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The approach based on variation principles for mathematical modeling of asymmetrical states in a power transformer

***Abstract**. In the paper a mathematical model of a power transformer has been presented. The transformer operates at asymmetric supply and load states. The differential equations which describe its working regime have been derived using an interdisciplinary method based on a modification of integral variational principle by Hamilton and Ostrogradsky by an extension of Lagrange's function. On the basis of the proposed model an analysis of functional dependencies for the transformer has been presented. The results of computer simulations in the graphical form have been given*

***Streszczenie.** W pracy przedstawiono model matematyczny transformatora mocy, który pracuje w stanach pracy niesymetrii zasilania i obciążenia. Równania różniczkowe opisujące stany pracy transformatora wyprowadzono z wykorzystaniem interdyscyplinarnej metody, która opiera się na modyfikacji integralnej zasady wariacyjnej Hamiltona-Ostrogradskiego drogą rozszerzenia funkcji Lagrange'a Na podstawie zaproponowanego modelu przedstawiono analizę zależności funkcjonalnych transformatora. Przedstawiano wyniki symulacji komputerowej w postaci graficznej. (**Wykorzystanie podejść wariacyjnych do modelowania matematycznego stanów asymetrycznych w transformatorze mocy**)*

Keywords: Hamilton rule, Euler-Lagrange's system, asymmetric working conditions, power transformer.
Słowa kluczowe: zasada Hamiltona, Euler-Lagrange'a system, stany asymetryczne, transformator mocy.

Introduction

The analysis of asymmetric states in electric devices is at present the subject of intensive research. The causes of lack of symmetry are different: deformation of supply voltage, change in resistance of supply cable, charging of large single-phase users to energy sources etc. It is obvious that the aforementioned causes influence the operation of devices to a large extent [2-5].

The asynchronous motors belong to the group of most prevailing consumers of electric energy [1,4]. They are supplied from secondary windings of power transformers. Therefore, in order to describe adequately the electromechanical processes in drive systems, it is necessary to take into account the processes in the power transformer itself. In order to develop a mathematical model, a generalized variational principle has been used in the present paper.

Theoretical foundations

The electric connection schemes for trqanformer windings are presented in Figs. 1 and 2. [3]

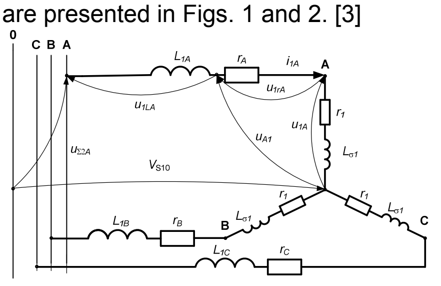


Fig. 1. Electrical connection scheme of the primary winding

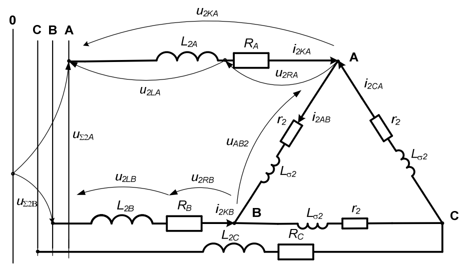


Fig. 2. Electrical connection scheme of the secondary winding

In Fig. 1 and 2 the following notation is introduced: u_{2AB}, u_{2BC}, u_{2CA} – phase voltages of secondary winding, u_{1LA}, u_{2LA} – voltage drop on inductance of phase A of the wire for both windings; L_{1A}, L_{1B}, L_{1C}, L_{2A}, L_{2B}, L_{2C} – phase inductances of both wires

(1) $\Psi_{jA}+\Psi_{jB}+\Psi_{jC}=0,\quad \psi_A+\psi_B+\psi_C=0,\quad j=1,2$;

(2) $u_{\Sigma 1A}+u_{\Sigma 1B}+u_{\Sigma 1C}\neq 0,\quad u_{\Sigma 2A}+u_{\Sigma 2B}+u_{\Sigma 2C}\neq 0$;

(3) $u_{1LA}+u_{1LB}+u_{1LC}\neq 0,\quad u_{2LA}+u_{2LB}+u_{2LC}\neq 0$;

(4) $u_{1rA}+u_{1rB}+u_{1rC}\neq 0,\quad u_{2RA}+u_{2RB}+u_{2RC}\neq 0$;

(5) $i_{2AB}+i_{2BC}+i_{2CA}=0,\quad i_{1A}+i_{1B}+i_{1C}=0$;

