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Sliding mode control of the DC drive with relative degree higher than one

***Abstract**. Certain problems concerning the higher order sliding mode (SM) controller have been presented. SM methodology has been applied to DC drive system. The control system consists of two control feedbacks in a cascade structure – conventional PI current controller and SM position and velocity controller. Due to the relative degree of sliding function is higher than one, the second order sliding controller has been suggested. To verify the theoretical results, simulation studies have been carried out.*

***Streszczenie.** W artykule przedstawiono zagadnienia syntezy regulatora ślizgowego wyższego rzędu. Metodę zastosowano dla układu napędowego prądu stałego o kaskadowej strukturze regulacji. Zaproponowano konwencjonalny PI regulator prądu, natomiast regulator ślizgowy został określony dla pętli regulacyjnej położenia i prędkości kątowej. Stopień relatywności dla funkcji ślizgowej jest większy od jeden, zastosowano więc regulator ślizgowy drugiego rzędu. Metodę zweryfikowano za pomocą badań symulacyjnych. **(Sterowanie ślizgowe w napędzie pozycyjnym prądu stałego przy stopniu relatywności większym od jeden)**.*

Keywords: variable structure system, sliding mode, DC drive, differential geometry, relative degree.
Słowa kluczowe: układ o zmiennej strukturze, ruch ślizgowy, napęd prądu stałego, geometria różniczkowa, stopień relatywności.

Introduction

Recently, much attention has been paid to the problem of designing feedback controllers for uncertain dynamical systems. It is very difficult to develop accurate mathematical models for many physical plants. There are inevitable uncertainties in many constructed models. These uncertainties result from imperfectly known structure or parameters of a plant as well as from unpredicted disturbances such as measurement noises and errors. Some assumption concerning uncertainties (matching condition, upper bounds on the norm) and the plant property (minimum-phase-ness) are usually made. At present, several basic approaches to uncertain nonlinear systems are available. It is suggested to solve the control problem with one of the following methods: adaptive control [1,2], robust control i.e. H∞ control [3,4] and variable structure control (VSC) [5-10]. In the first one, the stabilization and the tracking goals are achieved during the parameter adaptation process, the self-tuning regulators and the learning systems. The H∞ control problems are defined in the frequency domain, whereas the VSC method in the time domain. They both require some information about the bounds on uncertainties, but have different meaning. These methods allow nonparametrised uncertainties and neglected dynamics, too. Although each of these methods offers more or less general solutions for uncertain systems, the VSC method has been chosen here for control synthesis of angular position (α) and velocity (ω) in a certain drive system, for example to the positioning of the robot arm. The rigid model of the robot arm is obviously incorrect and can be treated as the source of uncertainties for the control system. On the other hand, the robot mechanical system ought to be provided with properly chosen drive system. The drive system stands for itself inaccurate modelled dynamics because of variable parameters of its electrical circuits. Moreover, the variable load torque coming from a robot arm can be regarded as a disturbance and a kind of nonparametrised uncertainty. Thus, as an example, DC drive system is taken into consideration in terms of suitability for the robotics.

The VSC method has been developed in the last two decades [8,10]. Higher order sliding mode control has been introduced to the control theory. New VSC method gives new possibilities of the control. This work is an attempt to verify this method for the selected object by means of simulation research.The paper is structured as follows. The first section presents the theory of the sliding mode control in which the 1st and the 2nd order sliding dynamics are distinguished. The following section presents the control structure of the DC drive with a current controller and a sliding controller defined in the (α,ω) – phase space. Because of the relative degree between control as an input and sliding function $S(\alpha,\omega)$ as an output is higher than one, the control synthesis in conventional meaning is not possible. For this reason, to realise the given reference trajectory $S(°)$, the second order sliding mode controller is suggested. The next section deals with the second order sliding control including additional assumptions such as matching conditions for parameter uncertainties and bounds on the norm of uncertainties for the variable load torque. The last section presents the simulation results of the designed control system.

The theory of the sliding mode control

The variable structure concept consists in defining such a surface in the state space, so that the system restricted to this surface has suitable dynamical properties. The feedback control law must guarantee that this surface will attract the system trajectory and moreover the trajectory reaching it will remain in its vicinity. Such a goal is realised by discontinuous control law. The control function has discontinuity on the mentioned surface called in the traditional meaning switching or sliding surface. The theory of VSC [6] can be interpreted in differential geometric terms, in general, for nonlinear control systems [11]. Then the switching surface stands for the output-nulling submanifold for the control system. The constrained dynamics corresponds to the zero dynamics of the system. The ideal sliding dynamics is governed by manifold invariance conditions, etc.

