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Sensorless Sliding Mode Control of PMSM Drives Using a High Frequency Injection Algorithm

***Abstract**. A high frequency injection (HF) technique for rotor position estimation using Sliding Mode Control (SMC) in Permanent Magnet Synchronous Machines (PMSMs) is presented. Since SMC is used, instead of the Field Oriented Control (FOC), the injected HF test signals for tracking the machine's saliency are naturally eliminated. Such angle information suppression is tackled using "the equivalent control" principle. Simulation results confirm not only the proper angle estimation but also the sensorless control at low speed reversal and load impact at zero speed.*

***Streszczenie.** W artykule przedstawiono technikę dodawania sygnału wysokoczęstotliwościowego (HF) w celu estymacji położenia wirnika w przypadku sterowania ślizgowego (SMC) maszyną synchroniczną z magnesami trwałymi (PMSMs). Od kiedy do sterowania ślizgowego, zamiast metody FOC, jest dodawany testowy sygnał wysokoczęstotliwościowy, asymetria magnetyczna maszyn zostaje w sposób naturalny wyeliminowana. Tłumienie informacji o kącie może być rozwiązane przy użyciu zasady „regulacji równoważnej". Wyniki symulacji potwierdzają nie tylko właściwą estymację kąta jak również, ale również możliwość sterowania bezczujnikowego przy bardzo niskich prędkościach i nawrocie maszyny.. (**Bezczujnikowe sterowanie ślizgowe napędem PMSM z zastosowaniem algorytmu z dodawaniem sygnału wysokoczęstotliwościowego**)*

Keywords: Sliding Mode Control. PMSM Sensorless Control. High Frequency Injection Methods.
Słowa kluczowe: Sterowanie ślizgowe, bezczujnikowe sterowanie PMSM, metoda dodawania HF.

Introduction

Permanent Magnet Synchronous Machines (PMSMs) have higher power density, higher efficiency and better dynamic performance than Induction Machines. Its control requires accurate rotor position information in order to implement the coordinate transformation and the speed (and position) vector control loops. Significant research efforts have been conducted in order to achieve vector control of PMSMs without encoders or resolvers. These techniques can be broadly divided into model based techniques, where the back-emf of the machine is used for rotor magnet flux detection, and injection techniques, where a test signal, either high frequency (HF) AC voltage or voltage pulse, is used to detect the rotor saliency (difference between LD and LQ) [1,2].

Model based techniques e.g. [3,4] successfully achieve sensorless control at medium and high rotor speed but fail at low excitation frequencies due to the reduction and eventual disappearance of the back-emf induced by the rotor magnets at low rotor speed.

Injection methods, on the other hand, detect the angular dependent saliency of the machine and its rotor position estimation is therefore fundamentally speed independent [1,2].

There are mainly two different injection techniques. The first one consists on the superposition to the fundamental voltage vector of a HF injection either in the alpha/beta frame [5,6,7,8] or in rotating d/q synchronous frame [9,10]. The second ones are based on the modification of the fundamental PWM pattern to include a voltage pulse test [11,12,13].

For all injection methods to function, some level of machine saliency is necessary. This makes the technique straightforward for salient machines such as the interior permanent magnet ones. Surface Mount PMSM, on the other hand, only have a saliency due to stator tooth saturation and it is generally of small magnitude.

Such mentioned injection techniques are mostly implemented using Field Oriented Control (FOC) schemes where the band width of the inner current control loops is typically of lower value than the injected frequency. Therefore, the system does not react against such a perturbation not eliminating the further position information. On the other hand, Sliding Mode Controllers (SMC) have a band width, (which strictly speaking is not defined since it is not a linear controller), within the range of the injected test signals and therefore the system tends to cancel it as any other perturbation.

In this paper, the HF injection technique for tracking the rotor position in a Surface Mount PMSM Controlled by Sliding Mode is addressed. The novelty of the paper relies on the use of SMC instead of the well known FOC technique and therefore the issue is to tackle the natural suppression of the injected signal. The well-know benefits of the SMC [14] [15], like fast torque response, robustness, reduction system order or high immunity against parameters drift are reached, improving the FOC dynamics.

Position signals and the injection algorithm are discussed and fully presented. Finally, Sensorless SMC speed reversal and load impact results are presented.

PMSM Model

The electrical equations that model the PMSM in the rotating reference frame (dq) are shown on (1) and (2).

