Srdjan MILOSAVLJEVIC[1], Jelena ANASTASOV[2], Mihajlo STEFANOVIC[3], Stefan PANIC[4,]

University of Pristina (1), University of Nis (2), University of Nis (3), University of Pristina (4)

Average BER of SSC receiver over correlated Rayleigh fading channels in the presence of interference

***Abstract**. In this paper switch-and stay combining (SSC) receivers over Rayleigh fading channels are observed. The case of interference-limited system in the environment with multiple co-channel interferers is considered. The proposed analysis gives the brand new expression for evaluating the probability density function (pdf) of signal-to–interference (SIR) at the output of combiner. Based on this, the average bit error probability (ABER) of the system, as one of the most important performance measure, is discussed.*

***Streszczenie.** W artykule przedstawiono wyniki obserwacji odbiornika SSC w kanale Rayleigha z zanikami. Rozważono system z ograniczonymi zakłóceniami i interferencją wielokanałową. Wyniki umożliwiają określenie funkcji gęstości prawdopodobieństwa i na ich podstawie określenia średniego błędu przypadkowego bitu. (**Średni błąd bitu odbiornika z kanałem Rayleigha z zanikami w obecności zakłóceń**)*

Keywords: average bit error rate, diversity combining, fading channel.
Słowa kluczowe: średni błąd bitu, kanał z zanikami..

Introduction

Fading phenomena is a major obstacle in a mobile wireless environment, introduced as random amplitude and phase distortion to the transmitted signal [1]. In addition to fading, interference is also present in communication systems. In the cellular mobile terrestrial and satellite communication systems, increasing of spectral efficiency is the main design goal. In such systems, signals from two or more channels and different locations operate at the same frequency and interfere for each other due to frequency reuse. This often result in co-channel interference as a general distortion in the performance of wireless communication systems [1], [2]. So, it is important to analyse how the interference affects well-accepted performance criteria of wireless systems, such as the outage probability, average output signal-to-interference ratio (SIR) and average bit-error probability (ABER) in order to implement practical system which satisfies predetermined minimum performance levels [3].

An efficient technique to diminish the bad influences of fading and co-channel interference is space diversity reception [1]. Space diversity reception provides the receiver with multiple faded replicas of the same information-bearing signal which can upgrade transmission reliability without increasing transmission power, bandwidth and channel capacity.

Among the well-known diversity schemes, switch-and-stay combining (SSC) scheme is one of the simplest to implement since the receiver processes the information from a single branch. In fading environments as cellular systems, where the level of the co-channel interference is sufficiently high compared to thermal noise, SSC receiver adheres to one branch as long as the signal-to-interference ratio (SIR) is greater than a specific threshold. Once the SIR-ratio drops below this threshold, the receiver switches to the other branch no matter whether its SIR-ratio is greater or less than the threshold.

Most of previously published papers assume independent fading between the diversity branches [4] and between the cochannel interferers in Rayleigh fading channel [5]. However, independent fading assumes antenna elements to be placed sufficiently apart, which is unusually realized in practice due to insufficient antenna spacing when diversity is applied in small terminals. So the correlation between the diversity branches should be take in consideration.

In a few published papers, the effect of co-channel interference on the performance of wireless communication system has been analysed [6-11]. In [6-7] performance analysis of optimum combining with multiple co-channel interferers over Rayleigh fading channels was presented. The outage probability analysis over mobile fading channel with multiple interferers was presented in [8] and similar analysis but in the case of single co-channel interferer was published in [9]. In [10] and [11], the performance analysis of Rician and α-μ correlated mobile radio channel, respectively, experiencing multiple co-channel interferers was considered.

In this paper, we consider correlated SIR-based SSC diversity system over Rayleigh fading channels in the presence of an arbitrary number of multiple co-channel interferences. In order to study the effectiveness of any modulation scheme and to evaluate the system performances over different channel conditions, the required formulae are derived. For proposed system model, through the short mathematical analysis, expression for evaluating the probability density function (pdf) of the output SIR is derived. Furthermore, expression for evaluating important performance measures such as the average output SIR and ABER are obtained. Effects of number of multiple interferers and correlation coefficients of desired signals as well as interferences to the system performances are shown. To the best author's knowledge, no similar results for the proposed system model has been reported in the literature.

