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Contact Problem of Disk on Shaft Fixed by Induction Shrink Fit

Abstract. *The problem of shrink fit between the disk and shaft is solved. The shrink fit realized by induction heating must transfer the prescribed torque and power. Both disk and shaft are considered elastic. The first step of the task is to find appropriate dimensions of the disk and interference. The second step is to suggest the parameters of its induction heating. The methodology is illustrated by a typical example.*

Streszczenie. *W artykule rozwiązano problem pasowania kurczowego między tarczą a wałem. Pasowanie kurczowe zrealizowane zostało metodą indukcyjną – musi ono przenieść założone moment i moc. Zarówno tarcza jak i wał zostały potraktowane jako elastyczne. Pierwszym krokiem w rozwiązaniu zadania jest znalezienie dogodnych rozmiarów tarczy. Drugim krokiem jest zasugerowanie parametrów jej nagrzewania indukcyjnego. Ta metodyka została zilustrowana typowym przykładem.* ***(Problem kontaktu tarczy na wale poprzez zastosowania pasowania kurczowego)***

Keywords: Induction shrink fit, contact problem, transfer of torque, magnetic field, field of temperature.
Słowa kluczowe: indukcyjne pasowanie kurczowe, problem kontaktu, przeniesienie momentu, pole magnetyczne, pole temperatury

Introduction

The paper deals with the contact problem of the disk on a shaft fixed by the induction shrink fit. Shrink fits represent firm connections of two metal parts and their principal task is to transfer prescribed mechanical forces or torques. They are widely used in numerous industrial and transport technologies (shrunk-on rings, crankshafts, tires of railway wheels, armature bandages in rotating electrical machines and so on, see, for instance [1]). Their realization is based on heating of the external part of the system, which leads to an enlargement of its dimensions (i.e., the radius of the internal bore of a wheel). The second part (i.e., a shaft whose radius at the room temperature is somewhat greater than the radius of the bore) is then inserted into it and the whole system is cooled. After cooling we obtain a firm joint characterized by a high pressure between both connected parts. The shaft is also considered elastic.

The process of heating is mostly realized by gas or induction. Induction heating is characterized by an easy control of the intensity of heating and its local distribution, no chemical changes in the surface layers of the heated material, and no products of combustion. For the above reasons, this way is preferred in all cases where it is possible.

Formulation of the technical problem

The shaft of external radius r_{A2} is manufactured with an interference Δr_{AB} with respect to the internal radius r_{B1} of the disk (see Fig. 1). The external radial force $\pm f_{r0}$ existing at the place of the contact at rest allows transferring

- mechanical torque $M = 2\pi r_C^2 h \left| f_{r0} \right| f_f$, where h denotes the width of the disk, r_C denotes the final common radius of the shaft and disk (see Fig. 1), and f_f is the coefficient of dry friction steel – steel
- and (provided the system rotates) power $P = M\omega$, where ω stands for the angular velocity of rotation.

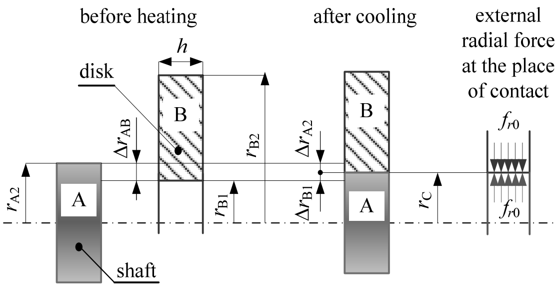

Fig. 1. Schematic view of manufacturing a shrink fit

Pressing of the disk on the shaft is considered thermoelastic. In other words, the disk is inductively heated as long as its internal radius r_{B1} dilates thermoelastically by a value $\Delta r_{B1,T} \geq \Delta r_{AB}$, then it is pushed on the shaft and cooled to its initial temperature.

The aim of the paper is to present a complete numerical algorithm for solution of such a contact problem respecting the deformations of both disk and shaft. The solution is carried out in three following steps:

- First, starting from the torque M that is to be transferred and dimensions r_{B1}, r_{B2} and h of the disk it is necessary to find a sufficient interference Δr_{AB}. This is, however, a very complicated inverse problem. The easiest way is, therefore, to find the dependence $M_{max} = M_{max}\left(\Delta r_{AB}\right)$ (M_{max} being the maximum transferable torque for the given interference Δr_{AB} at rest, which is the most unfavorable case) and then to estimate the value Δr_{AB} from this curve.
- Second, we have to check the mechanical stress of the disk after its pressing on the shaft. This starts from knowledge of the external radial force $\pm f_{r0}$ at rest. This value then serves for computing the reduced stress σ_{red} (for example, $\sigma_{red,Mi}$ by the von Mises hypothesis) and its comparison with the yield stress of the steel used. For growing revolutions, the effect of the centrifugal forces (acting mainly in the disk) leads to changes of the reduced stress that has to be checked as well.
- Third, we have to map the process of induction heating of the disk. Its purpose is to find the parameters of the field current in the inductors (amplitude and frequency) that would secure that the required dilatation of the internal bore of the disk reaches a value $\Delta r_{B1,T} \geq \Delta r_{AB}$ in a reasonable time and still acceptable temperature.

Continuous mathematical model

The mathematical model of the problem consists of two independent submodels. The first of them is purely mechanical and serves for finding the radial stresses in the disk and shaft after pressing, corresponding value of the maximum transferable torque and von Mises stress. Provided that these values are acceptable, we apply the second submodel for the description of induction heating. This task represents a nonlinear triply coupled problem characterized by the interaction of magnetic field, temperature field and field of thermoelastic displacements. The physical properties of material, moreover, depend on the temperature.

The first mechanical submodel is described by the isothermic Lamé equation in the form [2]

$$(1)\qquad (\varphi+\psi)\cdot\operatorname{grad}(\operatorname{div}\boldsymbol{u})+\psi\cdot\Delta\boldsymbol{u}+\boldsymbol{f}_{\mathrm{L}}=\boldsymbol{0},$$

$$\varphi=\frac{\nu\cdot E}{(1+\nu)(1-2\nu)},\ \psi=\frac{E}{2\cdot(1+\nu)},$$

where E denotes the modulus of elasticity, ν is the Poisson coefficient of the contraction, symbol $\boldsymbol{u}=(u_r,u_\varphi,u_z)$ represents the vector of the displacement, and $\boldsymbol{f}_{\mathrm{L}}$ stands for the vector of the volumetric (for example gravitational) forces. But in comparison with the thermoelastic strains and stresses they are very small and may be neglected. The boundary conditions follow from Fig. 2.

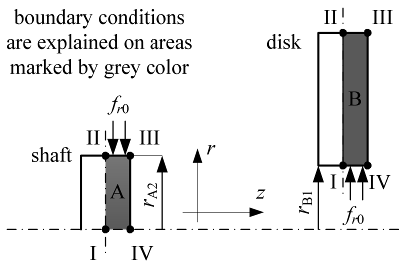

Fig. 2. Boundary conditions (f_r, f_z denoting external radial and axial forces)

Shaft: I-II – $u_z=0,\ f_r=0$, II-III – $f_r=-f_{r0},\ f_z=0$,
III-IV – $f_r=0,\ f_z=0$, IV-I – $u_r=0,\ f_z=0$.

Disk: I-II – $u_z=0,\ f_r=0$, II-III – $f_r=0,\ f_z=0$,
III-IV – $f_r=0,\ f_z=0$, IV-I – $f=f_{r0},\ f_z=0$.

Unfortunately, the value of force f_{r0} (see Fig. 1) is not known beforehand. The task must be, therefore, solved iteratively in the following way:

- Choice of external radial stress f_{r0},
- Solution of (1) for both disk and shaft in order to obtain the values of displacements Δr_{A2} and Δr_{B1} (Fig. 1),
- Calculation of $\delta=|\Delta r_{\mathrm{A2}}|+|\Delta r_{\mathrm{B1}}|-|\Delta r_{\mathrm{AB}}|$,
- If $|\delta|<\delta_0$, where δ_0 is a prescribed tolerance, the computations stop, otherwise the value of external radial stress f_{r0} must be changed appropriately.

After finishing this iterative process, we can easily find the maximum transferable torque at rest and also the von Mises stress.

The second submodel describing the process of induction heating (see Fig. 3) consists of three second-order partial differential equations describing the distribution of the three above mentioned fields.

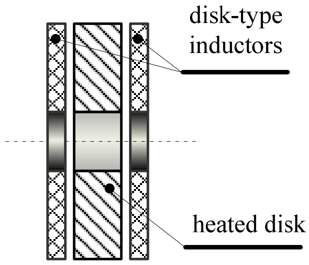

Fig. 3. Induction heating of the disk

Magnetic field in the system is described by the solution of a well-known parabolic equation for magnetic vector potential $\boldsymbol{A}$ in the form [3]

$$(2)\qquad \operatorname{curl}\left(\frac{1}{\mu}\operatorname{curl}\boldsymbol{A}\right)+\gamma\frac{\partial\boldsymbol{A}}{\partial\boldsymbol{t}}=\boldsymbol{J}_{\mathrm{ext}},$$

where μ denotes the magnetic permeability, γ is the electric conductivity and $\boldsymbol{J}_{\mathrm{ext}}$ stands for the vector of the external harmonic current density in the field coils.

