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Parameters of Superconducting Magnets with Racetrack-Shaped Coils and Support Structure Placed Inside Torus

***Abstract**. Toroidal SMES composed of racetrack-shaped coils with spokes inside torus is considered. The volume of superconducting winding in comparison with usual O-shaped coils is considerably lesser. The support system volume requirements do not exceed the corresponding values for O-shaped torus with spokes and D-shaped toroidal system. For these reasons proposed configuration of SMES may be alternatives to conventional coil systems.*

***Streszczenie.** Artykuł opisuje SMES o kształcie toru wyścigowego (racetrack-shaped), które jest wspomagane szprychami wewnątrz torusa. Objętość uzwojenia nadprzewodzącego jest znacząco mniejsza niż w przypadku cewek o kształcie O (O-shaped). Wymagania wytrzymałości konstrukcyjnej nie przekraczają odpowiednich wartości dla torusa ze szprychami o kształcie O oraz systemu toroidalnego o kształcie D (D-shaped). (**Parametry magnesu nadprzewodzącego o cewkach w kształcie toru wyścigowego i wspornikach umieszczonych wewnątrz torusa**)*

Keywords: superconducting magnetic energy storage, toroidal solenoid, racetrack-shape coils, support structure, parametrical analysis.
Słowa kluczowe: nadprzewodzący magnes magazynujący energię, cewki o kształcie tory wyścigowego, analiza parametrów

Introduction and SMES configuration

Superconducting magnetic energy storage (SMES) with elements of mechanical support system placed inside torus is considered. As it shown in [1] the toroidal systems with spokes placed inside each O-shaped coil solve two mechanical problems: eliminating bending moments in the support system and provide uniform mechanical stress in all the spokes and supporting structure. As a result, the volume of the support system is significantly below compare with traditional O-shaped coils with belts around coils, and this volume is about the same for more complicated D-shaped toroidal system. On the other hand the O-shaped torus has larger volume of superconducting winding than the D-shaped toroidal system.

The main purpose of this paper is to investigate the volumes of superconducting and structural materials for racetrack-shaped toroidal system.

The toroidal system with radius R formed by many number of coils N with current I_1.

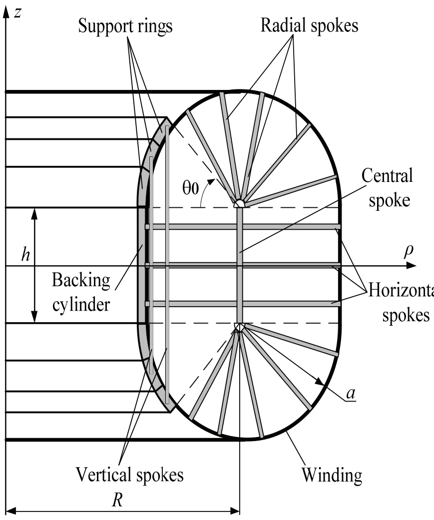


Fig. 1.Racetrack-shaped coil and main components of support system

Each coil has upper and lower semicircular parts with radius a and central straight part with length h (Fig. 1). The set of spokes are in tension. The sum of forces in the radial spokes must be equal to zero. Therefore the radial spokes are absent in a sector with the angular size $2\theta_0$. Here as well as at straight part the electromagnetic forces are balanced by reaction compression forces in support rings and backing cylinder.

Basic equations and main results

Usually for SMES initial parameters are the following: energy of magnetic field W permissible maximum values of induction of magnetic field B_m and density current j_m of superconducting winding, construction material properties of the support system (permissible values of stresses in tension σ_c and compression σ_t). In this paper, the following required SMES parameters are considered: volume of superconducting winding V_{sc}, volume of the mechanical support system material counteractive to forces of tension V_t and compression V_c.

The geometrical parameters which characterize the toroidal magnet configuration are relative size of torus $\varepsilon = a/R$ and ratio of vertical size to radial size of torus cross-section $\lambda = (2a+h)/2a$.

Let's define dependencies between initial and required parameters. The basic equations are given by [2] where dimensional parameters W, B_m, j_m, σ_c, σ_t and dimensionless ones are separated. It allows to make the parametrical analysis of magnets system, including effect of initial parameters and geometrical configuration which depends on parameters ε and λ.