(1) $$\frac{di_D}{dt}=\frac{v_D}{L_D}-\frac{R}{L_D}\cdot i_D+\omega_e\frac{L_Q}{L_D}i_Q$$

(2) $$\frac{di_Q}{dt}=\frac{v_Q}{L_Q}-\frac{R}{L_Q}\cdot i_Q-\omega_e\cdot\frac{L_D}{L_Q}i_D-\omega_e\cdot\frac{\phi_M}{L_Q}$$

where: v_D, v_Q - Stator *dq* Voltages, i_D, i_Q - Stator *dq* currents, L_D, L_Q -Stator *dq* Inductancse, R - Stator Resistance, Φ_M - Permanent Magnet Flux, ω_e - Electrical angular speed, p - Pole pairs number, B - Friction coefficient, J - Moment of inertia, T_L - Load torque, ω_m - Mechanical angular speed.

The related electromagnetic torque equation to *dq* current components is expressed at:

(3) $$T_E=\frac{3\cdot p\cdot\phi_M}{2}\left[i_Q-(L_D-L_Q)\cdot i_D\cdot i_Q\right]$$

where:

(4) $$k_T=\frac{3\cdot p\cdot\phi_M}{2}$$

The PMSM motion equation is:

(5) $$\frac{d\omega_m}{dt}=\frac{1}{J}Te-\frac{B}{J}\omega_m-\frac{1}{J}T_L$$

From the expressions (3), (4) and (5) the third state space system equation can be obtained.

PMSM Sliding Mode Control

The SMC is applied to the system defined by equations (1) and (2). The SMC is executed above a control variable, which will take two discrete values at any time, choosing the proper one depending on the system state. The control forces the systems states trajectories towards the switching surface at any time. The switching surfaces are defined by the systems errors, where the errors are the difference between the desirable and the real values of the controllable variable.

In order to control PMSM, the chosen controllable variables are *dq* currents, and consequently the control variables are *dq* voltages.

The switching surfaces (*S*) for controllable variables are defined, using directly the error of the reference value (x^*) and real value (x) of a given variable x, then:

(6) $$S_D = i_D^{\,*} - i_D = e_{ID}$$

(7) $$S_Q = i_Q^{\,*} - i_Q = e_{IQ}$$

Using the expression (1), (2), (6) and (7), and knowing that the equilibrium point of the system is forced by the SMC, the errors system dynamics is rewritten as follows:

(8) $$\frac{de_{ID}}{dt} = \frac{v_D}{L_D} - \frac{R}{L_D} \cdot e_{ID} + \omega_e \frac{L_Q}{L_D} \cdot e_{IQ}$$

(9) $$\frac{de_{IQ}}{dt} = \frac{v_Q}{L_Q} - \frac{R}{L_Q} \cdot e_{IQ} - \omega_e \cdot \frac{L_D}{L_Q} \cdot e_{ID}$$

The control will be defined by two discrete values, depending on the *S* sign. The discrete *dq* voltages values are defined as:

(10) $$v_D \in \{v_{D0}, -v_{D0}\}$$

(11) $$v_Q \in \{v_{Q0}, -v_{Q0}\}$$

The SMC has to ensure the system trajectories are always directed towards the sliding surface, and this is achieved when (12) and (13) are fulfilled:

(12) $$S_D \cdot \dot{S}_D < 0$$

(13) $$S_Q \cdot \dot{S}_Q < 0$$

Using the equivalent control definition (v_{DEQ}, v_{QEQ}), which is defined as the input values of control variable that satisfy a given system condition, expressions (16) and (17) are extracted from (14) and (15). The physical meaning of the equivalent control is that the system variables are on the switching surfaces and remain there.

(14) $$S_D \cdot [v_D - v_{DEQ}] < 0$$

(15) $$S_Q \cdot \lfloor v_Q - v_{QEQ} \rfloor < 0$$

That yields:

(16) $$v_D = -v_{D0} \cdot sign(S_D)$$

(17) $$v_Q = -v_{Q0} \cdot sign(S_Q)$$

Taking a look of the control law expression summarized in (16) and (17), it could be noted that the control does not depend on any system state variable or parameters value (*R* or *L*), which proves the high robustness against parameters drifts.

In order to ensure that the control can be successfully employed, the sliding domain must be guaranteed. Indeed, the two variable control discrete values have to be able to produce the equivalent control values, necessary to satisfy the references values for any given conditions.

(18) $$-v_{D0} < v_{Deq} < v_{D0}$$

(19) $$-v_{Q0} < v_{Qeq} < v_{Q0}$$

After the SMC has been deduced and developed, the PMSM can be controlled in terms of *dq* current with the control law obtained.

