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Evaluation of the boundaries of the solutions space for the Harmonic Cancellation Technique

***Abstract**. The Harmonic Cancellation Technique (HCT) is a modulation technique able to reduce the output voltage THD in Auxiliary Railway Power Supplies which supply both a linear and a well-known non linear load, helping the manufacturer to achieve the output voltage THD specifications. It is based on the pre-distortion of the inverter output voltage, through the analytical determination of the IGBTs switching events. So, along this paper the limitations of the available solutions space for the HCT is presented, along with some design parameters which may help to enlarge it.*

***Streszczenie.** Technika redukcja harmonicznych (HCT) jest metodą modulacji umożliwiającą zmniejszenie wartości współczynnika THD w napięciu wyjściowym trakcyjnego źródła potrzeb własnych, zasilającego zarówno odbiorniki liniowe jak i dobrze znane odbiorniki nieliniowe. Metoda ta, umożliwiająca producentom osiąganie wymaganego w specyfikacji współczynnika THD, bazuje na ocenie odkształcenia napięcia wyjściowego falownika, poprzez analityczne wyznaczenie chwil przełączeń IGBT. W artykule przedstawiono ograniczenia techniki HCT oraz kilka parametrów konstrukcyjnych, umożliwiających rozszerzenie możliwości jej zastosowania. (**Ocena możliwych obszarów zastosowania techniki modulacji HCT**).*

Keywords: Auxiliary Railway Power Supplies, inverters, low THD, inverters modulation.
Słowa kluczowe: pomocnicze źródło kolejowe, falowniki, niski THD, modulacja falowników.

Introduction

Auxiliary Railway Power Supplies (ARPS) implemented by means of an inverter is a widespread application of the power electronics. The interface characteristics that must be satisfied by an ARPS are detailed in [1]. Regarding the ARPS output voltage characteristics, it is said that the maximum allowable THD at its output voltage shall be established as a trade-off between the car builder and the converter manufacturer; since it is a compromise between the cost of the inverter filter, maintainability and the harmonic content that the loads are able to accommodate.

Therefore, depending on the loads connected to the inverter output, the maximum allowable THD may be different. As an example, three commonly used maximum THD values are: 4%, 7% and 14%.

Keeping the output voltage THD within the limits is an important challenge, especially when the inverter must feed simultaneously linear and nonlinear loads, since the nonlinear loads operation provokes additional low frequency harmonic content which distorts the power supply output voltage and so increases significantly the THD.

Along the work presented in this paper, a typical auxiliary railway power supply (ARPS) has been considered in order to develop and check the proposed modulation technique. The auxiliary power supply is used to feed low and medium voltage equipment from the catenary (lightning, air conditioning, battery charger, etc.).

The typical ARPS is formed by an input filter which provides overvoltage protection to the converter placed downstream, a Voltage Source Inverter (VSI), an AC output filter (L_F, C_F) which attenuates the switching ripple and finally the auxiliary services of the train (modelled as an AC linear load) and a battery charger. See Fig. 1.

Although some other options can be considered for the battery charger (PWM rectifiers [2], [3]), it is usually implemented by means of a three phase full bridge thyristor rectifier due to its robustness and cost effective characteristics, despite its non linear nature.

The presence of this nonlinear load provokes additional low frequency harmonics content which flow back through the impedances of the system and distort the AC filter output voltage. Thus the output voltage THD increases and doesn't comply with the specifications.

State of the art solutions reduce the output voltage THD by means of either multi-loop strategies ([4]-[6]) or active filtering ([8]-[10]). However neither multi-loop strategies nor active filtering are applicable to the case of an ARPS due to stability constraints or unaffordable increase of cost, size and complexity, respectively [11].