The starting point of the whole analysis of the sliding motion, in this paper is the notion of relative degree of the system. From then on, the subject of our research is single-input single-output system of the form:

$$\frac{dx(t)}{dt} = f(x) + \delta f(x) + g(x)u(x) + p(x)w(t) \qquad (1)$$
$$y = S(x,t)$$

where: x is a state variable, $x \in \mathbf{X}$, $\mathbf{X}$ is an open set in $\Re^n$, f, g, p - are smooth vector fields locally defined on $\mathbf{X}$ with $g(x) \neq 0, \forall x \in \mathbf{X}$, $u(x): \mathbf{X} \to \Re$ is the control input function; $w(t)$ is a scalar external disturbance signal affecting the system behaviour, and represents a state independent, unstructured uncertainties. The vector $\delta f(x)$ represents parametric uncertainties of the nominal vector field $f(x)$. The output function S : $\mathbf{X}\times\Re \rightarrow \Re$ is a locally smooth function of the state variable and in certain cases also of the time. The ideal dynamics of the nominal system (1) is independent of the perturbations like $\delta f(x)$ and $w(t)$ if the matching conditions are satisfied:

(2) $\delta f(x) \in$ span$\{g(x)\}$ $\quad p(t) \in$ span$\{g(x)\}$

Definition 1. System (1) has at point x relative degree **r** with respect to the vector field $g(x)$ if :

(i) $L_g L^k_f S(x(t)) = 0$ for all x and all k < r -1

(ii) $L_g L^{r-1}_f S(x(t)) \neq 0$

where: $L_g S(x)$ is the Lie derivative of a real-valued function $S(x)$ along a vector field g.

If the relative degree is equal r, the rth-derivative of the output function $y(t)$ with respect to time is given by (3)

(3) $y^{(r)}(t) = L^r_f S(x(t)) + L_g L^{r-1}_f S(x(t))\, u(x)$

It is seen that, the $y^{(k)}(t)$ for ($0 \le k < r$) is independent of $u(x)$. When $L_g L^k_f S(x) = 0$ for all x and all $k \ge 0$, then the output of the system (1) is not dependent on any input $u(x)$, for all x and t. For example, if the relative degree equals 2, the derivative of the output function $y(t)$ with respect to time is directly: $y^{(1)}(t) = L_f S(x(t))$, because $L_g S(x(t)) = 0$. Changes in the value of function y are not possible by any control input $u(x)$ directly.

Similar reasoning can be carried out for a disturbance signal $w(t)$. If $L_p S(x(t)) = 0$ i.e, relative degree r_p of the output function $S(x)$ with respect to vector field $p(x)$ is greater than one, so this is a good case because $S(x)$ is independent of the disturbance signal $w(t)$.

The level set $S^{-1}(0) = \{ x \in \mathbf{X} : S(x) = 0 \}$ defines a smooth (n-1)-dimensional manifold $\mathbf{S_S}$ of constant rank. This manifold is usually called the sliding manifold. By assumption, the dynamics of system (1) is constrained to such manifold.

The variable structure control law is a feedback control law on the manifold $\mathbf{S_S}$ and it is determined as follows:

(4) $$u(x) = \begin{cases} u^+(x) & \text{for } S(x) > 0 \\ u^-(x) & \text{for } S(x) < 0 \end{cases} \qquad u^+ \neq u^-$$

It means that the crossing of the $\mathbf{S_S}$ from each side of $\mathbf{S_S}$ is guaranteed by use of the switching method (4). The feedback control $u(x)$ has discontinuous nature with respect to variable $S(x)$. In order to a sliding motion could take place on $\mathbf{S_S}$, the trajectory of (1) starting from arbitrary points of $\mathbf{X}$, must reach this manifold and then lasts in its vicinity. Hence, there are two problems: 1^o the reaching and 2^o the sliding problem, both must be solved to realise the sliding mode control. The reaching problem can be solved by setting the control (state space trajectory) outside the 'sliding'. It means that the trajectory of the system with continuous control $u(x)$ should get $\mathbf{S_S}$ after a finite time interval. This problem, in general, has been solved as a stabilisation problem. On the other hand, the sliding motion locally exists in a direct vicinity of $\mathbf{S_S}$ whenever (5) is fulfilled.