For setting the reference current values, it is necessary to understand clearly the physical meaning of the *dq* model. The *dq* motor model, as it was noted before, is a rotating reference frame system aligned with the motor PM. The *d* current component is aligned in the flux PM direction, and the *q* current component, its 90° phase-shifted from *d* component. From this point of view, the *dq* PMSM model equation is highly similar to a stator equation of a DC motor. Besides, from this deduction, is clear that the preferred desired current value on *d* component has to be set to 0, in order to avoid fluxes that could demagnetize the PM. Torque current component, that is completely perpendicular to PM flux, is created by i_Q, as it can be deduced from (3).

Sliding Mode Control and High Frequency Injection

a. Analyzing the Control Band Width

The injection of a rotating HF voltages for saliency tracking when using FOC has been proposed by several authors [1,3].

At this point, analyzing the SMC proposed on section II, a big difference with FOC control arises. Meanwhile the bandwidth of the FOC control can be easily known by the PI controllers, the SMC have, theoretically, an infinite bandwidth (BW).

Typically, HF carrier signal frequency for angle estimation is fixed around 1-2 kHz (between fundamental electrical frequency and PWM frequency). Fig. 1, shows how the HF injection is inside SMC BW. This means that the SMC will perform a determined action control against this HF injection, being treated as an external disturbance.

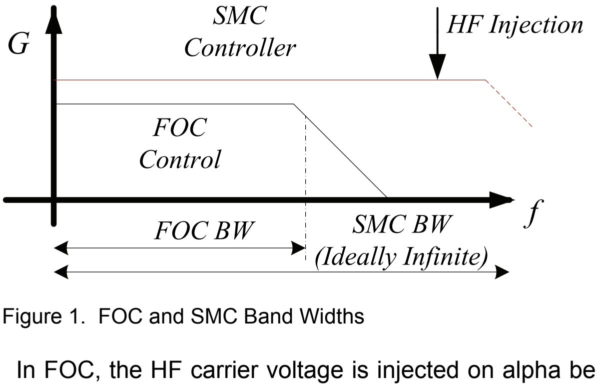

Figure 1. FOC and SMC Band Widths

In FOC, the HF carrier voltage is injected on alpha beta (αβ) plane, the PI controllers are not sensible at this frequency and the system does not perform any action against this disturbance. The HF signals pass through the PMSM, and the HF currents outputs have the desired angle information. When the same injection technique is used with SMC, the HF voltage passing through the PMSM is almost reduced to zero as well as the HF output currents because of the higher BW of the control and the capability to react against the perturbation.

Taking into account the idea explained before, is reasonable to choose a HF current carrier for the system. In the same way, the control will perform against the perturbation, but in this case, the current signal generated by the control, ideally equal to the disturbance with an opposite signal, are passing through the motor. In this injection method, the signal processing to obtain the angle estimation must be done above the αβ voltages. On Fig. 2, the proposed system with the HF current injected and HF voltage output processed is presented.

Note that an outer speed control loop has been implemented, based on the slower mechanical motion PMSM described on equation (4). A classical PI linear controller is used, which will give the q current reference to the SMC, keeping the d current component always equal to zero. If the sliding domain is ensured, we can extract the dynamics of the system related to the angular speed from equation (4).

b. HF PMSM System Model

In the PMSM with saliency, there are variations of the inductance values depending on the rotor position. These changes are caused by the different flux levels on the corresponding PMSM areas, depending on PM position. The inductance factor change in front of PM position is modelled:

$$(20)\quad L_\theta = \begin{bmatrix} Ls - \Delta Ls \cdot cos(2\theta_r) & -\Delta Ls \cdot sin(2\theta_r) \\ -\Delta Ls \cdot sin(2\theta_r) & Ls + \Delta Ls \cdot cos(2\theta_r) \end{bmatrix}$$

Where

$$(21)\quad \Delta Ls = \left(L_Q - L_D\right)/2$$

$$(22)\quad Ls = \left(L_Q + L_D\right)/2$$

Note that in a non-salient PMSM (L_D=L_Q), the inductance versus angle variation is not produced.