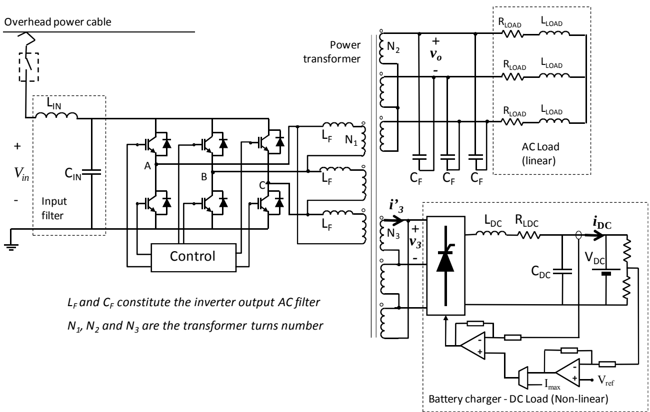

Fig. 1: Simplified scheme of a typical railway converter. VSI is used to feed simultaneously the auxiliary services of the train (AC linear load) and to charge some back-up batteries (nonlinear load).

Therefore, the pre-distortion of the inverter output voltage is a convenient strategy which is able to cancel out the low frequency content due to the non linear load without any significant increment of the size and cost of the system. The switching events are calculated analytically as it is summarized in Section II.

The present paper deals with the boundaries that limit the feasible solutions space of the Harmonic Cancellation Technique. The boundaries for a given design have been explored and the critical operating point, in terms of existence of solution, is identified. It should be remarked that the feasible solutions space is not only determined by the operating conditions but it is also influenced by the converter design parameters. So, an insight of the converter design parameters which may help to enlarge the ranges of operating conditions for which HCT can be applied are also considered in this paper.

Along Sections IV and V the limitations of the feasible solutions space and the design parameters which might help to enlarge the solutions space are described. In section VI the experimental performance of the Harmonic Cancelation Technique is shown. Finally some conclusions are extracted.

Operation Principle

The Harmonic Cancellation Technique (hereinafter HCT) aims to reduce the output voltage THD through the proper pre-distortion of the inverter output voltage. Since the pre-distortion of the inverter output voltage is calculated analytically for each given set of operating conditions, the HCT is a technique suitable in those cases in which the non linear load is concentrated and well known.

Let's consider the per phase equivalent circuit (Fig. 2) of the AC power supply in Fig. 1. The current drawn by the non linear load, referred to the primary side of the transformer is represented by means of a current source. The low frequency current harmonics drawn by the nonlinear load flow through the filter inductance and provoke a voltage drop across it (v_F) which distorts the filter output voltage (v_o).

The proposed modulation technique is able to generate an inverter output voltage formed by a controlled fundamental component as well as a set of low frequency harmonics which added to the ones across the filter inductance, they cancel each other out. Thus, the inverter output voltage has been represented qualitatively through a sinusoidal voltage source (V_{AB_1}) which represents the fundamental voltage and a non sinusoidal voltage source (v_F) that represents the low frequency content intended to cancel out the harmonics due to the non linear load operation.

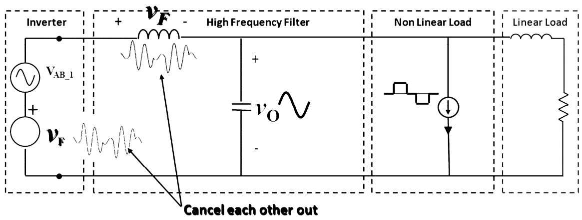

Fig. 2: Principle of operation of the Harmonic Cancellation Technique (HCT) on the per phase equivalent circuit of a Three phase AC power supply by means of an inverter, which supplies both a linear and a nonlinear load.

The implementation of the proposed modulation technique is based on the analytical determination of the switching events for a given set of operating conditions, in order to cancel out the harmonic content due to the non linear load. Those switching events are stored within a look-up table.

Applying the superimposing principle on the circuit represented in Fig. 2, the mathematical expression of the filter output voltage (v_o) can be obtained (1).

(1) $$v_o = v_{AB} \cdot G_F - i_3 \cdot G_F \cdot X_F$$

where:

$G_F = \dfrac{Z_o}{Z_o + X_F}$ Filter Gain

v_{AB} - Complex nth harmonic components of the line to line inverter output voltage, i_3 - Non linear load current referred to the primary side of the transformer, v_o - Line to line output voltage of the group inverter-filter, X_F - Complex impe-dance of the filter inductance at the nth harmonic frequency, Z_o - Complex output impedance at the nth harmonic frequency. It is formed by the parallel of the linear load and the filter capacitor.

