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# Approximate BEM analysis of a thin electromagnetic shield of variable thickness

***Abstract***. *The paper concerns an approximate analysis of an electromagnetic shield of variable thickness. The method is based on the boundary element method (BEM), but to avoid nearly singular integrals the analysis uses a semi-analytical solution for the shield. This makes the numerical errors smaller, and also reduces the memory used and the computation time.*

***Streszczenie.*** *Praca dotyczy przybliżonej analizy ekranu elektromagnetycznego o zmiennej grubości. Proponowana metoda wykorzystuje metodę elementów brzegowych (MEB), ale w celu uniknięcia całek prawie osobliwych w obszarze ekranu stosuje się rozwiązanie półanalityczne. Powoduje to nie tylko zmniejszenie błędów numerycznych, ale także redukuje zapotrzebowania na pamięć operacyjną i czas obliczeń. (**Przybliżona analiza cienkościennego ekranu elektromagnetycznego o zmiennej grubości za pomocą MEB**).*

**Keywords:** boundary element method, electromagnetic shielding, thin shells.
**Słowa kluczowe:** metoda elementów brzegowych, ekranowanie elektromagnetyczne, cienkie warstwy.

## Introduction

Thin layers are elements of many real configurations, e.g. magnetic or electromagnetic shields, human skin, dirty or dusted surfaces. Material properties of thin layer and the surrounding environment can differ much and significantly affect the electromagnetic field distribution. Owing to geometrical complexity, and maybe other aspects, such configurations can be analyzed or optimized only via numerical methods. Unfortunately, thin layers are almost always sources of difficulties in numerical methods. The first solution coming on mind is to use a very fine discretization. This is not always a remedy, because it implies a relatively large system of equations, and numerical errors of the solution can be significant or even unacceptable. Therefore thin layers require a special treatment, e.g. using air gap finite element or transforming the governing equations to more applicable form for the considered thin layer.

Many of the considered types of problems could be efficiently solved with use of the Boundary Element Method (BEM) [1-4]. Although from the mathematical point of view the resulting system of BEM equations is not singular, yet from the numerical point of view it is ill conditioned [2]. This is because BEM equations for corresponding points lying on the opposite surfaces of the layer contain coefficients of similar values, resulting in nearly linear dependence of some rows of the main matrix. In addition, the elements of the matrix are integrals which must be often evaluated numerically, therefore they may be quite inaccurate. It occurs especially if the integrals are nearly singular [2]. Such a situation takes place in the case of a thin layer, because the observation and source points can be very close. Small changes in small distance result in abrupt changes in the integrand, which is a function of the reciprocal of the distance.

Concluding, modeling thin layers in BEM requires special treatment. Some approaches were proposed in [2, 5-8] (among others). In this paper one of them is developed and used for thin electromagnetic (EM) shield of variable thickness. It may be regarded as a generalization of the approach described in [7,8].

## Problem description

A closed electromagnetic shield, $\Omega_1$, is placed in free space, $\Omega_0$, and encloses a protected region, $\Omega_2$ – Fig. 1. The external and internal surfaces of the shield are referred to as $S_1$ and $S_2$, respectively. The shield region has relative permeability $\mu_r$ = const, electric conductivity $\gamma_1$ = const, and is considered to be very thin; its thickness, $d$, can vary from point to point. The protected region and the free space are non-magnetic and non-conductive.

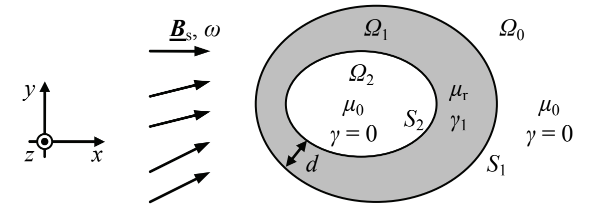


Fig. 1. Problem diagram – electromagnetic shield $\Omega_1$ in time-harmonic magnetic field (cross-section)