System model

We consider wireless communication system with dual-branch SSC receiver operating over Rayleigh fading channels. The observed system is presented in Fig. 1.

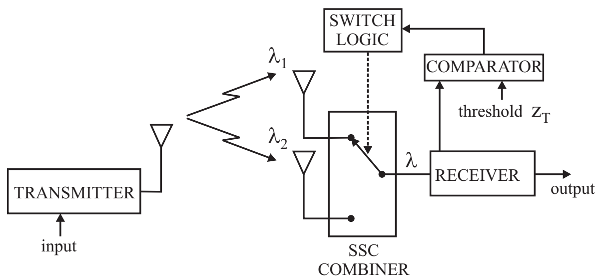

Fig.1. System model

The envelope of desired signal at the i-th input branch (i =1,2) has Rayleigh pdf [6] in this form

(1) $$p_{R_i}(R_i) = 2\left(\frac{1}{\Omega_d}\right) R_i \times e^{-\frac{R_i^2}{\Omega_d}}$$

where $\Omega_d = \mathrm{E}(R_i^2)$ is the average desired signal power at both branches.

The pdf of M interferers on each input branch, $r_i = \sum_{k=1}^{M} r_{ik}, i = 1,2$ is also Rayleigh

(2) $$p_{r_i}(r_i) = 2\left(\frac{1}{\Omega_{ck}}\right)^M \frac{r_i^{2M-1}}{\Gamma(M)} \times e^{-\frac{r_i^2}{\Omega_{ck}}},$$

where Ω_{ck} is the average power of single co-channel interference (Ω_c=$M\Omega_{ck}$, k=1, 2, …, M is the total average interference power, $\Omega_c = \mathrm{E}(r_i^2)$).

The pdf of instantaneous SIR, defined as $\lambda = \frac{R^2}{r^2}$ can be evaluated [12]:

(3) $$p_\lambda(\lambda) = \frac{1}{2\sqrt{\lambda}} \int_0^\infty r p_R\left(r\sqrt{\lambda}\right) p_r(r) dr = \frac{S^M M}{(\lambda + S)^{M+1}}$$

where $S = \frac{\Omega_d}{\Omega_{ck}}$ is the average SIR at both input branches.

When diversity system is applied on small terminals with multiple antennas, correlation arises between branches. The following analysis is based on the results available in literature for the Nakagami-*m* constant correlation model [13].

Due to insufficient antennas spacing, both desired and interfering signal envelopes experience correlated Rayleigh fading. The joint pdfs of two correlated signal and interference envelopes are given by, respectively:

(4) $$\begin{aligned} p(R_1, R_2) &= \left(1 - \sqrt{\rho_d}\right) \sum_{k_1,k_2=0}^{\infty} \frac{4\Gamma(k_1 + k_2 + 1)\rho_d^{\frac{k_1+k_2}{2}}}{\Gamma(k_1+1)\Gamma(k_2+1)k_1!k_2!} \\ &\times \left(\frac{1}{1+\sqrt{\rho_d}}\right)^{k_1+k_2+1} \left(\frac{1}{\Omega_d\left(1-\sqrt{\rho_d}\right)}\right)^{k_1+k_2+2} \\ &\times R_1^{2k_1+1} R_2^{2k_2+1} e^{-\frac{R_1^2+R_2^2}{\Omega_d\left(1-\sqrt{\rho_d}\right)}}, \end{aligned}$$

(5) $$\begin{aligned} p(r_1, r_2) &= \frac{\left(1-\sqrt{\rho_c}\right)^M}{\Gamma(M)} \sum_{l_1,l_2=0}^{\infty} \frac{4\Gamma(l_1 + l_2 + 1)\rho_c^{\frac{l_1+l_2}{2}}}{\Gamma(l_1+1)\Gamma(l_2+1)l_1!l_2!} \\ &\times r_1^{2M+2l_1-1} r_2^{2M+2l_2-1} e^{-\frac{M\left(r_1^2+r_2^2\right)}{\Omega_c\left(1-\sqrt{\rho_c}\right)}} \\ &\times \left(\frac{1}{1+\sqrt{\rho_c}}\right)^{l_1+l_2+M} \left(\frac{M}{\Omega_c\left(1-\sqrt{\rho_c}\right)}\right)^{l_1+l_2+2M}. \end{aligned}$$

