [7]. However, the topology that uses series connected cells inverters presents some advantages, as smaller voltage rate (dU/dt) due to existence of higher number levels, producing less common-mode voltage across motor windings [8]. Furthermore, this topology is simple and its modular configuration makes it easily extensible for any number of desired output voltage levels. Fig.1a shows the basic diagram of this topology with k partial cells represented by Fig.1b. The j^{th} single-phase inverter is supplied by a dc-voltage source U_{dj} ($j = 1 \dots k$). The relationship between the number of series-connected single-phase inverters in each phase and the number of output voltage levels generated by this topology, respectively k and N, is given by: $N = 2k + 1$, in the case where there are equal voltages in all partial inverters.

In all the well-known multilevel converter topologies, the number of power devices required depends on the output voltage level needed [9]. However, increasing the number of power semiconductor switches also increases the converter circuit and control complexity and the costs. To provide a large number of output levels without increasing the number of converters, a uniform step asymmetrical multilevel inverters (USAMI) can be used [10].

The key issue in designing an effective multilevel inverter is to ensure that the Total Harmonic Distortion (THD) of the output voltage waveform is within acceptable limits. Harmonic Elimination Strategy (HES) has been intensively studied in order to achieve low THD [11]. The output voltage waveform analysis using Fourier theory produces a set of non-linear transcendental equations. The solution of these equations, if exists, gives the switching angles required for certain fundamental component and selected harmonic profile. Iterative procedures such as Newton-Raphson method has been used to solve these sets of equations [12]. This method is derivative-dependent and may end in local optima, and a judicious choice of the initial values alone guarantees conversion [13]. Another approach based on converting the transcendental equation into polynomial equations is presented in [14, 15], where resultant theory is applied to determine the switching angles to eliminate specific harmonics. That approach, however, appears to be unattractive because as the number of inverter levels increases, so does the degree of the polynomials of the mathematical model. This is likely to lead to numerical difficulty and substantial computational burden as well.

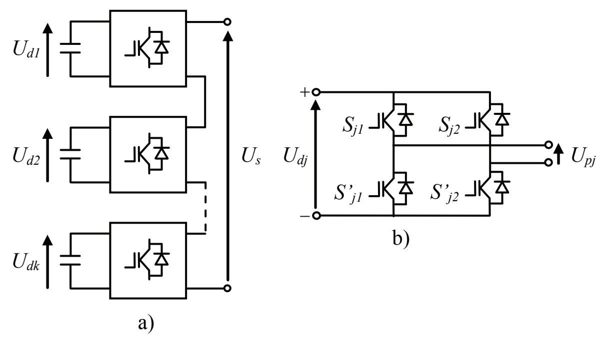

Fig.1. a) A series-connected multilevel inverter topology with k partial cells, b) Partial cell configuration

Genetic algorithms (GA) have been used to obtain optimal solutions for inverter circuits supplied from constant dc sources [16, 17]. Despite their effectiveness in harmonic elimination strategy, they are complicated and their parameters such as crossover and mutation probability, population size and number of generations are usually selected as common values given in literature or by means of a trial and error process to achieve the best solution set.

Heuristic algorithms such as Particle Swarm Optimization (PSO) [18] have the ability to combat the above drawbacks. As an optimization technique, PSO is much less dependent on the start values of the variables in the optimization problem when compared with the widely used Newton-Raphson or mathematical programming techniques such as Sequential Quadratic Programming (SQP). In addition, PSO does not rely on the guidance of the gradient information, such as the Jacobian matrix, hence it is more capable of determining the global optimum solution. PSO deal with all problems that usually considered very hard for researchers, such as integer variables, non-convex functions, non-differentiable functions, domains not connected, badly behaved functions, multiple local optima, and multiple objectives [19, 20]. For these reasons, PSO has been adopted in this study.

This paper presents an optimal minimization technique assisted with PSO algorithm in order to highly reduce the computational burden associated with the solution of the non-linear transcendental equations of the harmonic elimination strategy of a seven-level USAMI. The performance of this HES-PSO approach is evaluated and compared to the SPWM technique. The proposed HES-PSO strategy is also evaluated when the inverter supplies an asynchronous machine. In this application, it is important that the implemented controller computes appropriate switching angles for the inverters in order to minimize the harmonics absorbed by the asynchronous machine. Performances where successfully achieved, the HES-PSO controller demonstrates a satisfying behavior and a good robustness.