Additionally, it should be taken into account the following considerations in order to determine the harmonic content of v_{AB}:

- ✓ The triple order harmonic components are cancelled naturally because of the composition of two voltages phased out 120º.
- ✓ The required fundamental component is given as an specification. So, applying this condition on (1), equation (2) is obtained.

(2) $$v_{o\_spec} = v_{AB\_1} \cdot G_F(j \cdot \omega_1) - i_{3\_1} \cdot G_F(j \cdot \omega_1) \cdot X_F(j \cdot \omega_1)$$

- ✓ In order to cancel out the low frequency harmonics, the voltage at the filter output for each of the low frequency harmonics considered must be zero. So, to achieve a successful cancelation, each harmonic component must satisfy (3).

(3) $$0 = v_{AB\_n} - i_{3\_n} \cdot X_F(jn \cdot \omega_1) \quad \text{for n>1}$$

Analytical determination of the switching events

Although the development of the HCT has been already published in [11], along this section a brief summary has been included in order to improve the understanding of the following ones.

The key idea of the modulation technique is grounded on the generation at the inverter output of a set of low frequency harmonics which cancel out those across the filter inductance due to the current drawn by the non linear load. Therefore, control over both the phase and module of each harmonic is needed. Additionally, in order to prevent the presence of even harmonics in the system, half wave symmetry is required. Thus, the inverter output voltage (v_A) can be described by a Fourier series formed by odd terms in sine and cosine.

The Fourier coefficients of v_A as a function of the switching angles (α_p) are known [12] and provided in (4) and (5).

(4) $$A_{n\_vA} = \frac{4}{\pi} \cdot \frac{V_{in}}{2} \cdot \frac{1}{n} \sum_{p=1}^{N} (-1)^p \cdot \sin(n \cdot \alpha_p)$$

(5) $$B_{n\_vA} = \frac{4}{\pi} \cdot \frac{V_{in}}{2} \cdot \frac{1}{n} \left[-1 + \sum_{p=1}^{N} (-1)^{p-1} \cdot \cos(n \cdot \alpha_p) \right]$$

where: N - Number of switching angles to be determined, V_{in} - Inverter DC input voltage, α_p - Switching angles, n - Harmonic order.

As in any other inverter, the voltage of the middle point of any leg reproduces the switching pattern of its upper switch. So, if the desired pre-distorted voltage at the middle point of a leg A (v_A) is determined, the switching patterns of the six switches can be derived.

Therefore, the first step to determine the switching events of the inverter must be the characterization of the desired inverter output voltage (v_A). In order to determine the inverter output voltage, it has been used the basic equations which define the system operation: (2) and (3).

After the characterization of the current drawn by the non linear load and some algebraic manipulation of the equations, the Fourier coefficients of v_A are obtained as a function of the operating point of the converter and its physical parameters. The complete theoretical development is already published in [11].

Finally, replacing the obtained Fourier coefficients in (4) and (5), the equation system in (6) and (7) is obtained. Solving this system of equations for a given set of operating conditions will lead to a switching pattern able to modulate the inverter output and cancel out the low frequency content due to the non linear load operation.

The switching patterns calculated for different sets of operating conditions are stored within a look-up table (control table) in order to provide harmonic cancellation at any time during the regular operation of the converter. The control table will be calculated taking into account the ranges of variation of the operating conditions involved.

(6) $$0-\sin(n\alpha_1)+\sin(n\alpha_2)-...+\sin(n\alpha_N)=\frac{A_{n\_vA}}{\frac{4}{\pi}\cdot\frac{V_{in}}{2}}$$

(7) $$1-\cos(n\alpha_1)+\cos(n\alpha_2)-...+\cos(n\alpha_N)=\frac{-B_{n\_vA}}{\frac{4}{\pi}\cdot\frac{V_{in}}{2}}$$

Limitations of the feasible solutions space

Let's consider a fully designed converter, since all its physical parameters must be known a priori. Then, it is important to determine the feasible solutions space in order to be able to provide a successful harmonic cancellation for the whole range of operation of the converter.