The vector magnetic potential has only a *z*-component ($\boldsymbol{A} = A\boldsymbol{1}_z$) in the coordinate system whose *z* axis is perpendicular to the shield's cross-section. The phasor of the *z*-component of the vector magnetic potential fulfills the following equations corresponding to particular regions:

$$\nabla^2 \underline{A}^{(0)} = 0, \quad \nabla^2 \underline{A}^{(1)} - \kappa^2 \underline{A}^{(1)} = 0, \quad \nabla^2 \underline{A}^{(2)} = 0\,, \tag{1}$$

where $\kappa^2 = \mathrm{j}\omega\mu_r\mu_0\gamma_1$. Field continuity conditions yield $\underline{A}^{(0)} = A^{(1)}$ on boundary $S_1$, $\underline{A}^{(1)} = \underline{A}^{(2)}$ on boundary $S_2$, and

$$\left.\frac{\partial \underline{A}^{(0)}}{\partial n}\right|_{S_1} = -\frac{1}{\mu_r}\left.\frac{\partial \underline{A}^{(1)}}{\partial n}\right|_{S_1}, \quad \left.\frac{\partial \underline{A}^{(2)}}{\partial n}\right|_{S_2} = -\frac{1}{\mu_r}\left.\frac{\partial \underline{A}^{(1)}}{\partial n}\right|_{S_2}, \tag{2}$$

$\underline{A}^{(0)} \to \underline{A}_s$ far from the magnetic shield, where $\underline{A}_s$ is the $z$-th component of the magnetic vector potential of externally applied magnetic field $\underline{\boldsymbol{B}}_s$ (i.e. $\boldsymbol{B}_s = \mathrm{curl}(A_s\boldsymbol{1}_z)$).

## Standard BEM model

The standard BEM procedure applied to this problem, after using the continuity conditions, leads to the following system of equations

$$\begin{bmatrix} \mathbf{H}_1^{(0)} & \frac{1}{\mu_r}\mathbf{G}_1^{(0)} & \mathbf{0} & \mathbf{0} \\ \mathbf{H}_1^{(1)} & -\mathbf{G}_1^{(1)} & \mathbf{H}_2^{(1)} & -\mathbf{G}_2^{(1)} \\ \mathbf{0} & \mathbf{0} & \mathbf{H}_2^{(2)} & \frac{1}{\mu_r}\mathbf{G}_2^{(2)} \end{bmatrix} \begin{Bmatrix} \mathbf{A}_1 \\ \mathbf{Q}_1^{(1)} \\ \mathbf{A}_2 \\ \mathbf{Q}_2^{(1)} \end{Bmatrix} = \begin{Bmatrix} \mathbf{A}_s \\ \mathbf{0} \\ \mathbf{0} \end{Bmatrix}, \tag{3}$$

where $\mathbf{H}_l^{(m)}$ and $\mathbf{G}_l^{(m)}$ – the BEM matrices corresponding to boundary $S_l$ connected with domain $\Omega_m$, $\mathbf{A}_l$ – vector of nodal values of magnetic vector potential ($z$-th component) $A$ on boundary $S_l$, $\mathbf{Q}_l^{(m)}$ – vector of nodal values of normal derivative $\partial_n A$ on boundary $S_l$ of domain $\Omega_m$, $\mathbf{A}_\mathrm{s}$ – vector of nodal values of magnetic vector potential ($z$-th component) $A_\mathrm{s}$ of the source magnetic field $\boldsymbol{B}_\mathrm{s}$ on boundary $S_1$. Details on forming $\mathbf{H}$ and $\mathbf{G}$ matrices can be found in [1-4].

**Approximate solution**

The problem with the above equations is that some elements of matrices $\mathbf{H}_l^{(1)}$ and $\mathbf{G}_l^{(1)}$ can be numerically very inaccurate (due to nearly singular integrals). To avoid this, an approximate approach is proposed. The fragment of the layer between two corresponding boundary elements on $S_1$ and $S_2$ can be approximately regarded as a fragment of infinite plate, for which

$$\text{(4)}\qquad \underline{A}(\zeta) = \frac{\underline{A}_i \sinh\kappa(d-\zeta) + \underline{A}_j \sinh\kappa\zeta}{\sinh\kappa d},$$

where $0 \le \zeta \le d$. Hence, for such a plate

$$\text{(5)}\qquad \left.\frac{\partial \underline{A}}{\partial n}\right|_{S_1} = -\left.\frac{\partial \underline{A}}{\partial \zeta}\right|_{\zeta=0} = \frac{\underline{A}_i\kappa\cosh\kappa d - \underline{A}_j\kappa}{\sinh\kappa d},$$

$$\text{(6)}\qquad \left.\frac{\partial \underline{A}}{\partial n}\right|_{S_2} = \left.\frac{\partial \underline{A}}{\partial \zeta}\right|_{\zeta=d} = \frac{-\underline{A}_i\kappa + \underline{A}_j\kappa\cosh\kappa d}{\sinh\kappa d}.$$