The *αβ* PMSM model it is defined by (23).

$$(23)\quad \begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix} = R\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} + \frac{d}{dt}L_\theta\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} + \cdot\phi_m \frac{d}{dt}\begin{bmatrix} \cos(\theta_r) \\ \sin(\theta_r) \end{bmatrix}$$

If we assume the disturbance frequency higher enough than the phenomena related to the motor rotation, the HF motor model can be simplified as stated in (24).

$$(24)\quad \begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix} \approx \frac{d}{dt}L_\theta \cdot \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix}$$

Angle Estimation in Sliding Mode Control Scheme

The current injection added on the *αβ* currents is:

$$(25)\quad \begin{bmatrix} i_{\alpha P} \\ i_{\beta P} \end{bmatrix} = -A_P \cdot \begin{bmatrix} -sin(\omega_i t) \\ cos(\omega_i t) \end{bmatrix}$$

Under sliding motion the current vector cancelling the disturbances can be modelled as (26):

$$(26)\quad \begin{bmatrix} i_{\alpha P} \\ i_{\beta P} \end{bmatrix} = A_P \cdot \begin{bmatrix} -sin(\omega_i t) \\ cos(\omega_i t) \end{bmatrix}$$

The equation (24) is solved using as an input currents the ones showed on (26). Solving the proposed expression, (27) is reached. These voltages can be obtained using a proper band pass filter, depending on the injection frequency chosen.

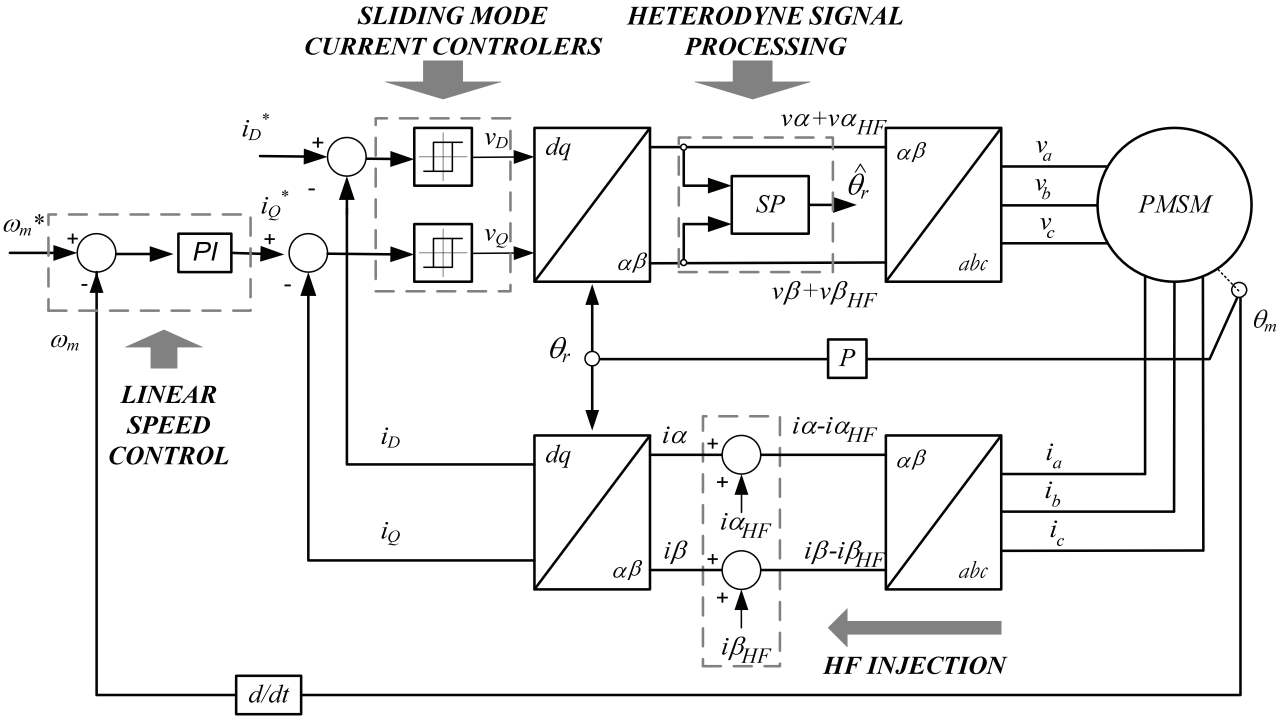

Figure 2. Sliding Mode Control with High Frequency Injection.

$$(27)\begin{bmatrix} v_{\alpha HF} \\ v_{\beta HF} \end{bmatrix} \approx \begin{bmatrix} v_O \cdot cos(\omega_i t) + v_1 \cdot cos(2\theta_r - \omega_i t) \\ v_O \cdot sin(\omega_i t) + v_1 \cdot sin(2\theta_r - \omega_i t) \end{bmatrix}$$

where $v_0 = -\omega_I \cdot A_P \cdot Ls$, $v_1 = \omega_I \cdot A_P \cdot \Delta Ls$