There are three different operating conditions which might vary within a range and must be taken into account to determine the feasible solutions space:

- ✓ V_{in} *Inverter supply voltage from the catenary.* The supply voltage of the inverter is not constant but it may vary within a pretty wide range. For instance, considering a nominal input voltage of 1500V the actual input voltage might vary within 1100V to 1800V.
- ✓ V_{DC} *Regulated output voltage of the battery charger.* The battery charger is a regulated device which output will be regulated to the nominal voltage of the batteries connected to its output. Some usual values might be: $24V_{dc}$, $72V_{dc}$ or $100V_{dc}$. Additionally, for a given nominal voltage, variation is allowed within a range as it can be seen in [1].
- ✓ P_{DC} *Output power delivered by the battery charger.* It is defined in (8) as a percentage of the nominal power delivered to the linear load by the inverter ($\%P_{AC}$) and it can vary up to 50% of the nominal power (P_{AC}).

(6) $$P_{DC}=\%P_{AC}\cdot P_{AC}$$

Since there are three separate ranges of variation involved, along the following paragraphs the influence of each of them on the feasible solution space is going to be illustrated. In order to achieve this aim, an actual converter has been considered and its characteristics are shown in Table I.

Table I. Converter characteristics

L_{AC}	C_{AC}	L_{DC}	C_{DC}	N_1	N_2	N_3
3.5mH	220µF	405uH	45mF	102	40	29
V_{RMS_nom}=400V			P_{AC_nom}=170kW			

The HCT is implemented in 7 notches. This is, 14 switching events per half period must be determined in order to control 7 harmonic components (fundamental component and 6 additional non triplen odd harmonics: 5th, 7th, 11th, 13th, 17th and 19th). In Fig. 3 the evolution of the 14 switching angles vs. the inverter input voltage for some different conditions at the battery charger output (output power and output voltage) can be observed.

It can be observed that the switching angles tend to get closer (the switching pulses gets narrower) as the input voltage decreases. Furthermore, below a given value of the inverter supply voltage, no feasible solution is found. Thus there is a lower limit for the feasible solutions space imposed by the inverter supply voltage.

Additionally, it is also noticeable that the minimum required input voltage is different depending on the non linear load operating conditions ($\%P_{AC}$ and V_{DC}):

- ✓ For a given non linear load output voltage (V_{DC}), the minimum inverter input voltage increases as the DC power delivered increases.
- ✓ For a given DC power, the minimum input voltage increases as the DC output voltage decreases.

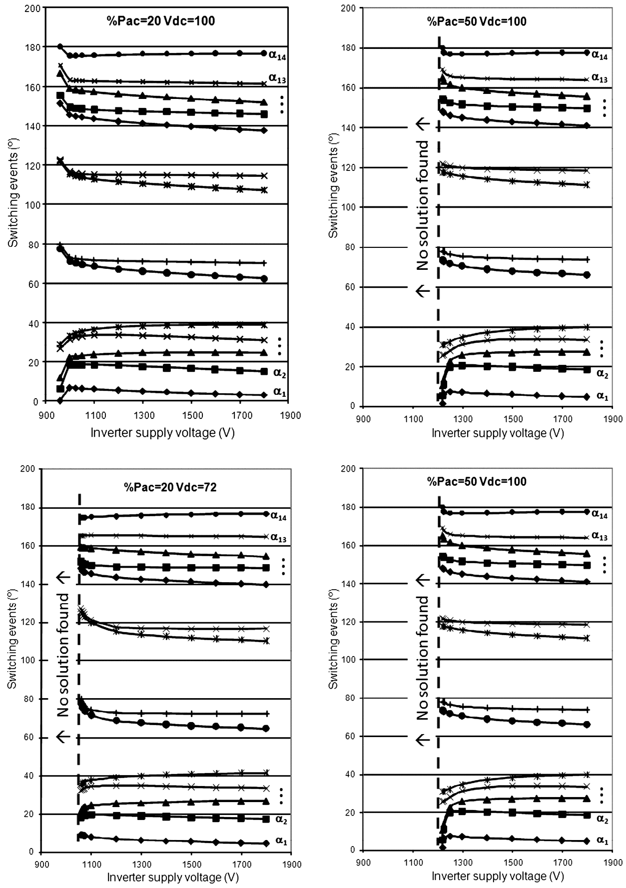

Fig. 3 Switching events evolution vs. inverter supply voltage for four different conditions at the battery charger output

Thus, it can be said that the limiting factor is the voltage drop across the filter inductance due to the current drawn by the non linear load. As the current drawn by the battery charger increases, the voltage drop across the filter inductance increases and the minimum required input voltage to achieve a feasible solution increases.