Volume of superconducting materials

Equation for energy of magnetic field may be written as:

(1) $$W = \frac{1}{4}\mu_0 I^2 R k_W(\varepsilon, \lambda),$$

where $I = NI_1$ is total current of all N coils of the toroidal magnetic system; dimensionless parameter k_W is

(2) $$k_W(\varepsilon, \lambda) = 2\left(1 - \sqrt{1-\varepsilon^2} + \frac{\varepsilon(\lambda-1)}{\pi} ln\frac{1+\varepsilon}{1-\varepsilon}\right).$$

The maximal value of induction of the magnetic field on winding takes place on the internal side of torus:

(3) $$B_{\rm m} = \frac{\mu_0 I}{2\pi R} k_B(\varepsilon),\; k_B(\varepsilon) = \frac{1}{1-\varepsilon}.$$

Total cross-section of all coils is $S = I / j_m$ and the volume of conductor winding $V_{sc} = Sl$ is given as

(4) $$V_{sc} = \frac{IR}{j_m} k_l(\varepsilon, \lambda) = \frac{W^{2/3}}{j_m B_m^{1/3} \mu_0^{1/3}} k_{VSC}(\varepsilon, \lambda),$$

here dimensionless parameter k_{VSC} is

(5) $$k_{VSC}(\varepsilon, \lambda) = 2 \frac{k_l k_B^{1/3}}{\pi^{1/3} k_W^{2/3}},$$

where parameter k_l connects length of coil perimeter l with large radius of torus R :

(6) $l(\varepsilon, \lambda) = k_l(\varepsilon, \lambda) R$, $k_l(\varepsilon, \lambda) = 2\varepsilon[\pi + 2(\lambda - 1)]$.

Parameter k_{VSC} is plotted in Fig. 2 as function of two geometrical parameters ε and λ .

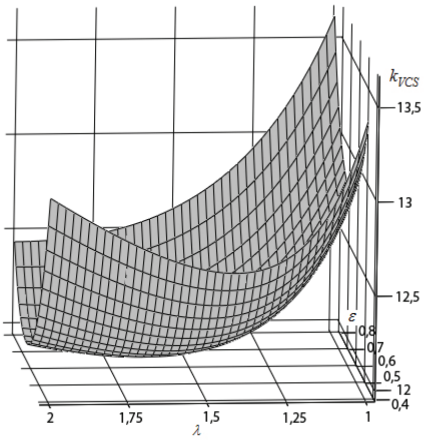


Fig. 2. The dependence of k_{VSC} by geometrical parameters ε and λ

The dimensionless parameter k_{VSC} has minimum value $k_{VSC}(\varepsilon_{\min}, \lambda_{\min}) = 11{,}99$ at $\varepsilon_{\min} = 0{,}63$, $\lambda_{\min} = 1{,}89$. Defined value $k_{VSC}(\varepsilon_{\min}, \lambda_{\min})$ is approaching to minimum analogous parameter for D-torus that equal to $k_{VSC\,D}(\varepsilon_{\min\,D}, \lambda_{\min\,D}) = 11{,}70$ at $\varepsilon_{\min\,D} = 0{,}68$, $\lambda_{\min\,D} = 1{,}75$ and corresponds to theoretical minimum for toroidal system [3].

Fig. 3 shows dependence of parameter k_{VSC} as function of the relative size ε for ratio of vertical size to radial size of torus cross-section λ for O-shaped, racetrack-shaped and D-shaped coils.

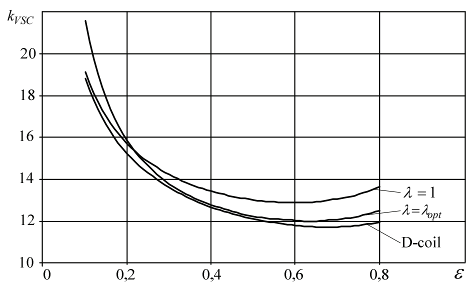


Fig. 3. Dimensionless parameter k_{VSC} as function of the relative size ε for ratio of vertical to radial size of torus λ

The dependences indicate that racetrack-shaped coil ($\lambda = \lambda_{opt}$) has considerably lesser volume of superconducting winding in comparison with usual O-shaped coil ($\lambda = 1$) and this value of volume is practically equal to volume of D-shaped coils.