The paper is organized as follows. USAMIs are described and modeled in Section 2. Section 3 briefly introduces the well-known Sinusoidal PWM and brings out the original HES using PSO. Section 4 evaluates the proposed HES-PSO strategy in computing optimal angles of 7-level inverter used to supply an asynchronous machine. Results show that the HES-PSO method cancels the harmonic distortions and supplies the machine with a well-formed sinusoidal voltage waveform. In Section 5, we conclude with final remarks.

2. Uniform step asymmetrical multilevel inverter

Fig.1b shows a detail of the partial cells and the main notations used. Each couple of switches S_{jx} and S'_{jx} ($x = 1, 2$; $j = 1\ldots k$) is controlled by a couple of switching functions M_{jx} and $M'_{jx} \in \{0, 1\}$ such that:

$$M_{jx} + M'_{jx} = 1 \quad (1)$$

The conversion of the switch commutations into a voltage is described by a conversion function, F_j such that:

$$F_j = M_{jx} - M'_{jx} \Rightarrow F_j \in \{-1, 0, +1\} \quad (2)$$

The output voltage of each cell is given by:

$$U_{pj} = F_j * U_{dj} \Rightarrow U_{pj} \in \{-U_{dj}, 0, +U_{dj}\} \quad (3)$$

Equation (3) shows that each partial cell can generate three different levels. The output voltage of the multilevel converter is given by:

$$U_s = U_{p1} + U_{p2} + \cdots U_{pk} \quad (4)$$

A series-connected multilevel inverter is known as asymmetric, if at least one to the dc-voltage sources feeding the partial inverters is different of the others. Three conditions have been established for the design of a uniform or regular step AMI (i.e., the steps ΔU between all voltage levels are equal, in this case the step is equal to the smallest dc-voltage U_{d1}) [10]:

1-the dc-voltage sources must be arranged in an increasing way $U_{d(h-1)} \le U_{dh}$, $\forall\, h = 2\ldots k$;

2-the ratio between two consecutive inverters must be an integer $U_{dh}/U_{d(h-1)} = \delta_h$, $\delta_h \in N^*$;

3-the j^{th} partial cell must be fed by the voltage U_{dj} such that:

$$U_{dj} \le 1 + 2\sum_{l=1}^{j-1} U_{dl} \quad (5)$$

If these three conditions are satisfied, the multilevel inverter will generate an output voltage U_s with N regular different levels such that:

$$N = 1 + 2\sum_{j=1}^{k} \frac{U_{dj}}{U_{d1}} \quad (6)$$

For example the generation of a seven-level output voltage can be achieved with the following dc-voltage sources: $U_{d1} = 1$ $p.u.$ and $U_{d2} = 2$ $p.u.$. According to (5) and (6), more than one possible dc-voltage setting can be chosen to generate the same number of levels. Fig.2 shows the possible output voltage of the two partial cells. There are two possible commutation sequences that result in $U_s = 1$ $p.u.$: $(U_{p1}, U_{p2}) \in \{(-1, 2), (1, 0)\}$. The dashed lines in Fig.2 show the commutation sequence $(U_{p1}, U_{p2}) = (1, 0)$. The possible redundant switching states in a multilevel converter are a degree of freedom which is usually used to optimise its performances with an appropriate modulation strategy [21].

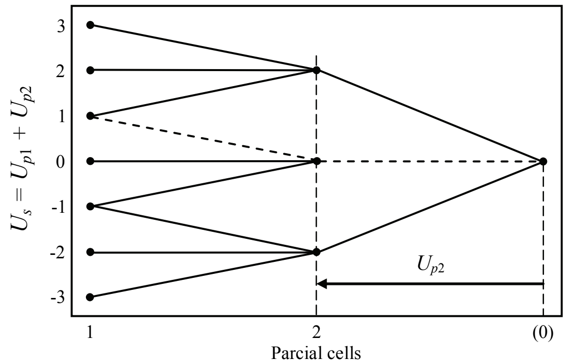

Fig. 2. Possible output voltages of each partial inverter to generate $N = 7$ levels with $k = 2$ cells, $U_{d1} = 1p.u.$ and $U_{d2} = 2p.u.$

3. Multilevel inverters control strategies

Among several modulation strategies, the multi-carrier sub-harmonic PWM technique has been receiving an increasing attention for symmetrical multilevel converters [10]. This modulation method can also be used to control asymmetrical multilevel power converters. Other kinds of modulation techniques can also be used in the case of AMIs [22].