Then, part of the available energy at the inverter input will be used to generate those low frequency harmonics. And so, as the low frequency harmonic content of the current drawn by the battery charger increases, the voltage across the filter inductance increases, and the low frequency harmonics at the inverter output will also be bigger. So a bigger amount of the available energy at the input will be injected at low frequencies.

Therefore, the worst case for the generation of the switching events will be the following combination of operating conditions:

- ✓ Minimum input voltage
- ✓ Maximum power delivered by the non linear load
- ✓ Minimum voltage at the non linear load

However, the feasible solutions space does not depend exclusively on the operating conditions, but it can be modified if the design of the converter is taken into account. This is, some parameters of the converter may help to enlarge the feasible solutions space. They are explained along the following section.

Design parameters which enlarge the feasible solutions space

Take into account the simplified per-phase equivalent of the converter shown in Fig. 4. The current drawn by the non linear load is represented by means of a current source (i'_3). Where, i'_3 is the current drawn by the battery charger seen at the primary side of the transformer. This current flows through the filter inductance provoking a voltage drop across that inductance (v_F). Therefore, it is easily observed that the size of the filter inductance directly affects the size of the low frequency harmonics which the inverter must generate to cancel out those due to the non linear load. Then, the effect of the size of this inductance is shown in Fig. 5.

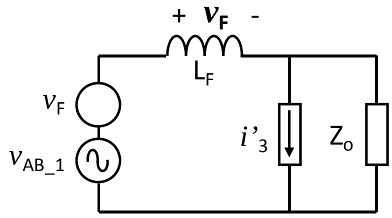

Fig. 4 Simplified per-phase equivalent of the Auxiliary Railway Supply which supplies a linear and a non linear load

On Fig. 5 the minimum input voltage required to achieve a feasible solutions is represented against the power delivered by the battery charger. Additionally, in order to illustrate the influence of the filter inductance on the feasible solutions space, three different values of filter inductance have been considered, as well as two different DC output voltages. It can be observed that the use of lower DC output voltages penalizes the feasible solutions space, as it has been previously established.

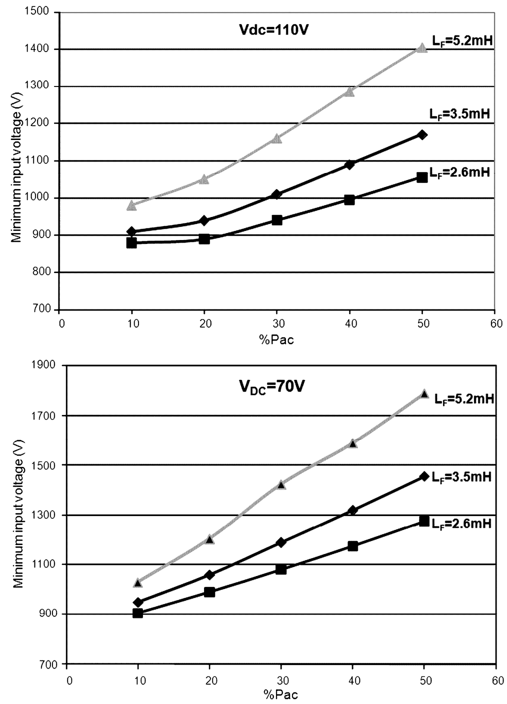

Fig. 5 Minimum input voltage required to achieve a feasible solution vs power delivered by the battery charger. Three different filter inductances have been considered as well as two different DC output voltages.

Regarding the influence of the filter inductance, the tendency is the same regardless the DC output voltage. This is, as the filter inductance value decreases the feasible solutions space is increased; since the minimum required voltage is lower.