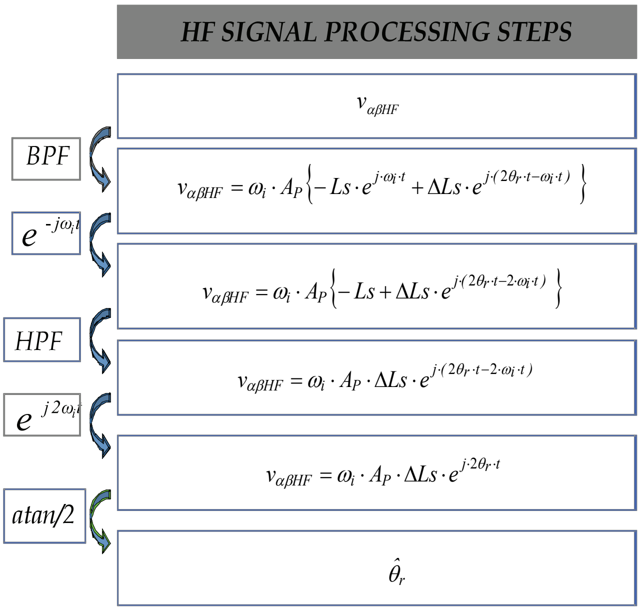

Figure 3. Signal Processing Algorithm steps

The resultant output voltages have the same structure than the output current voltages resultant when a HF voltage signal is injected in FOC application. Therefore, the signal processing, shown in Fig. 3, for angle estimation (θe) is almost equal.

Results

The simulation results showed have been obtained using the PMSM characterized by the table II parameters. The speed PI control and SMC were adjusted for a good dynamic response and acceptable error on the angle estimation obtained.

TABLE I. PERMANENT MAGNET SYNCHRONOUS MACHINE

Surface Mount PMSM	
Rated power	3.8 kW
Poles number	6
Nominal speed / Rated torque	314.15 rad/s / 12.2 Nm
R / Ld / Lq	0.94 Ω / 7 mH / 8.3 mH
Magnetic flux linkage [Wb]	0.2515
Friction Coefficient [N·m·s]	0.03833
Moment of Inertia [kg·cm^2]	20.5

TABLE II. ANGLE ESTIMATION BLOCK PARAMETERS

Injection Frequency	1 kHz
Injection Signal Amplitude	250 mA
Band-Pass Filter	4th Order Butterworth Type LP = 800Hz HP= 1200Hz
High-Pass Filter	1st Order Butterworth Type HP= 60 Hz

a. *Position Signals*

Fig. 4 and Fig. 5 correspond to steady state conditions, at 1% of the nominal speed (3.14 rad/s) and full load torque (12.2 Nm), which implies that the frequency of the position signals is at 3 Hz, considering the number of pole pairs and the fact that the position signal frequency is twice the fundamental electrical frequency. The results showed on Fig. 4 are directly the signals obtained at the end of the Signal Process, not applying any additional filter. In order to validate the quality of the position signals the FFT is performed as shown in Fig. 5. The quality of the signal is remarkable since any sub harmonic appears which might cause an error in the estimated angle.

From the *αβ* position signals the *arc tangent* function is executed and the estimated angle is obtained.

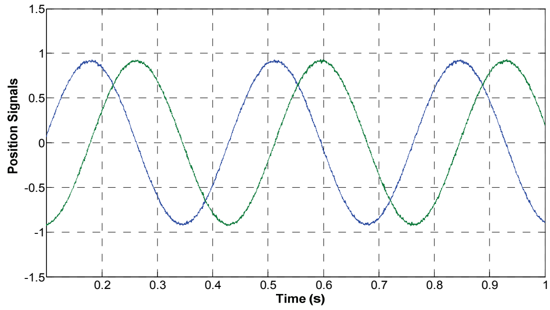

Fig. 4. Alpha and beta position signals.

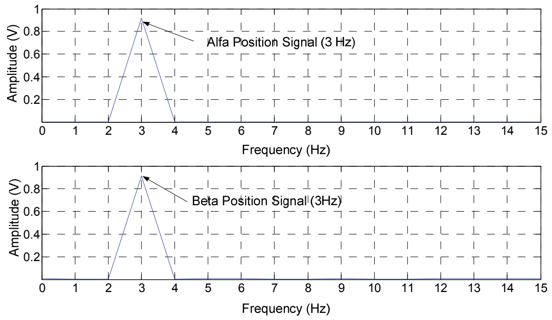

Fig. 5. Position signals' FFT.

b. *Speed Reversal*

Fig. 6 illustrates the performance under a step speed change (+/-10 rad/s) at full load torque (12.2Nm), under sensorless conditions.