(5) $$\lim_{S \to 0^+} L_{f+\delta f+gu^+ +pw} S(x) < 0$$
$$\lim_{S \to 0^-} L_{f+\delta f+gu^- +pw} S(x) > 0$$

Ideal sliding dynamics, without crossing of $\mathbf{S_S}$, is defined by manifold invariance conditions [6] for unperturbed system (1). They are as follows:

(6) $$S(x) = 0 \text{ and } \dot{S}(x) = 0$$
$$L_{f+gu_{eq}} S(x) = 0$$

The control variable u_{eq} (named equivalent control) is a smooth function for which every point of the trajectory (1) starting from the $\mathbf{S_S}$ does not leave it. The control u_{eq} is well defined [6] whenever it exists and it is uniquely determined from (6). This indicates that $L_g S(x) \neq 0$ (relative degree equals one) and further, vector field g cannot be tangential to the sliding manifold $\mathbf{S_S}$ This fact underlines that trajectory (1) along y=0 exists, but does not exclude the capability of control by any other kind of the input $u(x)$.

In the case of the selected output function $y=S(x)$ has relative degree higher than one, the relation $L_g S(x(t)) = 0$ holds, $u(t)$ does not changes output y. (Then also the equivalent control u_{eq} does not exist). To overcome such difficulties and solve the 'sliding problem', an auxiliary output function should be find for which the relative degree equals one and simultaneously zeroing of $S(x)$ is possible. The simplest (trivial) way is definition of the (r-1) derivative of $S^{(r-1)}(x)$ as output y, assuming that the other derivatives for k= 0,1,…, (r-2) too, equal zero.

The output function $y=h(x)$ that always satisfies the 'relative degree condition' can be as well selected as a function (7):

(7) $h(x) = S^{(r-1)}(x) + c_{r-2}\, S^{(r-2)}(x) + \ldots + c_1 S^{(1)}(x) + c_0 S(x)$

where c_k for k=0,1,..., r-2 are constant coefficients.

The function (7) can define the new sliding manifold $h^{-1}(0)$, denoted by $\mathbf{S_h}$, and then, the equivalent control for output $h(x)$ exists and is computed from (8):

(8) $$u_{eq(2)}(x) = -\frac{L_f h(x)}{L_g h(x)}$$

Suitable choice of the parameters c_k defines asymptotically stable movement restricted to the manifold $\mathbf{S_h}$.

It is convenient to write the system dynamic equation in normal form coordination (9), what for output function $y=S(x)$ with relative degree r could take the form:

(9)
$$\begin{cases} \dot{z}_1 = z_2 = S^{(1)}(x) \\ \dot{z}_2 = z_3 = S^{(2)}(x) \\ \ldots\ldots \quad \ldots \quad \ldots \quad \ldots \\ \dot{z}_{r-1} = z_r = S^{(r-1)}(x) \\ \dot{z}_r = b(\theta,\xi) + a(\theta,\xi)\, u(t) + W(w(t), w^{(1)}(t), \ldots w^{(r-1)}(t)) \\ \dot{\xi} = q(\theta,\xi) \end{cases}$$
$$y = z_1 = S(x)$$
$$h(x) = 0$$

where: $\theta = \begin{vmatrix} z_1 \\ \ldots \\ z_r \end{vmatrix} \quad \xi = \begin{vmatrix} z_{r+1} \\ \ldots \\ z_n \end{vmatrix} \qquad a(\theta,\xi) \neq 0$

and for an equilibrium point

$(\theta,\xi) = (0,0) \quad b(\theta,\xi) = 0$ and $q(\theta,\xi) = 0$.

According to previous assumptions about the zeroing of S, the problem of the zeroing output z_1 by means of an auxiliary function $h(x)$ can be expressed by the zero dynamics problem. The zero dynamics for system (9) is described by following conditions (10):

$$\begin{aligned} &if \;\; y(t) = z_1(t) = 0, \;\; then \; \forall t \\ (10) \qquad &\dot{z}_1(t) = \dot{z}_2(t) = \ldots = \dot{z}_r(t) = 0 \\ &\theta(t) = 0 \end{aligned}$$

If the $\theta(t)$ is being identically zero, the behaviour of $\xi(t)$ is governed by the differential equation

$$(11) \qquad \dot{\xi}(t) = q(0, \xi(t))$$

If the output $y(t)$ has to be zero, then necessarily, the initial state of the system (9) must be set to values $\theta(0)=0$, while $\xi(0)=\xi_0$ can be chosen arbitrarily.