This section briefly presents the sinusoidal PWM technique. We also propose a HES based on PSO. These control strategies will be compared by computer simulations. The objective is to elaborate optimized switching angles for a 7-level USAMI in order to supply an asynchronous machine.

3.1. Sinusoidal Pulse-Width Modulation (SPWM)

The SPWM is also known as the multi-carrier PWM because it relies on a comparison between a sinusoidal reference waveform and vertically shifted carrier waveforms. $N-1$ carriers are therefore required to generate N levels. The carriers are in continuous bands around the zero reference. They have the same amplitude A_c and the same frequency f_c. The sine reference waveform has a frequency f_r and an amplitude A_r. At each instant, the result of the comparison is 1 if the triangular carrier is greater than the reference signal and 0 otherwise. The output of the modulator is the sum of the different comparisons which represents the voltage level. The strategy is therefore characterized by the two following parameters [23], respectively called the modulation index and the modulation rate:

(7) $$m = \frac{f_c}{f_r}$$

(8) $$r = \frac{2}{N-1}\frac{A_r}{A_c}$$

We propose to develop a 7-level inverter composed of $k = 2$ partial inverters per phase with the following dc-voltage sources: $U_{d1} = 300$V and $U_{d2} = 600$V. The output voltage U_{ab} and its frequency representation are respectively presented by Fig. 3 and Fig. 4.

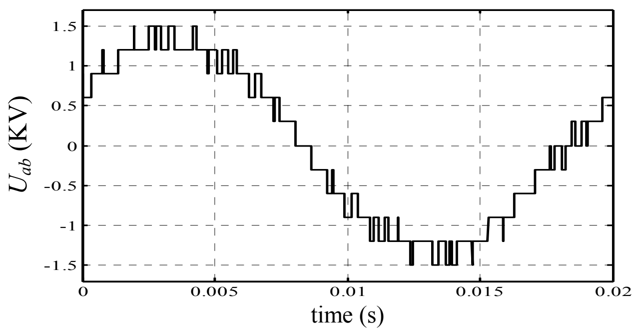

Fig. 3. Output voltage U_{ab} of the 7-level USAMI controlled by the SPWM (with $m = 18$ and $r = 0.85$)

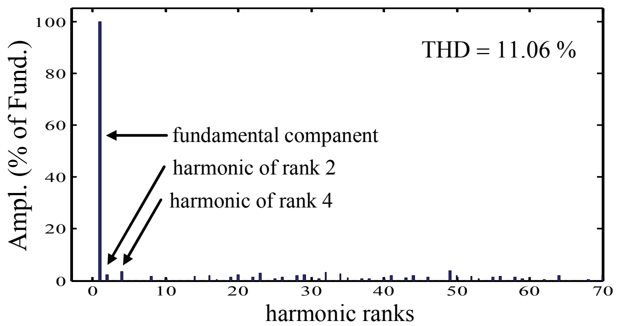

Fig. 4. Frequency content of the output voltage U_{ab} with the SPWM strategy (with $m = 18$ and $r = 0.85$)

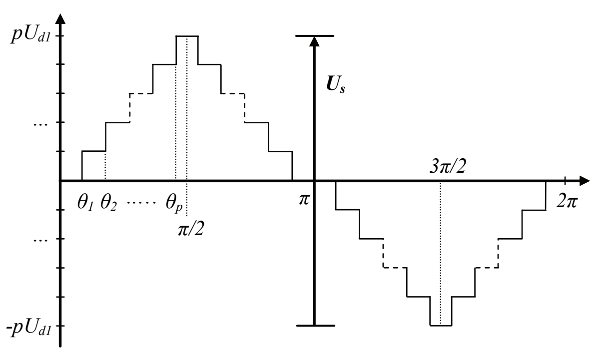

Fig. 5. Output voltage waveform of an N-level USAMI

3.2 Harmonics Elimination Strategy with PSO

A) Harmonics Elimination Strategy (HES): The HES is based on the Fourier analysis of the generated voltage U_s at the output of the USAMI [22]. This voltage shown in Fig. 5 is symmetric in a half and a quarter of a period. As a result, the even harmonic components are null. The Fourier series expansion for the U_s voltage is thus:

(9) $$U_s = \sum_{n=1}^{\infty} U_n \sin(n\omega t), \text{ with } U_n = \frac{4U_{d1}}{n\pi}\sum_{i=1}^{p}\cos(n\theta_i)$$

Where U_n is the amplitude of the harmonic term of rank n, $p = (N-1)/2$ is the number of switching angles per quarter waveform, and θ_i is the ith switching angle.