Support system requirements

The support system of O-shaped torus has following elements: radial, vertical spokes and support rings [1]. Racetrack-shaped coils have additional elements: central spoke, horizontal spokes, baking cylinder (Fig. 1). As well as for O-shape torus, all construction elements have no bending moments, additional spokes have constant stresses in tension and backing cylinder has constant stresses in compression.

The volume of radial spokes is the same as for O-shaped coils [1]:

(7) $$V_{t1} = \frac{2a^2 N}{\sigma_t} \int_{\theta_0}^{\pi} f(\theta) d\theta = \frac{\mu_0 I^2 R}{2\sigma_t} k_{t1}, \; k_{t1} = \frac{\varepsilon^2}{\pi}(\pi - \theta_0).$$

Here $f(\theta) = \mu_0 I^2 / 4\pi(R - a\cos\theta)$. All radial spokes are connected at one central unit. As the central unit has no radial support element, the sum of radial forces of these spokes must be equal to zero $\int_{\theta_0}^{\pi} f(\theta)\cos\theta d\theta = 0$ and this condition is determined angle the θ_0 where radial spokes are absent:

(8) $$\pi - \theta_0 - \frac{2}{\sqrt{1-\varepsilon^2}} \tan^{-1}\left[\frac{\sqrt{\frac{1-\varepsilon}{1+\varepsilon}}}{\tan\left(\frac{\theta_0}{2}\right)}\right] = 0.$$

Within angles $0 \le \theta \le \theta_0$ support rings compensate the suitable electromagnetic forces.

The volume of the vertical spokes for racetrack-shaped coils is given by analogous equation for O-shaped coils:

(9) $$V_{t2} = N \int_{-\theta_0}^{\theta_0} l_{||}(\theta) dS_2,$$

where length of spokes are $l_{||}(\theta) = 2(a\sin\theta + h/2)$ and dS_2 is the area of all this elements cross sections which determined from equation for the mechanical stress in each spoke $\sigma_t = \frac{f(\theta) a d\theta}{dS_2}$. So, after integration (9)

(10) $$V_{t2} = \frac{\mu_0 I^2 R}{2\sigma_t} k_{t2},$$

where

$$k_{t2} = \frac{\varepsilon^2}{\pi}\left(\frac{\pi}{1+\sqrt{1-\varepsilon^2}} + \frac{\sin\theta_0}{\varepsilon} + \theta_0 - \pi + \frac{(\lambda-1)}{\varepsilon} ln\frac{1-\varepsilon\cos\theta_0}{1-\varepsilon}\right).$$

The horizontal spokes compensate the electromagnetic forces at straight part of winding on external side of torus. If the spokes are located densely enough, their volume is the following

(11) $$V_{t3} = \frac{2aN}{\sigma_t} \int_h f dl = \frac{\mu_0 I^2 R}{2\sigma_t} k_{t3}, \; k_{t3} = \frac{2\varepsilon^2(\lambda-1)}{\pi(1+\varepsilon)}.$$

The volume of the central spokes $V_{t4} = hS_4$ is determined from the condition of equilibrium in the zdirection. Cross-section of all central spokes $S_4 = \frac{F_z}{\sigma_t}$ is determined of total force F_z that acts on upper half torus in sector with the angular size $\theta_0 < \theta < \pi$. Therefore, this volume will be as following

(12) $$V_{t4} = \frac{hN}{\sigma_t} \int_{\theta_0}^{\pi} f \, sin\,\theta dl = \frac{\mu_0 I^2 R}{2\sigma_t} k_{t4} ,$$

where

$$k_{t4} = \frac{\varepsilon(\lambda - 1)}{\pi} ln \frac{1+\varepsilon}{1 - \varepsilon \, cos\,\theta_0} .$$

The total requirements of structure materials that are in tension is the sum of the radial, vertical and central spokes volumes $V_t = V_{t1} + V_{t2} + V_{t3} + V_{t4}$. Utilizing the factor $\mu_0 I^2 N^2 R$ appearing in the equation for stored energy *W* in (1), finally, the volume of spokes is as follows:

(13) $$V_t = \frac{W}{\sigma_t} Q_t ,\ Q_t = \frac{k_{t1} + k_{t2} + k_{t3} + k_{t4}}{k_W} .$$

The volume of the support rings and backing cylinder comprises the volume of the structure materials in compression.