When the sliding mode takes place on the manifold $\mathbf{S_h}$, the original output $y = S(x,t) = z_1$ and its first (r-1) derivatives asymptotically tend to zero (9). Initial conditions mentioned above, stand for the points belonging to the manifold $\mathbf{S_h}$.

However, the use of the auxiliary function (7) implies two possibilities of realising the dynamics (10). One of them is a measure of all the state variables, and the second one is availability of (r-1) derivatives of the original output $y=S$.

This brief introduction to the VSC should be complemented by higher order sliding mode (HOSM) definition. There are several definitions. The definition introduced by Fridman and Levant [8] will be used:

Definition 2. When the first r successive total time derivatives of $S(x,t)$ are smooth functions, and a set given by the equalities (12)

$$(12) \quad S(x,t) = S^{(1)}(x,t) = \ldots = S^{(k-1)}(x,t) = 0 \quad for \;\; k=1,\ldots,r$$

is locally an integral set in Filippov's sense, then the movement mode existing on this set is called a *sliding mode with sliding order (r)* with respect to the constraint function $S(x,t)$. The rth total time derivative $S^{(r)}$ is not a continuous function of the state variables or does not exists.

From the above definition we conclude that output function $h(x)$ and its components, as smooth derivatives of $S(x)$ determine the HOSM condition (12) with zero dynamics (10). The sliding order not always coincides with the relative degree notion and then the control $u(x)$ is discontinuous function on sliding manifold. This is the case, when first order sliding motion cannot be affected by any control function. Then second order sliding mode is realised on the manifold $S=S^{(1)}=0$ and $S^{(2)}$ is discontinuous. Actually, the HOSM method is taken into account for arbitrary relative degree, not necessarily higher than one, but mainly to reducing the 'chattering' effects [10].

DC drive control system

There are different, more or less precise models of dc drive. Here, the separately excited motor is fed from three-phase converter. The converter is considered as a black box with certain gain, perhaps constant. The commutation effect is neglected in the dynamic model. Hence, almost ideal mathematical model of the motor is obtained as [12]:

$$(13) \qquad \begin{aligned} \frac{d\omega(t)}{dt} &= \frac{\psi_e}{J} i(t) - \frac{1}{J} M_m(t) \\ \frac{di(t)}{dt} &= -\frac{R}{L} i(t) - \frac{\psi_e}{L}\omega(t) + \frac{1}{L} u(t) \\ \frac{d\alpha(t)}{dt} &= \omega(t) \end{aligned}$$

where: $\alpha(t)$ and $\omega(t)$ are the angular position and velocity of the motor shaft, respectively, ψ_e is the exciting flux linkage, $i(t)$ and $u(t)$ are the armature current and voltage suitably. R - generalized resistance and L – total inductance represent armature circuit parameters. J is a moment of inertia of the rotor and mechanical load (for example of a robot arm). $M_m(t)$ is the load disturbance torque. The parameters R and J constitute parameter uncertainties, while the torque $M_m(t)$ stands for unstructured uncertainties; ψ_e is by assumption constant.

Most frequently, the synthesis of the control system for the motor drive consist of creating, at least two feedback controllers (cascade control) – armature current and angular velocity (sometime angular position as well). Hence, the first stage of this task is the designing of an armature current controller in a closed loop system.

Typically, the current controller is of PI type with two adjusted, according to selected criterion, parameters K_R and T_R. We have chosen as a criterion the functional Q involving function $e(t)$ and its derivative in the square. This functional is presented in (14). The variable $e(t)$ corresponds to the current error in the control loop. Constant T_z is a weighting factor. We look for a weak local minimum of the Q:

$$(14) \qquad Q = \int_0^{t_k} (e^2(t) + T_z^2 \, \dot{e}^2(t))dt$$

with boundary conditions shown in (15):

$$(15) \qquad e(t) \in C^2_{[0\,\infty)}, \quad e(0) \neq 0, \;\; e(t_k) = 0, \;\; t_k = var$$

The extreme of the functional Q must satisfy the Euler-Lagrange equation subjected to the conditions (15). The only real solution of min Q with (15) is a function $e(t)$:

$$(16) \qquad e(t) = e(0)\exp(-\frac{t}{T_z}) \quad \text{and} \quad t_k = \infty$$