(6) $i_{2AB}\equiv i_{2A},\quad i_{2BC}\equiv i_{2B},\quad i_{2CA}\equiv i_{2C}$;

(7) $u_{2AB}+u_{2BC}+u_{2CA}=0,\quad i_{2KA}+i_{2KB}+i_{2KC}=0$;

(8) $R_A\neq R_B\neq R_C,\ L_{1A}\neq L_{1B}\neq L_{1C},\ L_{2A}\neq L_{2B}\neq L_{2C}$;

(9) $i_{2KA}=i_{2A}-i_{2C},\quad i_{2KB}=i_{2B}-i_{2A},\quad i_{2KC}=i_{2C}-i_{2B}$;

(10) $u_{2AB}=u_{2A}-u_{2B},\ u_{2BC}=u_{2B}-u_{2C},\ u_{2CA}=u_{2C}-u_{2A}$.

where $\mathbf{\Psi}_1,\mathbf{\Psi}_2$ – column vectors of total magnetic windings of the primary and the secondary winding of the transformer; $\mathbf{r}_1,\mathbf{r}_2$ – matrices of resistances of the primary and the secondary winding; ψ – column vector of fundamental (main) magnetic windings of the primary and the secondary winding; $\mathbf{i}_1,\mathbf{i}_2$ – column vectors of currents in the primary and the secondary winding, $\mathbf{u}_1,\mathbf{u}_2$ – column vectors of voltages of the primary and the secondary winding. Lagrangian shall be written in the following form: [2]

(11) $$L^*(q,\dot{q},t)=T^*-P^*+\Phi^*-D^*,\ \ \Phi^*=\int_0^t \Phi_p^*(t)\big|_{t=\tau}d\tau ,$$

where L^* – modified non-force Lagrange' function [2]; $\tilde{T}^*$ – total (full) kinetic coenergy of the electromechanical system [2]; P – potential energy concentrated in the system, D^* – energy of active and passive non-potential forces, Φ^* – function of internal and external dissipation energy; Φ_p^* – dissipative function of the system; τ – additional integration variable

In generalized coordinates the electric charges in the windings of three-phase transformer are denoted as $q_{1-6}=Q_{1-6}$, currents in these windings as $\dot{q}_{1-6}=i_{1-6}$, respectively: three currents in the primary winding: i_{1A},i_{1B},i_{1C}, and three windings in the secondary winding: i_{2A},i_{2B},i_{2C}; $\mathbf{q}\equiv(Q_{1A},Q_{1B},Q_{1C},Q_{2A},Q_{2B},Q_{2C})^{\mathrm{T}}$; $\dot{\mathbf{q}}\equiv(i_{1A},i_{1B},i_{1C},i_{2A},i_{2B},i_{2C})^{\mathrm{T}}$.

$$\tilde{T}^* = \sum_{j=1}^{2}\left(\int_0^{i_{jA}} \Psi_{jA}(i_{jA})di_{jA} + \int_0^{i_{jB}} \Psi_{jB}(i_{jB})di_{jB} + \int_0^{i_{jC}} \Psi_{jC}(i_{1C})di_{jC}\right) +$$

$$+\frac{1}{2}\left(L_{1A}i_{1A}^2 + L_{1B}i_{1B}^2 + L_{1C}i_{1C}^2\right) + \frac{1}{2}\left(L_{2A}(i_{2A}-i_{2C})^2 +\right.$$

$$\left.+L_{2B}(i_{2B}-i_{2A})^2 + L_{2C}(i_{2C}-i_{2B})^2\right); \tag{12}$$

$$\Phi^* = \int_0^t \frac{1}{2}\Big((r_1+r_A)i_{1A}^2 + (r_1+r_B)i_{1B}^2 + (r_1+r_C)i_{1C}^2 +$$

$$r_2(i_{2A}^2 + i_{2A}^2 + i_{2A}^2) + R_A(i_{2A}-i_{2C})^2 +$$

$$+R_B(i_{2B}-i_{2A})^2 + R_C(i_{2C}-i_{2B})^2\Big)d\tau\,; \tag{13}$$

$$D^* = \int_0^t (u_{\Sigma 1A}i_{1A} + u_{\Sigma 1B}i_{1B} + u_{\Sigma 1C}i_{1C})d\tau -$$

$$-\int_0^t V_{01}(i_{1A}+i_{1B}+i_{1C})d\tau + \int_0^t \Big((u_{\Sigma 2A}-u_{\Sigma 2B})i_{2A} +$$

$$+(u_{\Sigma 2B}-u_{\Sigma 2C})i_{2B} + (u_{\Sigma 2C}-u_{\Sigma 2A})i_{2C}\Big)d\tau\,, \tag{14}$$

where the second integral denotes the virtual energy.