The volume of support rings is given in [1]:

(14) $$V_{c1} = \frac{\mu_0 I^2 R}{2\sigma_t} k_{c1} ,\ k_{c1} = \frac{\varepsilon \, sin\theta_0}{\pi} .$$

The backing cylinder is under the action of uniform distributed forces, which are directed to the center of the cylinder. This support element compensates difference between forces $F_R = F_1 - F_2$, acting on the straight part of winding: F_1 at $\rho = \rho_1$ and F_2 at $\rho = \rho_2$ of the torus. The backing cylinder is in compression with stresses $\sigma_c = \frac{F_R N}{2\pi S_c}$, where S_c is the area of cross-section of backing cylinder wall. From this equation the volume of backing cylinder is given by

(15) $$V_{c2} = 2\pi\rho_1 S_c = \frac{\mu_0 I^2 R}{2\sigma_c} k_{c2} ,\ k_{c2} = \frac{2\varepsilon^2 (\lambda - 1)}{\pi (1+\varepsilon)} .$$

The total volume of structure materials being in compression is the sum volumes of the support rings and backing cylinder, $V_c = V_{c1} + V_{c2}$.

Using (1), the volume of structural materials in compression is:

(16) $$V_c = \frac{W}{\sigma_c} Q_c ,\ Q_c = \frac{k_{c1} + k_{c2}}{k_W} .$$

In [4, 5] have been determined that for toroidal solenoids with constant tension and momentless coils is right equation $Q_t = Q_c + 1$. This equation is also valid for magnets with O-shaped coils [1]. Comparing (13) and (16) gives the same condition and in our case for racetrack-shaped coils it may be written as

(17) $$\sum k_t = \sum k_c + k_W .$$

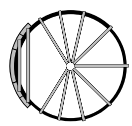
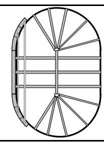
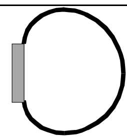
The total dimensionless parameter, which characterizes requirements of structural materials is sum of parameters Q_t and Q_c

(18) $$Q = Q_t + Q_c .$$

The dependence of parameter Q from torus relative size ε for various support systems is presented in Table. The parameter Q is compared for: toroids with spokes inside O-shaped coils, with spokes inside racetrack-shaped coils, and D-shaped coils with support belts around the coils.

The calculation results show that for racetrack-shaped coils the parameter Q practically does not depend on λ and support system volume requirements do not exceed at the same ε values for D-shaped toroidal system as well as for O-shaped coils with spokes.

Table 1. Dependence Q from ε for various shapes of coils

ε	O-Coils with Spokes	Racetrack Coils	D Coils
			
0,1	2,81	2,812	2,813
0,2	2,638	2,643	2,649
0,3	2,481	2,488	2,5
0,4	2,338	2,345	2,362
0,5	2,204	2,21	2,232
0,6	2,079	2,081	2,104
0,7	1,958	1,953	1,976
0,8	1,836	1,823	1,838

Conclusions

The fulfilled parametrical analysis gives the possibility to make comparison of magnetic systems by criterion of the minimum volume materials of superconducting winding and mechanical support system for different torus configurations and to choose the optimal form of coils.

The toroidal magnetic system composed of racetrack-shaped coils with spokes inside torus has considerably lesser volume of superconducting winding in comparison with usual O-shaped coils. The volume of structural materials does not exceed the corresponding values for O-shaped torus with spokes and D-shaped toroidal system. So, the toroidal SMES with spokes and racetrack-shaped coils could provide attractive alternatives to conventional coil systems in the development of SMES devices.

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