But solution to (2) is, in this case, practically unfeasible. The reason consists in the deep disproportion between the frequency f (tens or hundreds Hz) of the field current I_{ext} and time of heating t_{H} (minutes). That is why the model was somewhat simplified using the assumption that the magnetic field is harmonic. In such a case it can be described by the Helmholtz equation for the phasor $\underline{\boldsymbol{A}}$ of the magnetic vector potential $\boldsymbol{A}$ [3]

$$(3)\qquad \operatorname{curlcurl}\underline{\boldsymbol{A}}+\mathrm{j}\cdot\omega\gamma\mu\underline{\boldsymbol{A}}=\mu\underline{\boldsymbol{J}}_{\mathrm{ext}},$$

where ω is the angular frequency. But the magnetic permeability of ferromagnetic parts needs not be a constant; it can always be assigned to the local value of magnetic flux density. It computation is, in such a case, based on an iterative procedure.

The conditions along the axes of the system and artificial boundary placed at a sufficient distance from it are of the Dirichlet type ($\underline{\boldsymbol{A}}=\underline{\boldsymbol{0}}$).

The temperature field is described by the heat transfer equation [4]

$$(4)\qquad \operatorname{div}(\lambda\cdot\operatorname{grad}T)=\rho c_{\mathrm{p}}\cdot\frac{\partial T}{\partial t}-p,$$

where λ is the thermal conductivity, ρ denotes the mass density and c is the specific heat (all of these parameters are temperature-dependent functions). Finally, symbol p denotes the time average internal volume sources of heat that generally consist of the volume Joule losses p_{J} due to eddy currents and magnetization losses p_{m}. So

$$(5)\qquad p=p_{\mathrm{J}}+p_{\mathrm{m}},$$

where

$$(6)\qquad p_{\mathrm{J}}=\frac{\left|\underline{\boldsymbol{J}}_{\mathrm{eddy}}\right|^2}{\gamma},\quad \underline{\boldsymbol{J}}_{\mathrm{eddy}}=\mathrm{j}\cdot\omega\gamma\underline{\boldsymbol{A}},$$

while p_{m} (if they are considered) are determined from the known measured loss dependence $p_{\mathrm{m}}=p_{\mathrm{m}}\left(|\underline{\boldsymbol{B}}|\right)$ for the used material (magnetic flux density $\boldsymbol{B}$ in every element is in this model also harmonic).

The boundary conditions take into account convection and radiation.

The solution of the thermoelastic problem is solved by means of the Lamé nonisothermic equations that read [2]

$$(7)\qquad \begin{aligned}&(\varphi+\psi)\cdot\operatorname{grad}(\operatorname{div}\boldsymbol{u})+\psi\cdot\Delta\boldsymbol{u}-\\ &-(3\varphi+2\psi)\cdot\alpha_{\mathrm{T}}\cdot\operatorname{grad}T+\boldsymbol{f}=\boldsymbol{0},\end{aligned}$$

where α_{T} is the coefficient of the linear thermal dilatability of material and T denotes the temperature. Other parameters are identical with those in (1). The boundary conditions correspond to the free disk.

Numerical solution

The numerical solution of the task was realized by codes QuickField (mechanical submodel) and COMSOL Multiphysics (induction heating submodel) supplemented with a number of own scripts and procedures. Attention was particularly paid to the convergence of results in the dependence on the density of discretization mesh and distance of the artificial boundary (in case of magnetic field). The results were required to reach 2–3 valid digits. The computation of one example takes (on a good PC) several hours.

Illustrative example

The nominal radii of the shaft and internal bore of the disk $r_{A2} = r_{B1} = 0.1$ m, $r_{B2} = 0.5$ mm, $h = 0.05$ m. The interference Δr_{AB} is tested within the range $0 - 0.3$ mm. Some physical parameters of disk material – steel 15300 – ($\mu, \gamma, \lambda, \rho c_p, \alpha_T$) are temperature-dependent functions, other parameters are constant ($E = 2.1 \times 10^{11}$ N/m^2, $\nu = 0.3$). Its yield stress $\sigma_k = 4.226 \times 10^8$ N/m^2 and the coefficient of friction $f_f = 0.55$.