Substituting the extended Lagrangian (11), whose components are energy terms given with relationships (12) – (14), into the Eulera-Lagrange equation [2]:

$$\frac{d}{dt}\frac{\partial L^*}{\partial \dot{q}_k} - \frac{\partial L^*}{\partial q_k} = 0, \quad k = 1,\dots,6 \tag{15}$$

the following relationships have been obtained

$$\frac{d\Psi_{1j}(i_{1j})}{dt} + (r_1+r_j)i_{1j} - u_{\Sigma 1j} + V_{10} + L_{1j}\frac{di_{1j}}{dt} = 0\,; \tag{16}$$

$$\frac{d\Psi_{2A}(i_{2A})}{dt} + r_2 i_{2A} + u_{\Sigma 2B} - u_{\Sigma 2A} + L_A\frac{d(i_{2A}-i_{2C})}{dt} -$$

$$-L_B\frac{d(i_{2B}-i_{2A})}{dt} + R_A(i_{2A}-i_{2C}) - R_B(i_{2B}-i_{2A}) = 0\,; \tag{17}$$

$$\frac{d\Psi_{2B}(i_{2B})}{dt} + r_2 i_{2B} + u_{\Sigma 2C} - u_{\Sigma 2B} + L_B\frac{d(i_{2B}-i_{2A})}{dt} -$$

$$-L_C\frac{d(i_{2C}-i_{2B})}{dt} + R_B(i_{2B}-i_{2A}) - R_C(i_{2C}-i_{2B}) = 0\,; \tag{18}$$

$$\frac{d\Psi_{2C}(i_{2C})}{dt} + r_2 i_{2B} + u_{\Sigma 2C} - u_{\Sigma 2B} + L_C\frac{d(i_{2C}-i_{2B})}{dt} -$$

$$-L_A\frac{d(i_{2A}-i_{2C})}{dt} + R_C(i_{2C}-i_{2B}) - R_A(i_{2A}-i_{2C}) = 0\,, \tag{19}$$

where $j = A, B, C$.

Adding the relationships (16) together, taking into account (1) – (10), the following dependence for the neutral primary voltage of the transformer was obtained [1], cf. Fig.1

$$V_{10} = \frac{1}{3}\bigg((u_{\Sigma 1A} + u_{\Sigma 1B} + u_{\Sigma 1C}) + (L_{1B} - L_{1C})\frac{di_{1B}}{dt} -$$

$$-(L_{1A} - L_{1C})\frac{di_{1A}}{dt} + (r_B - r_C)i_{1B} - (r_A - r_C)i_{1A}\bigg). \tag{20}$$

The relationship (20) introduced into equation (16), taking into account the dependencies (1) – (10) allowed us to obtain the differential equations of electromagnetic state in the primary and the secondary winding of the power transformer in the form:

$$\frac{d}{dt}\begin{bmatrix}\Psi_{1A}\\ \Psi_{1B}\end{bmatrix} = \frac{1}{3}\left(\begin{bmatrix}2u_{\Sigma 1A} - u_{\Sigma 1B} - u_{\Sigma 1C}\\ 2u_{\Sigma 1B} - u_{\Sigma 1A} - u_{\Sigma 1C}\end{bmatrix} - \begin{bmatrix}2L_{1A}+L_{1C} & L_{1C}-L_{1B}\\ L_{1C}-L_{1A} & 2L_{1B}+L_{1C}\end{bmatrix}\times\right.$$

$$\left.\times\frac{d}{dt}\begin{bmatrix}i_{1A}\\ i_{1B}\end{bmatrix} - \begin{bmatrix}3r_1+2r_A+r_C & r_C-r_B\\ r_C-r_A & 3r_1+2r_B+r_C\end{bmatrix}\begin{bmatrix}i_{1A}\\ i_{1A}\end{bmatrix}\right); \tag{21}$$

$$\frac{d}{dt}\begin{bmatrix}\Psi_{2A}\\ \Psi_{2B}\end{bmatrix} = \begin{bmatrix}u_{\Sigma 2A} - u_{2\Sigma B}\\ u_{\Sigma 2B} - u_{2\Sigma C}\end{bmatrix} - \begin{bmatrix}2L_A+L_B & L_A-L_B\\ L_C-L_B & 2L_C+L_B\end{bmatrix}\frac{d}{dt}\begin{bmatrix}i_{2A}\\ i_{2B}\end{bmatrix} -$$

$$-\begin{bmatrix}2R_A+R_B-r_2 & R_A-R_B\\ R_C-R_B & 2R_C+R_B-r_2\end{bmatrix}\begin{bmatrix}i_{2A}\\ i_{2B}\end{bmatrix}. \tag{22}$$