The p switching angles in (9) are calculated by fixing the amplitude of the fundamental term and by canceling the $p - 1$ other harmonic terms. Practically, three switching angles (θ_1, θ_2 and θ_3) are necessary for canceling the two first harmonics terms (i.e., harmonics with a odd rank and non multiple of 3, therefore 5 and 7) in the case of a three phase 7-level USAMI composed of $k = 2$ partial inverters per phase supplied by the dc-voltages $U_{d1} = 300$V and $U_{d2} = 600$V. These switching angles can be determined by solving the following system of non linear equations:

(10) $$\begin{cases} \sum_{i=1}^{p=3}\cos(\theta_i) = \frac{3\pi}{4}r \\ \sum_{i=1}^{p=3}\cos(n\theta_i) = 0 \text{ for } n \in \{5, 7\} \end{cases}$$

where $r = U_1/3U_{d1}$ is the modulation rate. The solution of (10) must also satisfy

(11) $$0 < \theta_1 < \theta_2 < \theta_3 < \frac{\pi}{2}$$

An objective function is then needed for the optimization procedure, which is selected as a measure of effectiveness of eliminating selected order of harmonics while maintaining the fundamental component at a pre-specified value. Therefore, this objective function is defined as:

(12) $$F(\theta_1, \theta_2, \theta_3) = \left(\sum_{n=1}^{3}\cos(\theta_n) - \frac{3\pi}{4}r\right)^2 + \left(\sum_{n=1}^{3}\cos(5\theta_n)\right)^2 + \left(\sum_{n=1}^{3}\cos(7\theta_n)\right)^2$$

The optimal switching angles are obtained by minimizing Eq. (12) subject to the constraint (11), and consequently the required harmonic profile is achieved. The main challenge is the non-linearity of the transcendental set of Eq. (10), as most iterative techniques suffer from convergence problems and other techniques such as elimination using resultant and GA are complicated.

B) Particle swarm optimization (PSO): Particle swarm optimization is an intelligent algorithm which relies on exchanging information through social interaction among particles. The PSO conducts searches using a swarm of particles randomly generated initially. Each particle i ($i = 1$ to swarm size) possesses a current position $p_i = [p_{i1}\ p_{i2}\ \ldots\ p_{iq}]$ and a velocity $v_i = [v_{i1}\ v_{i2}\ \ldots\ v_{iq}]$, q is the dimension of search space. The position of the particle represents a possible solution of the problem. The velocity indicates the change in the position from one step to the next. Each particle memorizes its personal best position ($pbest_i$) which corresponds to the best objective function value in the searched places. Each particle can also access to the global best position ($gbest$) that is the overall best place found by one member of the swarm. Namely, particles profit from their own experiences and previous experience of other particles during the exploration, to adjust their velocity, in direction and amount [24, 25]. The concept of a moving particle is illustrated in Fig. 6.

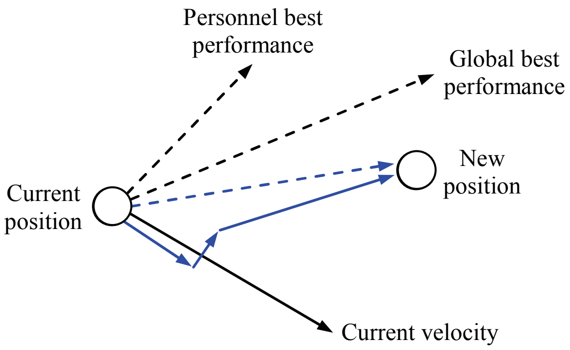

Fig. 6. Concept of modification of searching points

The velocity of each particle can be updated iteratively according to the following rule:

$$v_i(j+1) = wv_i(j) + c_1 r_1 (pbest_i - p_i(j)) + c_2 r_2 (gbest - p_i(j)) \quad (13)$$

where $v_i(j)$ is the current velocity of particle i at iteration j and $p_i(j)$ is the current position of particle i at iteration j.