Therefore, the current control subsystem can be identified as the following transfer function (Fig.1):

$$(17) \qquad G(s) = \frac{I(s)}{U_z(s)} = \frac{K_z}{T_z s + 1} \quad and \quad U_{z\,max} K_z < I_{max}$$

Reference signal U_z must be constrained and designed with respect to the gain of the feedback loop and other parameters of the controller and motor. As results from (17) the time derivative of the armature current is constrained, what is described by inequality (18)

$$(18) \qquad \left.\frac{di(t)}{dt}\right|_{max} = \frac{I_{max}}{T_z} \leq pI_n$$

where I_n - rated current, p – multiplicity of the rated current per one second.

Current controller synthesis based on the performance index Q is preferred to guarantee of the current and its time derivative limiting without special individual limiter. Control system retains the linear characteristics, not enters into the saturation. The example of the dc motor data and the calculated current controller parameters are found at the end of this paper (see Appendix).

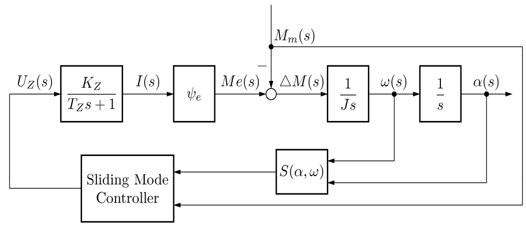

Fig.1. Structure of the control system

Drive system (13) involved the current corrector can be described in the form of the equations (1). Appropriate variables and vector fields are as follows:

$x(t) = [\ \alpha(t)\ \omega(t)\ i(t)\ ]^T \qquad u(t) = U_z(t) \qquad w(t) = M_m(t)$

$$(19)\quad f(x) = \begin{vmatrix} \omega(t) \\ \frac{\psi_e}{J} i(t) \\ -\frac{1}{T_z} i(t) \end{vmatrix} \quad g(x) = \begin{vmatrix} 0 \\ 0 \\ \frac{K_z}{T_z} \end{vmatrix} \quad p(x) = \begin{vmatrix} 0 \\ -\frac{1}{J} \\ 0 \end{vmatrix}$$

The next stage of the control synthesis is the design of the position and velocity controller.

Second order sliding controller for angular position and velocity

This controller is designed based on the sliding mode method. So the dynamical equation (19) should be supplemented by output function

$$(20)\quad S(\alpha,\omega) = c\alpha(t) + \omega(t)\text{ , } c > 0 \text{ - constant coefficient.}$$

This simplest linear function, describes an asymptotically stable system trajectory in the subspace $(\alpha,\omega) \in \mathbf{X}$. For system (19) with output function (20) the relative degree r with respect to U_z equals two, and therefore the function $S(x)$ cannot be directly changed by this control input. The relative degree r_p with respect to $w(t)$ is equal to one, so that this disturbance will cause changes in function S. Moreover, the matching conditions (3) for input $w(t)$ are not satisfied. This yields finally the sliding motion disturbed by the load torque $M_m(t)$. Matching conditions for the parametric uncertainties $\delta f(x)$ are satisfied only for the armature circuit parameters included in the time constant T_z i.e. R, L, K_P.

Because of r=2, in order to realise trajectory $S(x) = 0$, we define new output function $h(x) = S^{(1)}(x) + c_0 S(x)$. According to (9) we choose new coordinates z and write dynamic equations (19) in the normal form:

$$(21)\quad \begin{aligned} \frac{dz_1}{dt} &= z_2 \\ \frac{dz_2}{dt} &= v(z) - \frac{c}{J} M_m(t) - \frac{1}{J}\frac{dM_m(t)}{dt} \\ \frac{dz_3}{dt} &= z_1 - c z_3 - \end{aligned}$$

$$z_3 = \alpha \qquad y = z_1 = S(x) \qquad h(z) = z_2 + c_0 z_1$$

where new control input $v(z)$ is expressed below:

$$v(z) = b(z(x)) + a(z(x))\, U_z$$

$$b = \frac{\psi_e}{J}\left(c - \frac{1}{T_z}\right) i(t) \qquad a = \frac{\psi_e K_z}{J T_z} \neq 0$$