Another design parameter involved, is the transformer ratio between its primary and tertiary sides (n_{13}). Since the modules of the harmonic components which flow through the filter inductance are affected by this transformer ratio.

As it has been illustrated previously, the larger the non linear load current, the bigger the minimum required input voltage. Then, if the number of turns of the transformer tertiary side is reduced, then the feasible solutions space will be enlarged.

Actual converters are subject to parasitic voltage drops such as parasitic resistances of the filter inductances, etc. Even though they must be taken into account within the calculation of the switching events; their effect on the feasible solutions space is negligible compared to the effect of the operating conditions and the aforementioned design parameters.

HCT Experimental Performance

The experimental performance of the Harmonic Cancellation Technique regarding the output voltage THD (9) has been carried out on a scaled-down prototype which characteristics are summarized in Table II. Considering an inverter input voltage range from 325V to 475V and a DC output power from 5% to 40%, the HCT is able to provide a successful performance in the whole range.

(9) $$THD = \sqrt{\frac{\sum_{n=2}^{40} V_{o\_n}^2}{V_{o\_1}^2}}$$

In order to illustrate the HCT performance, in Fig. 6 a comparison between the well known Selective Harmonic Elimination (SHE) versus HCT has been carried out. As it can be seen, the THD is reduced when the inverter is modulated with the HCT.

Table II. Prototype characteristics

L_{AC}	R_{LAC}	C_{AC}	L_{DC}	R_{LDC}	C_{DC}	V_{DC}	N_1	N_2	N_3
22mH	1.61 Ω	4.5 µF	5.4 mH	0.3 Ω	16.4 mF	75 V	255	286	90
V_{RMS_nom}=400V						P_{AC_nom}=1280W			

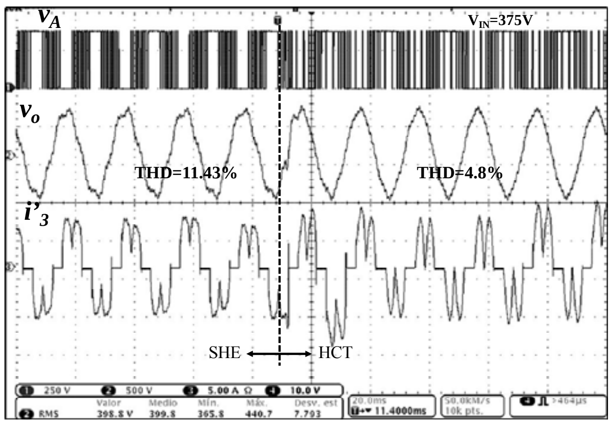

Fig. 6: Comparison of SHE vs. HCT for I=5.2A (CCM). vA voltage in the middle point of the branch A of the inverter referred to ground, vo line to line voltage on the secondary side of the transformer, i3 current drawn by the nonlinear load

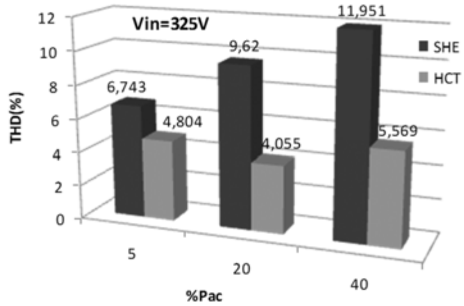

Fig. 7: Output voltage THD vs. %Pac for an inverter input voltage of 325V

Fig. 6 only shows a trial for an inverter input voltage of 375V, but it has been carried out a sweep regarding the input voltage and the DC output power. The obtained results are shown in Fig. 7 and Fig. 8 for an inverter input voltage of 325V and 475V, respectively. Three different DC output powers have been considered for each voltage.

It can be observed that the THD is reduced in all cases. Since the high frequency THD is higher for higher input voltages, the THD reduction is slightly better for low input voltages than for high input voltages.