The inertia weight w governs how much of previous velocity should be retained from the previous time step. The acceleration coefficients $c_1 > 0$ and $c_2 > 0$ influence the maximum size of the step that a particle can take in a single iteration. r_1 and r_2 are two independent random sequences uniformly distributed in [0, 1]. These sequences are used to affect the stochastic nature of the algorithm.

The first term of right-hand side of the velocity update equation is the inertia velocity of particle, which reflects the memory behavior of particle. The second term in the velocity update equation is associated with cognition since it only takes into account the private thinking and own experiences of particles. This component is a linear attraction toward the local best position ever found by the given particle. But the third term in the same equation represents the social collaboration and interaction between the particles. This component is a linear attraction toward the global best position found by any particle.

Each particle investigates the search space from its new local position using the following equation:

$$p_i(j+1) = p_i(j) + v_i(j) \quad (14)$$

After a number of iterations, the particles will eventually cluster around the area where fittest solutions are.

C) Implementation of PSO for HES: In order to describe the implementation of the PSO in harmonic elimination strategy of multilevel inverters, the following algorithm is adopted.

Step 1: Initialization
for each particle:
– Initialize the position $\theta_i(0) = [\theta_{i1}(0)\ \theta_{i2}(0)\ \ldots\ \theta_{ip}(0)]$ of each particle with random angles that respect the constraint (11);
– Initialize the velocity $v_{\theta i}(0) = [v_{\theta i1}(0)\ v_{\theta i2}(0)\ \ldots\ v_{\theta ip}(0)]$ of each particle to random values;
– Initialize the best objective function F_pbest_i of particle i.
end for
– Initialization of the best objective function F_gbest of the swarm.
Loop
{
for each particle
Step 2: Objective function evaluation
– Compute the F_i value of each particle i of the swarm using the cost function given by (12);
Step 3: Personal best position updating
if $F_i < F\_pbest_i$
then $F\_pbest_i = F_i$ and $\theta_{pbesti} = \theta_i$
end if
Step 4: Global best position updating
if $F_i < F\_gbest$
then $F\_gbest = F_i$ and $\theta_{gbest} = \theta_i$
end if
end for
for each particle
Step 5: Position and velocity updating
$v_{\theta i} = wv_{\theta i} + c_1 r_1(\theta_{pbesti} - \theta_i) + c_2 r_2(\theta_{gbest} - \theta_i)$
$\theta_i = \theta_i + v_{\theta i}$
end for
} Until a sufficiently good objective function value is reached.

This algorithm was used to find the switching angles for each phase in the case of a seven-level USAMI. The parameters selected for the implementation of PSO are: the population size = 20 particles, w = 0.75 and $c_1 = c_2 = 1.8$. The results for phase *a* are plotted in Fig. 7 versus r, where $r \in [0.3, 1.3]$ with a step of 0.001. One can notice that for r in the interval [0.488, 1.069] there is at least one solution. Furthermore, there are two different sets of solutions in the interval [0.632, 0.785]. On the other side, for $r \in [0, 0.339]$, $r \in [0.352, 0.483]$, and $r \in [1.073, 1.175]$ there are no solutions. Interestingly, for $r \approx 0.339$, $r \approx 0.343$ and $r \approx 1.171$ there are isolated solutions.

In the case of two possible solutions of an angle θ_i, one clear way to choose a particular solution is simply to pick the one that results in the lowest THD given by

$$\mathrm{THD} = \frac{\sqrt{\sum_{n=5,7,\ldots}^{\infty} \left(\frac{1}{n} \sum_{i=1}^{p=3} \cos(n\theta_i) \right)^2}}{\sum_{i=1}^{p=3} \cos(\theta_i)} \quad (15)$$

The THD corresponding to the solutions given in Fig. 7 is represented by Fig. 8. Selecting the switching angles that lead to the lowest THD results in the solutions given by Fig. 9. The corresponding THD is shown on Fig. 10.