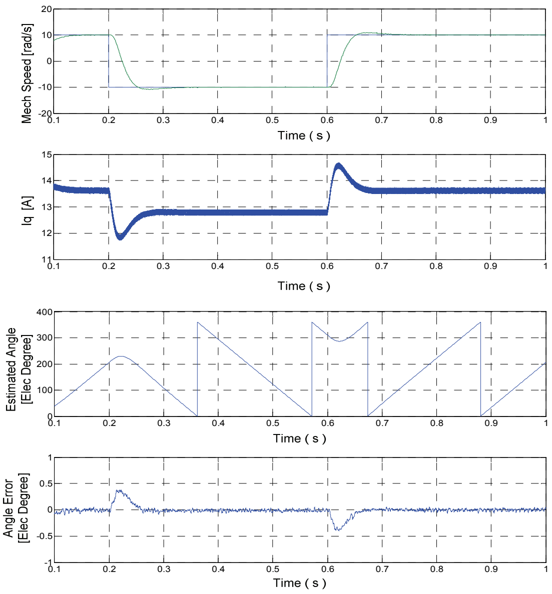

Fig. 6 Sensorless Speed Reversal (±10 rad/s) at full load (12.2 Nm). From top: Estimated (green) and reference speed (blue) (rad/s); current torque demand (A); estimated angle; angle error. All angles in electrical degrees.

The angle error is kept within ±0.5 electrical degrees (±0.16 mechanical degrees) even during the speed reversal and when the speed is set to zero. The angle estimation is good enough and the SMC used eliminates properly the perturbation included. The i_Q current has an acceptable ripple value caused by the SMC, and any undesired effect caused by the perturbation signal appears. The external PI controller speed loop works properly in speed reversal as shown in Fig. 6.

c. Load Impact

The second experiment involves a Sensorless load impact test with the speed reference set at zero. Fig. 7 shows the speed response for a load impact of 100% and back to 0 again. The angle error is kept at smaller values than ±4 electrical degrees (<±1.3 mechanical degrees) taking into account the worst situation under the transient response.

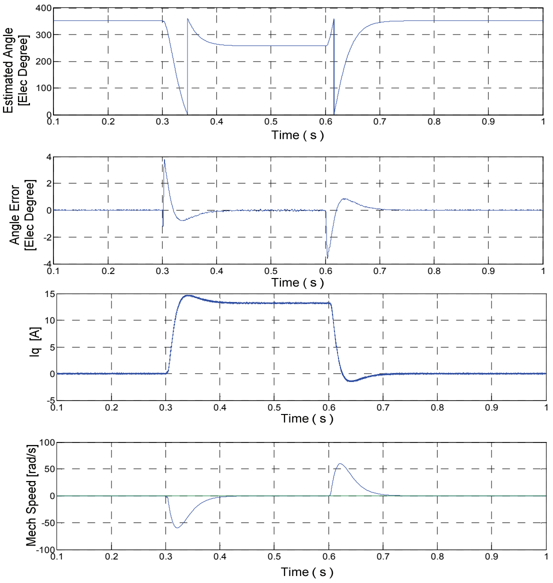

Fig. 7. Sensorless at zero Speed response to 100% load impact. From top: estimated rotor position (electrical degrees); angle error (electrical degrees) ; i_Q current(A); mechanical speed (rad/s)

Conclusions

This work has introduced an angle estimation algorithm for Permanent Magnet Synchronous Machines (PMSMs) at low and zero speed based on the high frequency injection technique to track the machine saliency. The novelty of the work relies on the fact that the PMSM drive is under SMC for the inner current loops instead of the traditional Field Oriented Control (FOC), based on PI controllers.

The traditional voltage injection process used under FOC schemes does not work in SMC due to the ideally infinite band width of SMC controllers. Alternatively, the high frequency test signal has been injected in the feedback *alpha- beta* currents. Despite the elimination of the injected HF current, the angle information can be obtained from the equivalent control voltage values generated by the SMC.

Simulation results (speed reversal and load impact) shows the validity of this angle estimation algorithm since the angle error value is at all times less than 4 electrical degrees. Further research is focused on modelling the VSI and a final implementation in order to obtain experimental results.

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