What may be written in the concise matrix-vector form as

$$\frac{d}{dt}\mathbf{\Psi}_1 = \mathbf{B}_2\mathbf{u}_{\Sigma 1} - \left(\mathbf{L}_1\frac{d}{dt} + \mathbf{r}_\Sigma\right)\mathbf{i}_1,\ \mathbf{u}_{\Sigma 1} \equiv \begin{bmatrix}u_{\Sigma 1A} & u_{\Sigma 1B} & u_{\Sigma 1C}\end{bmatrix}^{\mathrm{T}},$$

$$\mathbf{L}_1 \equiv \frac{1}{3}\begin{bmatrix}2L_{1A}+L_{1C} & L_{1C}-L_{1B}\\ L_{1C}-L_{1A} & 2L_{1B}+L_{1C}\end{bmatrix},\ \mathbf{B}_2 \equiv \frac{1}{3}\begin{bmatrix}2 & -1 & -1\\ -1 & 2 & -1\end{bmatrix},$$

$$\mathbf{r}_\Sigma \equiv \frac{1}{3}\begin{bmatrix}3r_1+2r_A+r_C & r_C-r_B\\ r_C-r_A & 3r_1+2r_B+r_C\end{bmatrix}; \tag{23}$$

$$\frac{d}{dt}\mathbf{\Psi}_2 = \mathbf{B}_4\mathbf{u}_{\Sigma 2} - \left(\mathbf{L}_2\frac{d}{dt} + \mathbf{R}_\Sigma\right)\mathbf{i}_2,\ \ \mathbf{u}_{\Sigma 2} \equiv \begin{bmatrix}u_{\Sigma 2A} & u_{\Sigma 2B} & u_{\Sigma 2C}\end{bmatrix}^{\mathrm{T}},$$

$$\mathbf{L}_2 \equiv \begin{bmatrix}2L_{2A}+L_{2B} & L_{2A}-L_{2B}\\ L_{2C}-L_{2B} & 2L_{2C}+L_{2B}\end{bmatrix},$$

$$\mathbf{B}_4 \equiv \begin{bmatrix}1 & -1 & 0\\ 0 & 1 & -1\end{bmatrix},\ \mathbf{R}_\Sigma \equiv \begin{bmatrix}2R_A+R_B-r_2 & R_A-R_B\\ R_C-R_B & 2R_C+R_B-r_2\end{bmatrix}. \tag{24}$$

The expressions for calculation of winding currents are determined in the usual way [1, 2, 5].

$$\mathbf{i}_j = \boldsymbol{\alpha}_{\sigma j}(\mathbf{\Psi}_j - \boldsymbol{\psi})\,,\quad \boldsymbol{\alpha}_{\sigma j} = L_{\sigma j},\quad j = 1, 2\,. \tag{25}$$

Differentiating with respect to time the Relationships (25), the following expressions are obtained [1, 5]:

$$\frac{d\mathbf{i}_1}{dt} = \boldsymbol{\alpha}_{\sigma 1}\left(\frac{d\mathbf{\Psi}_1}{dt} - \frac{d\boldsymbol{\psi}}{dt}\right),\ \frac{d\mathbf{i}_2}{dt} = \boldsymbol{\alpha}_{\sigma 2}\left(\frac{d\mathbf{\Psi}_2}{dt} - \frac{d\boldsymbol{\psi}}{dt}\right); \tag{25}$$

$$\frac{d\boldsymbol{\psi}}{dt} = \mathbf{G}_1\left(\mathbf{B}_2\mathbf{u}_{1\Sigma} - \left(\mathbf{L}_1\frac{d}{dt} + \mathbf{r}_\Sigma\right)\mathbf{i}_1\right) +$$

$$+\mathbf{G}_2\left(\mathbf{B}_4\mathbf{u}_{\Sigma 2} - \left(\mathbf{L}_2\frac{d}{dt} + \mathbf{R}_\Sigma\right)\mathbf{i}_2\right), \tag{26}$$

where [2]:

$$\mathbf{G} = \begin{bmatrix}(\alpha_{\sigma 1} + \alpha_{\sigma 2} + \rho_A)^{-1} & \\ & (\alpha_{\sigma 1} + \alpha_{\sigma 2} + \rho_B)^{-1}\end{bmatrix},$$

$$\mathbf{G}_1 = \boldsymbol{\alpha}_{\sigma 1}\mathbf{G},\quad \mathbf{G}_2 = \boldsymbol{\alpha}_{\sigma 2}\mathbf{G},\quad \boldsymbol{\rho} \equiv \frac{\partial(\mathbf{i}_1 + \mathbf{i}_2)}{\partial\boldsymbol{\psi}}\,;$$

$$\boldsymbol{\rho} = \mathrm{diag}(\rho_A;\rho_B),\quad \rho_j = \left(\frac{\partial\psi_{m,j}}{\partial i_{m,j}}\right)^{-1},\quad j = A, B\,, \tag{27}$$

where: $\psi_m = \psi_m(i_m)$ – magnetization curve of transformer bars.

Substituting the dependencies (26) into (25), taking into account the expressions (1)-(10), the mathematical model of the transformer in the coordinates system related to phase currents, which takes into account the resistance and inductance of cable line, as well as asymmetry of supply voltage for the winding connection Y/Δ. Further the dependencies are presented in the Cauchy form.

$$\frac{d\mathbf{i}_1}{dt} = \left(\mathbf{1} + \mathbf{A}_1\mathbf{L}_1\right)^{-1}\Big(\mathbf{A}_1(\mathbf{B}_2\mathbf{u}_{\Sigma 1} - \mathbf{r}_\Sigma\mathbf{i}_1) +$$

$$+\mathbf{A}_{12}(\mathbf{B}_4\mathbf{u}_{\Sigma 2} - \mathbf{L}_2\mathbf{X} - \mathbf{R}_\Sigma\mathbf{i}_2)\,; \tag{28}$$

$$\frac{d\mathbf{i}_2}{dt} \equiv \mathbf{X} = \left[\mathbf{1} + \left(\mathbf{A}_2 + \mathbf{A}_{21}\mathbf{L}_1\left(\mathbf{1} + \mathbf{A}_1\mathbf{L}_1\right)^{-1}\mathbf{A}_{12}\right)\mathbf{L}_2\right]^{-1}\times$$

$$\times\left[\mathbf{A}_{21}\left(\mathbf{B}_2\mathbf{u}_{\Sigma 1}-\mathbf{L}_1\left(\mathbf{1}+\mathbf{A}_1\mathbf{L}_1\right)^{-1}\left(\mathbf{A}_1\left(\mathbf{B}_2\mathbf{u}_{\Sigma 1}-\mathbf{r}_\Sigma\mathbf{i}_1\right)+\right.\right.\right.$$

$$\left.\left.\left.+\mathbf{A}_{12}\left(\mathbf{B}_4\mathbf{u}_{\Sigma 2}-\mathbf{R}_\Sigma\mathbf{i}_2\right)\right)-\mathbf{r}_\Sigma\mathbf{i}_1\right)+\mathbf{A}_2\left(\mathbf{B}_4\mathbf{u}_{\Sigma 2}-\mathbf{R}_\Sigma\mathbf{i}_2\right)\right], \quad (29)$$

where the matrix **X** is introduced for concise notation of (28). The coefficients A are described with the relationships:

$$\mathbf{A}_{11}=\boldsymbol{\alpha}_{\sigma 1}(\mathbf{1}-\mathbf{G}_1),\ \mathbf{A}_{12}=\mathbf{A}_{21}=-\boldsymbol{\alpha}_{\sigma 1}\boldsymbol{\alpha}_{\sigma 2}\mathbf{G},$$

$$\mathbf{A}_{22}=\boldsymbol{\alpha}_{\sigma 2}(\mathbf{1}-\mathbf{G}_2)\ . \quad (30)$$

Simulations

The results of simulations of transients are presented for a three phase power transformer working in the following regimes: supply asymmetry (I), single phase short-circuit for phase A of the secondary winding (II), two-phase short-circuit for phases *A* and *B* (III)*,* three-phase shortcircuit (IV).