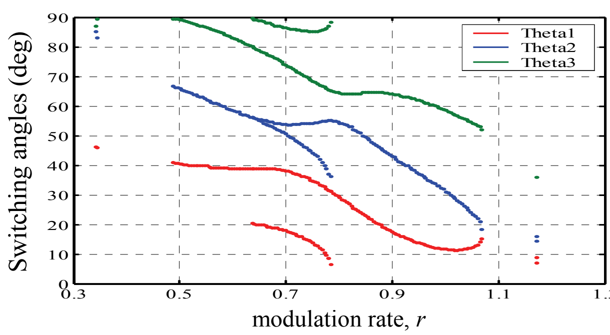

Fig. 7. All switching angles versus r for a 7-level USAMI

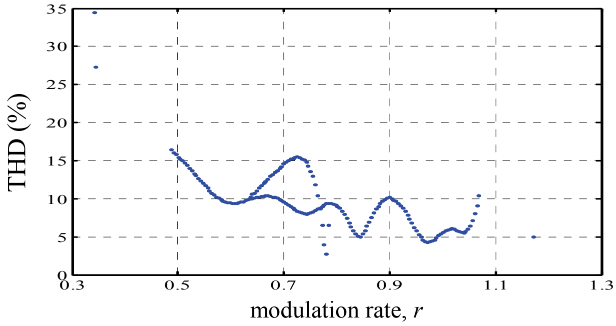

Fig. 8. THD versus r for all switching angles

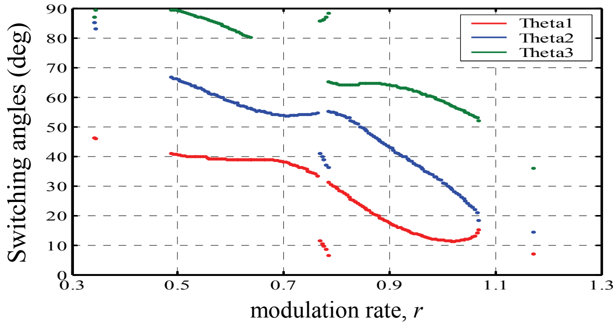

Fig. 9. Switching angles versus r which give the lowest THD

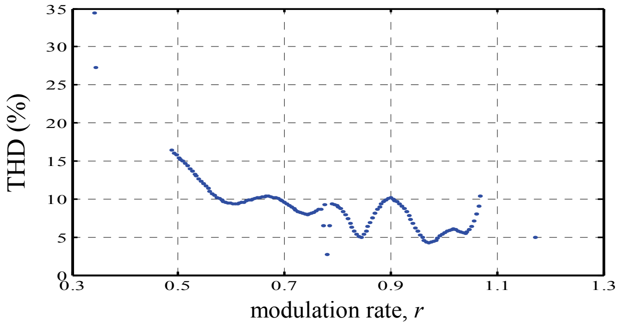

Fig. 10. THD for the switching angles that give the lowest THD

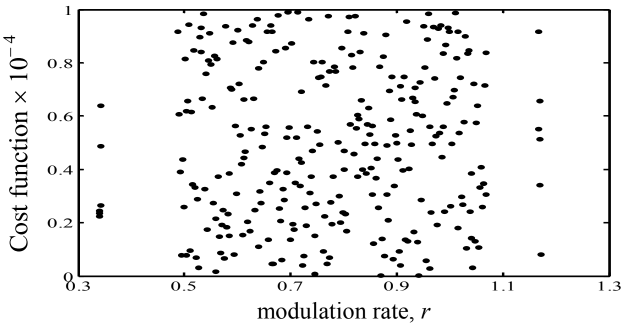

Fig. 11. Cost function versus r

As seen on Fig. 11, any solution that yields a cost function less than 0.0001 is accepted. We clearly notice that the number of solutions for each r increases or decreases in according to precision constraint value by which solutions are calculated.

Using the optimal switching angles calculated above, simulations have been conducted to verify that the fundamental frequency switching can achieve high control performance.

Fig. 12 shows the output voltage U_{ab} for $r = 0.85$ which corresponds to $\theta_1 = 22.7632^o$, $\theta_2 = 49.3781^o$ and $\theta_3 = 64.5567^o$. Fig. 13 shows the frequency content of U_{ab} which can be compared to Fig. 4 obtained with the SPWM.