So, the joint pdf of instantaneous SIRs at two input branches, denoted by $\lambda_1 = \frac{R_1^2}{r_1^2}$ and $\lambda_2 = \frac{R_2^2}{r_2^2}$, can be find using [12]

(6) $$\begin{aligned} p_{\lambda_1\lambda_2}(\lambda_1, \lambda_2) &= \frac{1}{4\sqrt{\lambda_1, \lambda_2}} \int_0^\infty\int_0^\infty p_{R_1R_2}\left(r_1\sqrt{\lambda_1}, r_2\sqrt{\lambda_2}\right) \\ &\times p_{r_1r_2}(r_1, r_2) r_1 r_2 dr_1 dr_2 \end{aligned}$$

Substituting (4) and (5) in (6), we get

(7) $$\begin{aligned} p_{\lambda_1\lambda_2}(\lambda_1, \lambda_2) &= \sum_{k_1,k_2,l_1,l_2=0}^{\infty} \mathrm{A} \times \left(S\frac{1-\sqrt{\rho_d}}{1-\sqrt{\rho_c}}\right)^{2M+l_1+l_2} \\ &\times \sum_{i=1}^{2} \frac{\lambda_i^{k_i}}{\left(\lambda_i\left(1-\sqrt{\rho_c}\right) + S\left(1-\sqrt{\rho_d}\right)\right)^{M+k_i+l_i+1}} \end{aligned}$$

with

(8) $$\begin{aligned} \mathrm{A} &= \left(1-\sqrt{\rho_d}\right)\left(1-\sqrt{\rho_c}\right)^M \frac{\Gamma(k_1+k_2+1)\Gamma(l_1+l_2+M)}{\Gamma(M)\Gamma(k_1+1)\Gamma(k_2+1)} \\ &\times \frac{\Gamma(M+k_1+l_1+1)\Gamma(M+k_2+l_2+1)}{\Gamma(l_1+M)\Gamma(l_2+M)k_1!k_2!l_1!l_2!} \rho_d^{\frac{k_1+k_2}{2}} \rho_c^{\frac{l_1+l_2}{2}} \\ &\times \left(\frac{1}{1+\sqrt{\rho_d}}\right)^{k_1+k_2+1} \left(\frac{1}{1+\sqrt{\rho_c}}\right)^{l_1+l_2+M}. \end{aligned}$$

According to SSC diversity combining, the pdf of output SIR can be evaluated using:

(9) $$p_{SSC}(\lambda) = \begin{cases} f_{SSC}(\lambda), \lambda \le z_T \\ f_{SSC}(\lambda) + p_\lambda(\lambda), \lambda > z_T \end{cases},$$

where

(10) $$\begin{aligned} f_{SSC}(\lambda) &= \int_0^{z_T} p_{\lambda_1\lambda_2}(\lambda, \lambda_2) d\lambda_2 = \\ &\sum_{k_1,k_2,l_1,l_2=0}^{\infty} \mathrm{A} \times \left(S\frac{1-\sqrt{\rho_d}}{1-\sqrt{\rho_c}}\right)^{M+l_1} \frac{\lambda^{k_1}}{\left(\lambda + S\frac{1-\sqrt{\rho_d}}{1-\sqrt{\rho_c}}\right)^{M+k_1+l_1+1}} \\ &\times \mathrm{B}\left(\frac{z_T}{z_T + S\frac{1-\sqrt{\rho_d}}{1-\sqrt{\rho_c}}}, k_2+1, M+l_2\right) \end{aligned}$$

and $p_\lambda(\lambda)$ is the pdf given by (3).

Moments

Starting from the basic definition, $m_n = \mathrm{E}(\lambda^n)$, the nth order moment of λ can be derived in closed-form as

(11) $$\begin{aligned} m_n &= \int_0^\infty \lambda^n p_{SSC}(\lambda) d\lambda = \int_0^\infty \lambda^n f_{ssc}(\lambda) d\lambda + \int_{z_T}^\infty \lambda^n p_\lambda(\lambda) d\lambda \\ &= I_1 + I_2 \end{aligned}$$

Using [14, eq. (3.194.3)] integral I_1 can be evaluated as

(12)

$$I_1 = \sum_{k_1,k_2,l_1,l_2=0}^{\infty} \mathrm{A} \times \left(S \frac{1-\sqrt{\rho_d}}{1-\sqrt{\rho_c}} \right)^n \mathrm{B}\left(k_1+n+1, M+l_1-n\right) \times \mathrm{B}\left(\frac{z_T}{z_T + S\frac{1-\sqrt{\rho_d}}{1-\sqrt{\rho_c}}}, k_2+1, M+l_2 \right), M > n$$

where $\mathrm{B}(z,a,b)$ is the incomplete Beta function [14, eq. (8.391)].