Figure 4 shows the dependences of the von Mises stress in the shaft and disk as functions of interference Δr_{AB}. The highest allowable interference for the disk (for which still $\sigma_{red,Mi} \le \sigma_k$) is 0.22 mm. This value also provides the maximum allowable torque (at rest) $M_{0,max} = 4.182 \times 10^5$ Nm, see Fig. 5.

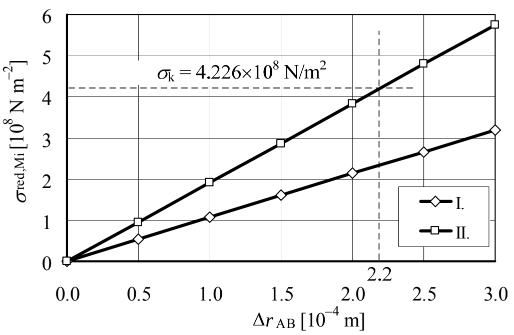

Fig. 4. Von Mises stress $\sigma_{red,Mi}$ as a function of Δr_{AB} at rest: I – shaft (here $\sigma_{red,Mi} = f_{r0}$), II – disk

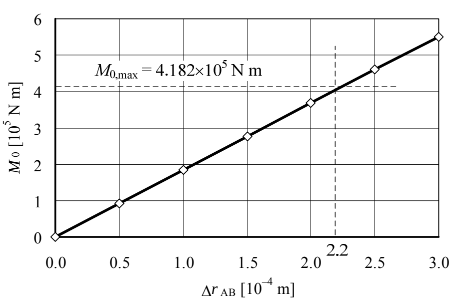

Fig. 5. Transferable torque M_0 (at rest) of the shrink fit as a function of Δr_{AB}

Fig. 6 shows the arrangement of the inductors, for which Fig. 7 shows the time evolution of the average temperature of the disk for various values of field current whose frequency is 50 Hz. Similarly, Fig. 8 shows the displacements of the internal radius of the disk, from which we can determine the time of heating for the chosen value of Δr_{AB}.

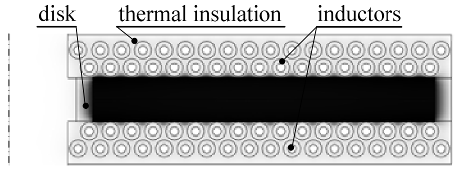

Fig. 6. Arrangement of the inductors around the disk

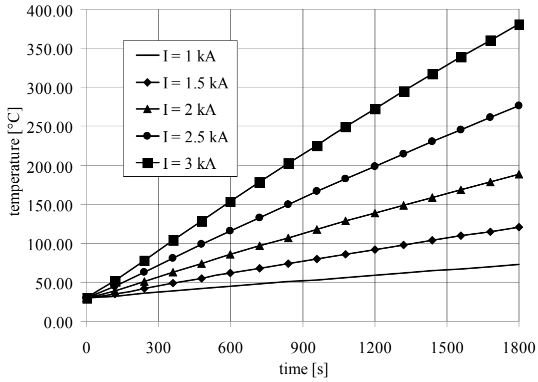

Fig. 7. Time evolution of average temperature of the disk

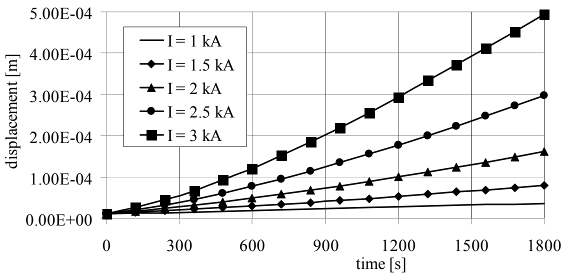

Fig. 8. Time evolution of radial displacement of the bore

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REFERENCES

[1] Kotlan, V., Karban, P., Ulrych, B., Doležel, I., Kůs, P.: *Hard-Coupled Modeling of Induction Shrink Fit of Gas-Turbine Active Wheel*. Proc. ISTET'11, Klagenfurt, Austria, July 2011, pp. 173–178.
[2] Boley, B., Wiener, J.: *Theory of Thermal Stresses*. NY, 1960.
[3] Kuczmann, M.: Iványi, A.: *The Finite Element Method in Magnetics*. Akademiai Kiado, Budapest, 2008.
[4] Holman, J.P.: *Heat Transfer*. McGrawHill, NY, 2002.

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