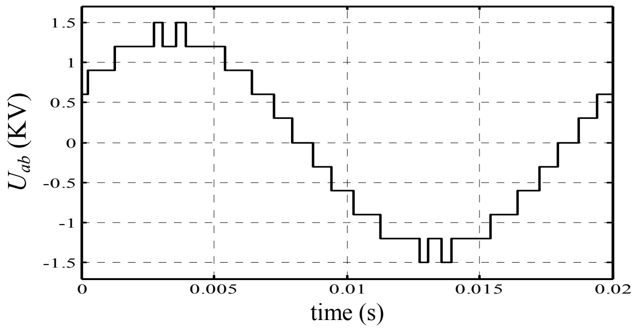

Fig. 12. Output voltage U_{ab} of the 7-level USAMI controlled by the HES-PSO (with $r = 0.85$)

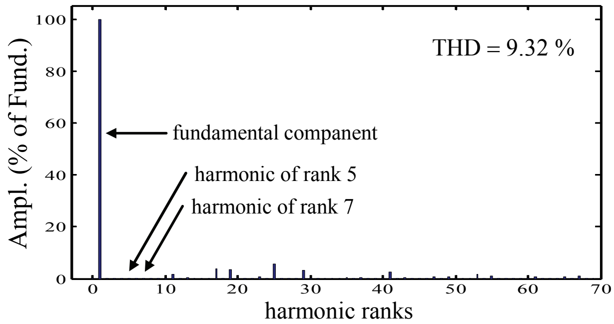

Fig. 13. Frequency content of the output voltage U_{ab} with the HES-PSO strategy (with $r = 0.85$)

4. Performance in Supplying an Asynchronous Machine

In order to evaluate the performance and the robustness of the proposed approach, a 7-level USAMI is used to supply an asynchronous machine with the following data: rated power P_n = 1MW, stator resistance R_s = 0.228Ω, rotor resistance R_r = 0.332Ω, stator inductance L_s = 0.0084H, rotor inductance L_r = 0.0082H, mutual inductance L_m = 0.0078H, number of pole pairs P = 3, rotor inertia J = 20kg.m^2, viscous friction coefficient K_f = 0.008 Nm.s.rad^{-1}.

The HES-PSO is compared to the SPWM strategy in controlling the 7-level USAMI. The objective is to use the proposed PSO strategy in order to minimize the harmonics absorbed by the asynchronous machine.

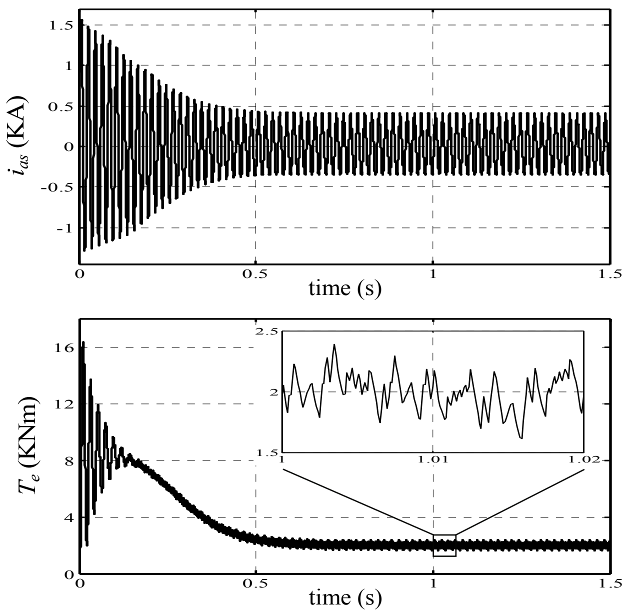

Fig. 14. Stator current (top) and electromagnetic torque (bottom) of the asynchronous machine fed by a 7-level USAMI controlled by the SPWM

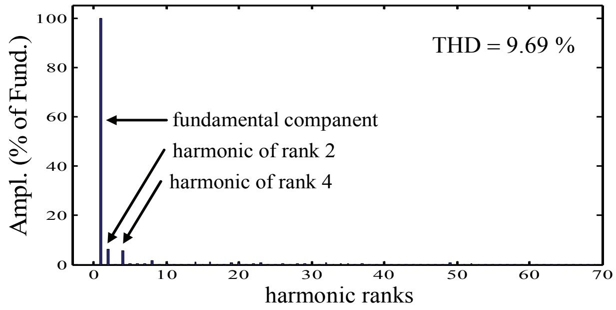

Fig. 15. Frequency content of the stator current of the asynchronous machine fed by a 7-level USAMI controlled by the SPWM

The results of the control based on the SPWM are presented by Fig. 14 and Fig. 15. This first figure shows the stator current and the electromagnetic torque with significant fluctuations. The second figure shows the frequency content of the stator current. Results by using PSO approach of the HES are presented by Fig. 16 and Fig. 17. By comparing Fig. 15 to Fig. 17, it can be deduced that the HES-PSO efficiently cancels the harmonics of ranks 5 and 7 from the output voltage U_{ab}. Moreover, the amplitudes of the harmonic distortions are very small compared to the amplitude of the fundamental component.