This allows obtaining the following relationships:

$$\text{(7)}\qquad \mathbf{Q}_1^{(1)} \approx \boldsymbol{\alpha}\mathbf{A}_1 - \boldsymbol{\beta}\mathbf{A}_2, \qquad \mathbf{Q}_2^{(1)} \approx -\boldsymbol{\beta}\mathbf{A}_1 + \boldsymbol{\alpha}\mathbf{A}_2,$$

where $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are diagonal matrices, whose diagonal elements are

$$\text{(8)}\qquad \alpha_i = \frac{\kappa\cosh\kappa d_i}{\sinh\kappa d_i}, \qquad \beta_i = \frac{\kappa}{\sinh\kappa d_i},$$

and $d_i$ is the thickness of the shield at node $i$. Such an approximation leads to the following system of equations:

$$\text{(9)}\qquad \begin{bmatrix} \mathbf{H}_1^{(0)} + \frac{1}{\mu_\mathrm{r}}\mathbf{G}_1^{(0)}\boldsymbol{\alpha} & \frac{1}{\mu_\mathrm{r}}\mathbf{G}_1^{(0)}\boldsymbol{\beta} \\ \frac{1}{\mu_\mathrm{r}}\mathbf{G}_2^{(2)}\boldsymbol{\beta} & \mathbf{H}_2^{(2)} + \frac{1}{\mu_\mathrm{r}}\mathbf{G}_2^{(2)}\boldsymbol{\alpha} \end{bmatrix} \begin{Bmatrix} \mathbf{A}_1 \\ \mathbf{A}_2 \end{Bmatrix} = \begin{Bmatrix} \mathbf{A}_\mathrm{s} \\ \mathbf{0} \end{Bmatrix},$$

System of Eqs. (9) contains half the number of equations and unknowns of Eq. (3), and no nearly singular integrals occur in it (for sufficiently regular boundary). If thickness $d$ is constant, matrices $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ can be replaced by constants from their diagonals, and Eq. (9) can be simplified to the form given in [7]. For magnetostatic field $\kappa = 0$; the limits of matrices $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ equal then matrix $\mathbf{w}$ from [8] (a diagonal matrix with elements $1/d_i$).

**Numerical results**

Both considered models, the standard BEM and approximate BEM, were implemented as a Mathematica 7.0 program. To be able to map the geometry of possible configurations a quadratic boundary element was used. In all cases a uniform external magnetic field was assumed.

Both models were verified in some benchmark problems. A cylindrical shield of radius $R_1$ was considered. First, thickness was constant and equaled $d$. Since there is an analytical solution in such a case, numerical results from both models were compared with theoretical ones for “thick” (Fig. 2a) and “thin” (Fig. 2b) EM shield. The following values of parameters were used: $\mu_\mathrm{r} = 1$, $\kappa R_1 = (1 + \mathrm{j})KR$, $KR = 10$, 16 + 16 quadratic elements. Results are presented in Figs. 3 to 6. The standard BEM model works well, if the layer is thick enough (Figs. 3 and 4), but errors become unacceptable for too thin layer (Figs. 5 and 6). The approximate BEM model gives more accurate results in such cases.

a) b)

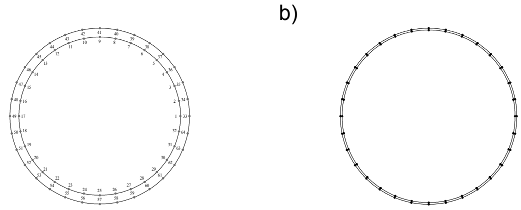

Fig. 2. “Thick” ($d/R_1$ = 0.1) (a) and “thin” ($d/R_1$ = 0.02) (b) EM shield

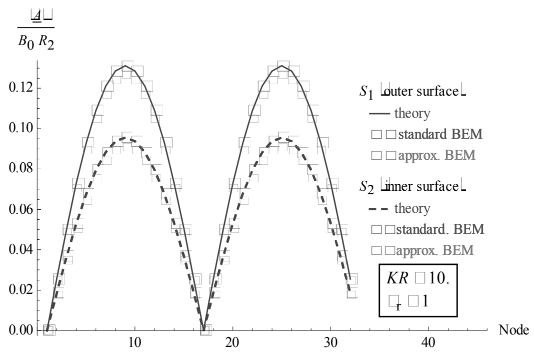


Fig. 3. Boundary values of $|\underline{A}|$ for “thick” cylindrical EM shield ($d/R_1$ = 0.1, $KR$ = 10, $\mu_\mathrm{r}$ = 1)