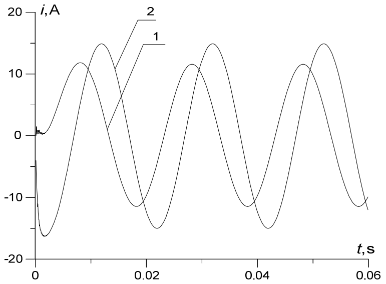


Fig. 3. Transient currents in the primary winding for the case I
1 -current in the phase A, 2 - current in the phase B

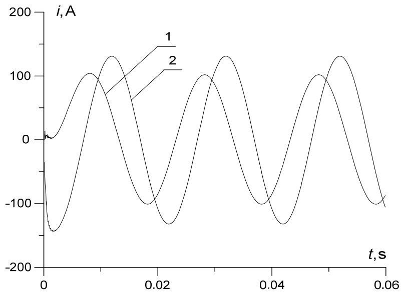


Fig. 4. Transient currents in the secondary winding for the case I
1 -current in the phase A, 2 - current in the phase B

In Figs. 3, 4 the currents of primary and secondary winding of the transformer are presented for the first considered case.

In Fig. 5 the currents secondary winding of the transformer are presented for the second considered case. At time instant $t = 0{,}06$ s a one-phase short-circuit has occurred for phases *A.*

In Fig. 6 the currents of secondary winding of the transformer for the third considered case are presented. Here, like previously, at time instant $t = 0{,}06$ s a two-phase short-circuit has occurred for phases *A* and *B.*

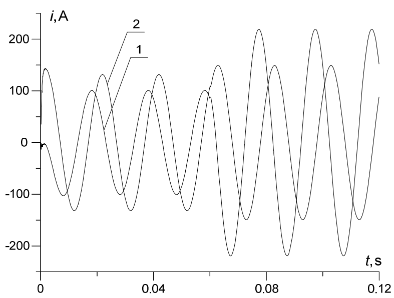


Fig. 5. Transient currents in the secondary winding for the case I
1 -current in the phase A, 2 - current in the phase B

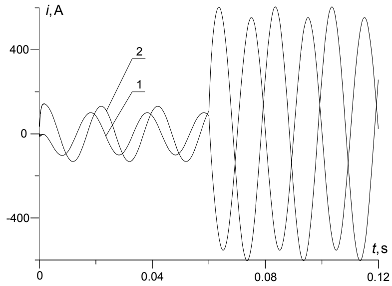
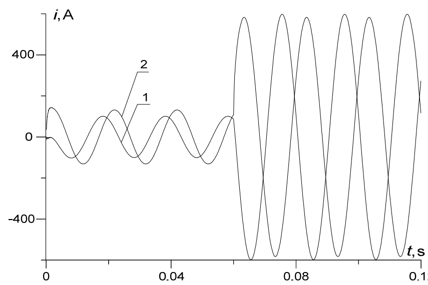


Fig. 6. Transient currents in the primary winding for the case III
1 -current in the phase A, 2 - current in the phase B



Rys. 7. Transient currents in the secondary winding for the case IV
1 -current in the phase A, 2 - current in the phase B

Fig. 7 depicts the transient currents in the secondary winding for the fourth considered case. In this case the short-circuit currents are practically the same.

Conclusions

On the basis of the proposed method a mathematical model of a dynamical system has been developed. Its components are an asymmetric cable line and a power transformer, supplied by an asymmetric supply source of electric energy. On the basis of the proposed model, the analysis of work conditions for different kinds of short-circuits of the secondary winding of the transformer has been carried out. Using the results of computer simulations the failure states in a nonlinear power transformer have been considered.

REFERENCES

[1] Chaban A., Rusek A. *Wykorzystanie zasady Hamiltona-Ostrogradskiego dla modelowania matematycznego stanów asymetrycznych w głębokożłobkowym napędzie asynchronicznym* // Technical News – 2011/1(33), 2(34). – S. 58 – 61.

[2] Czaban A. *Modelowanie matematyczne procesów oscylacyjnych w systemach elektromechanicznych*. – Lwów: W-wo T. Soroki 2008. – 328 s. (w języku ukraińskim).

[3] Kopyłow I. *Modelowanie matematyczne maszyn elektrycznych* – M.: Wyższa Szkoła, 2001. – 327 s. (w języku rosyjskim).

[4] Popenda A., Rusek A. *Model matematyczny i wybrane stany nieustalone trójfazowej maszyny indukcyjnej z nasyconym obwodem magnetycznym*, Przegląd Elektrotechniczny 3'2009, s. 223-226, 2009, PL ISSN 0033-2097.

[5] Tchaban V. *Podstawy teorii procesów przejściowych w ukladach elektromaszynowych*. – Lwów: High School, 1980. – 200 s. (w języku rosyjskim).

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