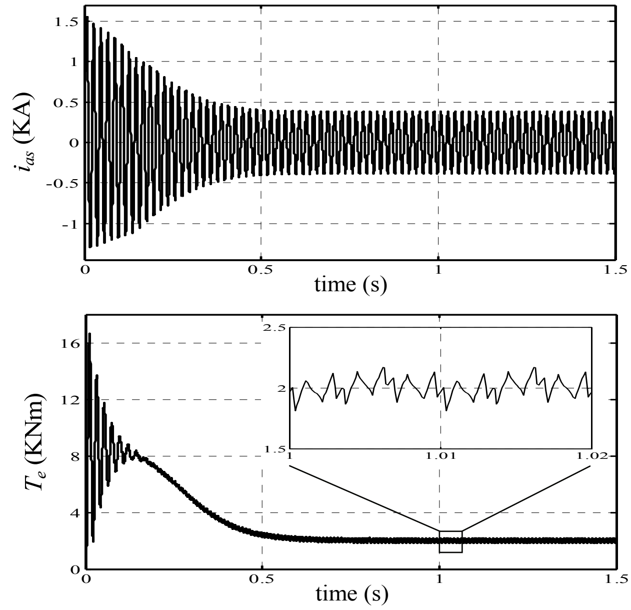

Fig. 16. Stator current (top) and electromagnetic torque (bottom) of the asynchronous machine fed by a 7-level USAMI controlled by the HES-PSO

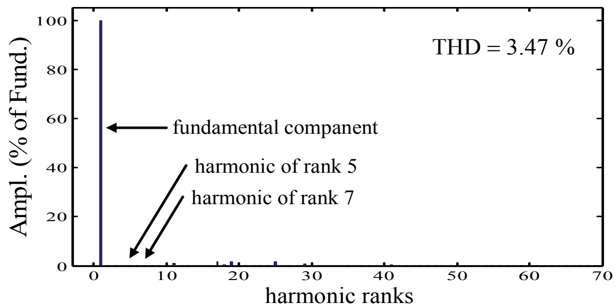

Fig. 17. Frequency content of the stator current of the asynchronous machine fed by a 7-level USAMI controlled by the HES-PSO

Table 1. Performances of the control methods

Control method	U_{ab} THD (%)	i_{as} THD (%)	f_{Te} (Hz)	Δ_{Te} (Nm)	Nb of θ_i
SPWM	11.06	9.69	f	775.66	$2m = 36$
HES-PSO	9.32	3.47	$2f$	360.87	$4p = 12$

Performances obtained with both methods are summarized in Table 1. The THD measured on U_{ab} and resulting from the PSO approach of the HES is smaller than the one obtained with the SPWM method. The THD measured on the stator current i_{as} is reduced by a factor 2.79 with the HES-PSO compared to the SPWM method. The control is thus optimized with the HES-PSO in order to avoid the asynchronous machine to absorb harmonics.

It can also be seen that the electromagnetic torque continuously oscillates at a frequency f with the SPWM method (because of the harmonics of rank 2 and 4 which are present in the output voltage). The torque oscillates at $2f$ with the HES-PSO approach. The HES-PSO method also reduced the number of switching angles by a factor 3 compared to the SPWM method which is highly appreciated for the electronic devices.

5. Conclusions

In this paper, a novel strategy to eliminate harmonics in USAMIs has been described which exploits the swarm intelligence. Particle swarm optimization is used to improve the harmonic elimination technique for USAMIs, which exhibits clear advantages in term of low switching frequency and high output quality. This study has shown that the particle swarm optimization is more suitable for USAMIs optimal control design. This optimization algorithm is simple to implement, effective and inexpensive in term of memory and time required. The proposed HES-PSO approach is compared to the SPWM strategy. Simulation results are given to show the high performance and technical advantages of the PSO implementation of the HES for the control of a uniform step asymmetrical 7-level inverter. The HES-PSO approach is used to feed an asynchronous machine. The harmonic distortions are efficiently cancelled and the torque undulations are thus significantly reduced.

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