Also, using [14, eq. (3.194.2[6])], integral I_2 can be derived in a closed form as

(13)

$$I_2 = \frac{z_T^{n-M} S^M M}{(M-n)} 2F1\left(-\frac{S}{z_T}; M+1, M-n; M-n+1 \right).$$

Based on derived expressions for evaluating moments, average SIR at the output of SC can be obtained as $\overline{S_{out}} = m_1$. $\overline{S_{out}}$ is very important performance criterion for SIR-based wireless communication systems operating in a cochannel interference environment. The optimum threshold, z_T, for max{$\overline{S_{out}}$} can be found by solving the equation $\partial m_1 / \partial z_T \Big|_{z_T^* = z_T} = 0$. Also, it can be numerically evaluated by using root-finding techniques available in Mathematica or Matlab software package.

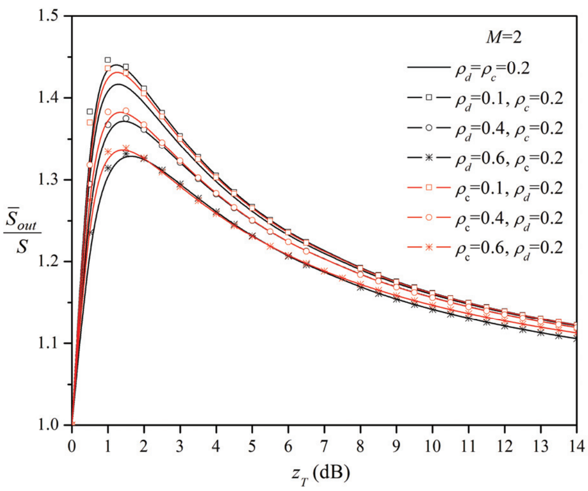

Fig.2. Average-output SIR for different values of correlation coefficients

Fig. 2 illustrates that $\overline{S_{out}}$ could be considerably degraded due to correlation among the branches. The increased correlation coeficient of desired signals gives larger degradation compared to the increased correlation coeficient of interferers.

Average bit error rate

The average bit error rate (ABER), $\overline{P_e}$, at the output for noncoherent binary signaling is derived by averaging the conditional error probability over the pdf of the output SIR, according to following expression

(14)

$$\overline{P_e} = \int_0^{\infty} p_{SSC}(\lambda) \frac{1}{2} e^{-h\lambda} d\lambda = \int_0^{\infty} f_{ssc}(\lambda) \frac{1}{2} e^{-h\lambda} d\lambda + \int_{z_T}^{\infty} p_{\lambda}(\lambda) \frac{1}{2} e^{-h\lambda} d\lambda = I_1 + I_2$$

Using [15, eq.(07.34.21.0086.01)] integral I_1 can be evaluated as

(15)

$$I_1 = \sum_{k_1,k_2,l_1,l_2=0}^{\infty} \frac{1}{2} \mathrm{A} \times \frac{1}{\Gamma(M+k_1+l_1+1)} \times \mathrm{B}\left(\frac{z_T}{z_T + S\frac{1-\sqrt{\rho_d}}{1-\sqrt{\rho_c}}}, k_2+1, M+l_2 \right) \times G_{1,1}^{1,1}\left[hS\frac{1-\sqrt{\rho_d}}{1-\sqrt{\rho_c}} \middle| \begin{matrix} -k_1 \\ M+l_1, 0 \end{matrix} \right]$$

Also, using [14, eq. (3.353.)], integral I_2 can be derived in a closed form as

(16)

$$I_2 = \frac{M}{2} S^M [e^{-z_T h} \sum_{k=1}^{M} \frac{(k-1)!(-h)^{M-k}}{M!(z_T+S)^k} - \frac{(-h)^M}{M!} e^{hS} \mathrm{E}_i((z_T+S)h)]$$

where $\mathrm{E}_i(x)$ is exponential-integral function defined as [14, eq. (8.211)]; h denotes modulation constant, i.e. h=1 for binary differential phase-shift keying (BDPSK) and h = 1/2 for noncoherent binary frequency-shift keying (BFSK).