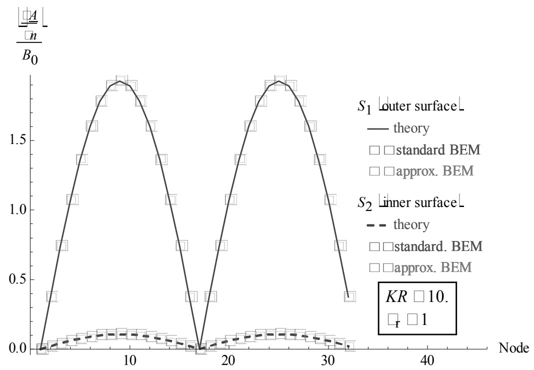


Fig. 4. Boundary values of $|\partial_n\underline{A}|$ for “thick” cylindrical EM shield ($d/R_1$ = 0.1, $KR$ = 10, $\mu_\mathrm{r}$ = 1)

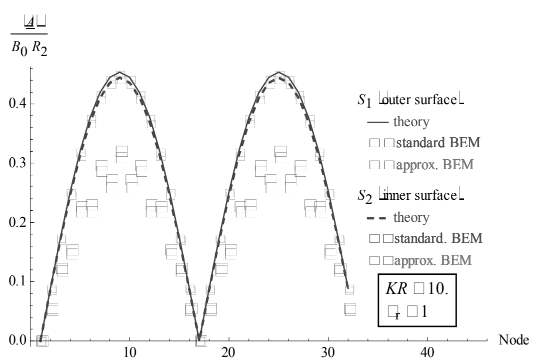


Fig. 5. Boundary values of $|\underline{A}|$ for “thin” cylindrical EM shield ($d/R_1$ = 0.02, $KR$ = 10, $\mu_\mathrm{r}$ = 1)

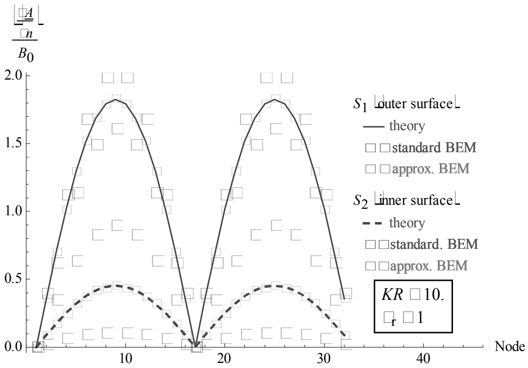


Fig. 6. Boundary values of $|\partial_n \underline{A}|$ for "thin" cylindrical EM shield ($d/R_1$ = 0.02, $KR$ = 10, $\mu_r$ = 1)

If the thickness of the shield is variable (Fig. 7), no analytical solution can be found. Therefore testing consists in using a "transparent shield" ($\mu_r$ = 1, $KR$ = 0) – Figs. 8 and 9. Results for exemplary value of $KR$ = 10 are shown in Figs. 10 and 11. It can be observed that the standard BEM model gives a "nasty" scattering of boundary nodal values at the nodes where the shield is thin. In the presented example the external surface is a cylinder of radius $R_1$ and the thickness varies from $0.02R_1$ to $0.08R_1$.

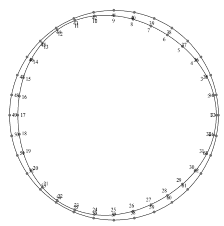

Fig. 7. Example of EM shield of variable thickness (cross-section)

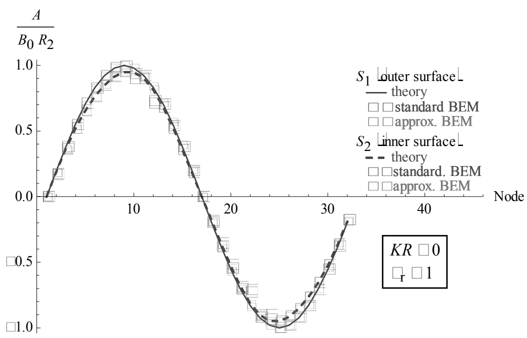


Fig. 8. Nodal values of $A$ on boundaries $S_1$ and $S_2$ for "transparent shield" ($KR$ = 0, $\mu_r$ = 1)