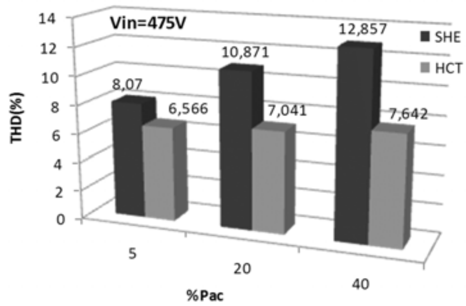

Fig. 8: Output voltage THD vs. %Pac for an inverter input voltage of 475V

Conclusions

Along the present paper, the feasible solutions space for the Harmonic Cancellation Technique (HCT) has been studied. The HCT is able to pre-distort the inverter output in order to cancel out the harmonic content due to a non linear load performance. And so reduce the THD at the output voltage of the auxiliary railway power supply. For each set of operating conditions (V_{in}, V_{DC}, P_{DC}) the proper switching pattern is determined analytically.

However, not all the combinations of the aforementioned variables lead to a feasible solution. Thus, the determination of the feasible solutions space is needed to guarantee a successful cancellation of the low frequency content along the whole range of variation of the converter operating conditions.

For any converter the most restrictive operating conditions in terms of achieving a feasible solution, are: minimum input voltage, maximum power delivered by the non linear load and minimum voltage at the non linear load. So, if a solution is found in these conditions then the HCT can be used for the whole converter range of operation.

However the feasible solutions space does not depend only on the operating conditions, some physical parameters of the converter must be taken also into account. In order to enlarge the feasible solutions space, the filter inductance and the primary-tertiary transformer ratio should be designed as small as possible.

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REFERENCES

[1] IEEE Std. 1476-2000 "IEEE Standard for passenger train auxiliary power systems and interfaces" ISBN 0-7381-1964-4

[2] S. Choi "A three phase unity power factor diode rectifier with active input current shaping" IEEE Transactions on Industrial Electronics, Volume 52, Issue 6, pp: 1711-1714, December 2005

[3] M. Baumann, J. W. Kolar, "Parallel Connection of Two Three-Phase Three-Switch Buck-Type Unity-Power-Factor Rectifier Systems With DC-Link Current Balancing" IEEE Transactions on Industrial Electronics*,* Volume: 54 Issue: 6, *pp*: 3042 - 3053, Dec. 2007

[4] H. Deng, R. Orungati, D. Srinivasan "Modeling and control of single phase UPS inverters: a survey" International Conference on Power Electronics and Drives systems (PEDS) 2005, pp. 848-853

[5] S. Buso, S. Fasolo, P. Matavelli "Uninterruptible power supply multiloop control employing digital predictive voltage and current regulators" IEEE Transactions in Industry Applications, vol 37, 2001, pp. 1846-1854

[6] S. Sae-Sue, V. Kinnares, C. Tangsiriworakul, S. Potivejkul "Comparative performance evaluation of fixed and adaptive hysteresis band delta modulation techniques for UPS" International Conference on Power Electronics and Drive Systems 1999 pp. 956-960.

[7] F. Z. Peng "Harmonic sources and filtering approaches" IEEE Industry Applicatoins Magazine, Volume 7, Issue 4, Jul/Aug 2001 pp. 18-25

[8] K. Nishida, Y. Konishi, M. Nakaoka "Novel current control scheme with deadbeat algorithm and adaptive line enhancer for three phase current source active power filter" IEEE Industry Applications Conference Volume 1, Issue 30, Sept-Oct 2001, pp. 194-201

[9] H. Akagi "Modern active filters and traditional passive filters" Bulletin of the Polish Academy of Science, Vol 45, no 3, 2006.

[10] S. Bhim, K. Al Haddad, A. Chandra "A review of active filters for power quality improvement", IEEE Transactions on Industrial Electronics, vol 46, no 5,1999, pp. 960 – 971.

[11] I. Quesada, A. Lazaro; C. Martinez; A. Barrado; M. Sanz, C. Fernandez, R. Vazquez, I. Gonzalez, "Modulation technique for low frequency harmonic cancellation in Auxiliary Railway Power Supplies", IEEE Transactions on Industrial Electronics, Vol. 58, No. 9, September 2011, pp.: 3976-3987, ISSN. 0278-0046.

[12] D. Grahame Holmes, Thomas A. Lipo, "Pulse Width Modulation for Power Converters. Principles and Practice" ISBN 0-471-20814-0

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