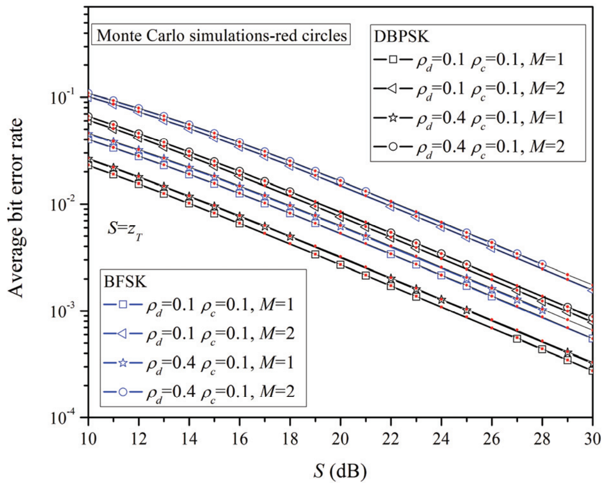

Fig.3. ABER for DBPSK and BFSK modulation schemes

The ABER as a function of average SIR at both input branches for two modulation schemes, DBPSK and BFSK, is obtained in Fig. 3. When number of interferers increases, ABER increases (performance degrades). The ABER performance also degrades when correlation coefficients increase. It is obvious that better performance gain is achieved when DBPSK modulation is used for any system condition. This figure also presents Monte Carlo simulation results that are in an excellent agreement with analytical results. Each ABER value is estimated on the basis of 3×10^3 bit errors. The minimum number of bits used during evaluation of any ABER value is 10^4, and maximum number of bits used in simulation is about 2×10^9.

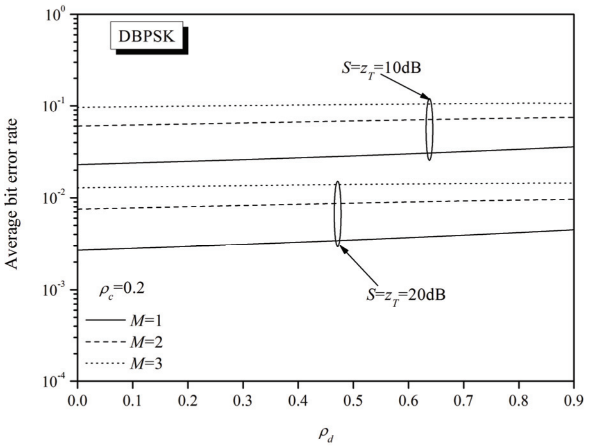

Fig.4. ABER as a function of correlation coefficient ρ_d

The ABER for dual branch SSC receiver is plotted in Fig. 4 versus correlation coefficient ρ_d for different values of average SIR/switching threshold. It is obvious that for both picked cases $S=z_T$=10dB and $S=z_T$=20dB, the ABER increases as the correlation coefficient increases. Also, when number of interferers increases, performance gain degrades. For $S=z_T$=20dB ABER performance is better.

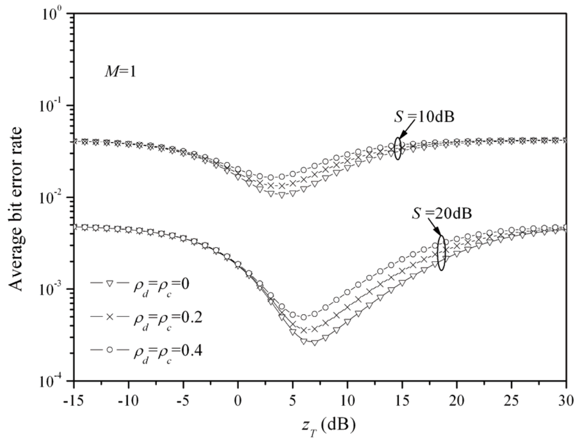

Fig.5. ABER as a function of switching threshold z_T

Fig. 5 presents ABER dependence on switching threshold. There is an optimal switching threshold in the minimum ABER sense and its value depends on the average input SIR. For higher value of S, the value of optimal z_T is larger.