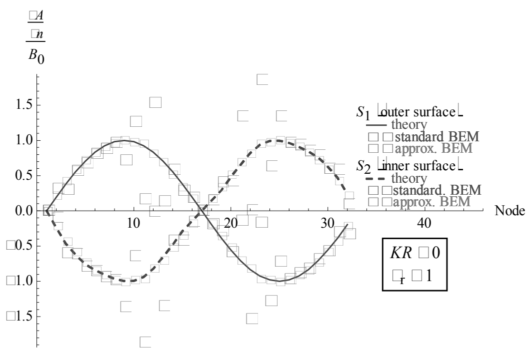


Fig. 9. Nodal values of $\partial_n A$ on boundaries $S_1$ and $S_2$ for "transparent shield" ($KR$ = 0, $\mu_r$ = 1)

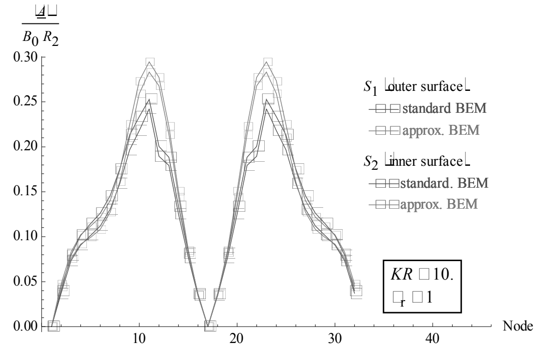


Fig. 10. Nodal values of $|\underline{A}|$ on boundaries $S_1$ and $S_2$ for arbitrary value of $KR$ = 10 ($\mu_r$ = 1)

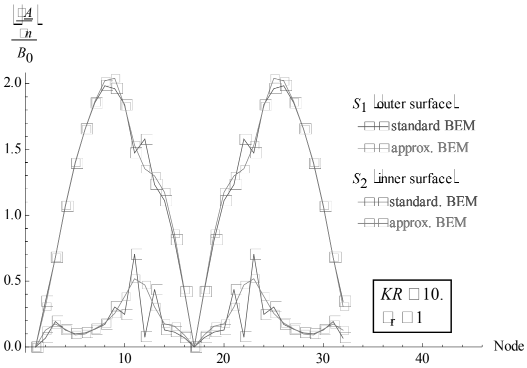


Fig. 11. Nodal values of $|\partial_n \underline{A}|$ on boundaries $S_1$ and $S_2$ for arbitrary value of $KR$ = 10 ($\mu_r$ = 1)

**Concluding remarks**

The approximate BEM model combines the standard BEM and approximate analytical solution, and can be efficiently used in thin EM shield analysis. It gives quite accurate results and produces half the number of equations when compared with standard BEM model. The idea can also be used in modeling thin layers of other types, e.g. weakly conducting walls of biological cells. Although it requires further testing, it can be an efficient method of analysis of an EM field in the considered class of problems.

REFERENCES

[1] Brebbia C.A.: *The boundary element method for engineers*. Pentech Press, London 1978.
[2] Kurgan E.: *Analiza pola magnetostatycznego w środowisku niejednorodnym metodą elementów brzegowych*. Rozprawy Monografie 81, Uczelniane Wyd. Nauk.-Dyd., Kraków 1999.
[3] Jabłoński P.: *Metoda elementów brzegowych w analizie pola elektromagnetycznego*. Wyd. Pol. Cz., Częstochowa 2003.
[4] Sikora J.: Podstawy metody elementów brzegowych. Zagadnienia potencjalne pola elektromagnetycznego, Wydawnictwo Książkowe Instytutu Elektrotechniki, W-wa 2009.
[5] Krähenbühl L., Muller D.: *Thin layers in electrical engineering. Example of shell models in analyzing eddy-currents by boundary and finite element methods*, IEEE Transactions on Magnetics, 29 (1993), 2, 1450-1455.
[6] Bao Z., Mukherjee S.: *Electrostatic BEM for MEMS with thin conducting plates and shells*, Eng. Anal. with Bound. El. 28 (2004), 1427-1435.
[7] Jabłoński P.: *Approximate BEM analysis of thin electromagnetic shield*, Proceedings of XXXIV IC-SPETO 2011, Gliwice-Ustroń, 18-21.05.2011, 17-18.
[8] Jabłoński P.: *Approximate BEM analysis of thin magnetic shield of variable thickness*, Proceedings of XXI Sympozjum PTZE, Zamek Lubliniec 5-8.06.2011, 81-83.

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