Conclusion

In this paper, the ABER analysis of dual branch SSC receiver operating over correlated and identically distributed Rayleigh fading channels with M Rayleigh interferers was presented. Assuming this diversity technique, infinite series expressions for the pdf, moments and ABER of output SIR were derived. The effects of branch correlation and co-channel interferers on average-output SIR and ABER were considered and numerically presented. The presented results can be used in design of cellular mobile system as determined optimal values of system parameters in order to achieve reasonable system's performance.

REFERENCES

[1] Simon M., Alouini K., M. S., Digital communication over fading channels. (2005). Wiley, New York, 2nd edn.
[2] Stavroulakis P., Interference analysis and reduction for wireless systems. (2003). Artech House, INC, London
[3] Lee W., C., Y. Mobile communications engineering. (2001). New York, Mc-Graw-Hill
[4] Young-Chai Ko, Tao Luo, Effect of noise Imbalance on dual-MRC over Rayleigh fading Channels, *IEEE Trans Comm.* 5, No 3, 514-518
[5] Zhang S.;Kam P. Y., Ho P., Performance of Differentially detected DPSK over nonselective Rayleigh fading channels with maximal ratio combining and multiple cochannel Interferers, *IEEE Trans. Comm.*, 55 (2007), No.1, 133-141
[6] Shah A., Haimovich A., Performance analysis of optimum combining in wireless communications with Rayleigh fading and cochannel interference, *IEEE Trans Comm.*, 46 (1998), No. 4, 473-479
[7] Viler, E. (1999). Performance analysis of optimum combining with multiple interferers in flat Rayleigh fading. *IEEE Trans Comm.* 47, 1503-1509
[8] Abu-Dayya A., Beaulieu N., Outage probabilities of celular mobile radio systems with multiple Nakagami interferers, *IEEE Trans Veh Technol* 40 (1991), No. 4, 757–768
[9] Karagiannidis G., Performance analysis of SIR-based dual selection diversity over correlated Nakagami-*m* fading channels. *IEEE Trans Veh Technol.* 52 (2003), No. 5, 1207–1216
[10] Panajotovic A., Stefanovic M., Draca D.,. Cochannel interference effect on BEP performance of SSC receiver in correlated Rician fading, *Journal of the Franklin Institute*, 347 (2010), No. 7, 1242-1252
[11] Spalevic P., Panic S., Dolicanin C., Stefanovic M., Mosic A., SSC diversiity receiver over correlated α-μ fading channels in the presence of co-channel interference, *EURASIP Journal on Wire. Comm. and Net.*, (2010). accepted for publication on 31 January 2010
[12] Helstrom C. W., Probability and Stohastic Processes for Engineers. (1991) New York, MakMillian
[13] J. Reig, Multivariate Nakagami-m distribution with constant correlation model, *Int. J. Electron. Commun.*, (2007), doi: 10.1016/j.aeue.2007.10.009
[14] Gradshteyn I. S. , Ryzhik I. M., Table of Integrals, Series, and Products, (1994), Academic Press, 5th ed., Orlando
[15] The Wolfram Functions Site, 2008. [Online] Available: http:/functions.wolfram.com

Authors

***Authors**: dipl. ing. Srdjan Milosavljevic, University of Pristina, Faculty of Economics, ul. Kolasinska 156, 38220 Kosovska Mitrovica, E-mail: srdjan.milosavljevici@pr.ac.rs; dipl. ing. Jelena Anastasov, University of Nis, Faculty of Electronic Engineering, ul. Aleksandra Medvedeva 14, 18000 Nis, E-mail: anastasovjelena@gmail.com, prof. dr Mihajlo Stefanovic, University of Nis, Faculty of Electronic Engineering, ul. Aleksandra Medvedeva 14, 18000 Nis, E-mail: mihajlo.stefanovic@elfak.ni.ac.rs, dr Stefan Panic, University of Pristina, Faculty of Natural Science and Mathematics, ul. Lole Ribara 29, 38000 Kosovska Mitrovica, E-mail: stefanpnc@yahoo.com*