On the manifold $\mathbf{S}_h$ and from assumption (10) on the manifold $\mathbf{S}_S$, system (21) behaves like a linear autonomous system with eigenvalues coinciding with the roots of the polynomial $N(s) = s^2 + (c_0 + c)s + c_0 c$. Since both coefficients c_0 and c are real and positive the system is asymptotically stable with time constants 1/c and $1/c_0$, respectively. There are no oscillations. It is evident that if $c_0 > c$ the sliding motion on $\mathbf{S}_h$ is performed faster than dynamics realized 'near' $\mathbf{S}_S$ .On the other hand, if $c_0 < c$ the asymptotic sliding mode near $\mathbf{S}_S$ cannot be faster because its steady state is achieved with larger time constant, that is $1/c_0$. The load torque M_m and its time derivative form new kind of disturbances. Of course, in this case the matching conditions are satisfied and hence only some information about $||M_m||$ and $||dM_m/dt||$ is needed to perform an undisturbed sliding dynamics. The equivalent control for $h(z)$ exists and it is well defined as $v_{eq(2)} = -c_0 z_2$. To determine sliding control by means of switching function, it remains to choose appropriate input function $v(z)$. For example, it can be the function (22):

$$(22)\quad v(z) = v_{eq}(z) + k_1 z_1 + k_2 z_2 + d\, sgn(h(z))$$

where $k_1 = \begin{cases} k_1^+ & for\ h z_1 \geq 0 \\ k_1^- & for\ h z_1 < 0 \end{cases} \qquad k_2 = \begin{cases} k_2^+ & for\ h z_2 \geq 0 \\ k_2^- & for\ h z_2 < 0 \end{cases}$

Each component of (22) is responsible for certain part of the dynamics in sliding mode. Control $v_{eq}(z)$ is a continuous function that maintains trajectory $x(t)$ in a tangential direction to manifold $\mathbf{S_h}$. It 'includes' disturbances of (21), so that the zero dynamics conditions (10) could be hold: $v_{eq}(z) = v_{eq(2)} + [M_m(t)]c/J + [dM_m(t)/dt]/J$. Components $k_1 z_1$ and $k_2 z_2$ can work together or alternately. They provide feedback signals to the correction of the system trajectory on both sides of the manifold $\mathbf{S_h}$. By suitable choice of feedback coefficients k_1 and k_2 we determine, on the one side $\mathbf{S_h}$ stable oscillations while on the other, unstable hyperboles. Also, they play a important role in solving the reaching problem of $\mathbf{S_h}$ from any initial conditions. The last component counteracts disturbances resulting from the load torque and its derivative. The constant d is selected according to the maximum of the disturbance norm.

In order to verify the theoretical results, simulation studies have been carried out.

Simulation results

The study has been made for the data included in the Appendix. These data has been supplemented with the load torque definition. The load torque is an active load of two different shapes (Fig.2), one sinusoidal and the other trapezoidal, both positive sign and of the same period T equals 1 [s]. In every case of simulation, initial points of the dynamical system (21) are governed by the conditions (10). Main simulation results are presented in figures 2-5.

Sliding controller (Fig.1) is a discrete time system that pursues a specific algorithm, processing input signals such as position, velocity, current, load torque and its derivative. It was appropriate to use load torque observer and accurate differentiation discrete systems for load torque and position, too.

Control algorithm performs switching on the hiperplane $\mathbf{S_h}$ with a finite frequency f_p ($=1/T_P$), hence chattering phenomena occurs on this plane. In general, this is a serious practical problem, but here it is only a part of digital algorithm, realized on auxiliary plane, which is not 'our' target. The reaching conditions are always satisfied for this hiperplane, because for the initial conditions (10), spiral trajectory (21) will intersect $\mathbf{S_h}$ in a finite time.

On the other hand the sliding dynamics of the 'first order' tends asymptotically, along the sliding manifold $\mathbf{S_S}$ to steady state behaviour. When t tends to infinity, function $S(t)$ tends to zero with time constant 1/c, but c_0 must be greater then c, as results from assumptions. The $S(x)$ is a smooth function without chattering, therefore α and ω are smooth functions, too.

Reference voltage signal U_z of the current controller is a discontinuous signal, switching with high frequency f_p associated with switching over to $\mathbf{S_h}$. The armature current, as planned is constrained, its derivative too. The current waveform shows, some harmonics are filtered by low pass filter with cut-off frequency equals $1/T_z$.

The variable load torque does not affect the dynamics of the sliding motion on $\mathbf{S_h}$ and $\mathbf{S_S}$, too. Trajectories (α,ω) are the same for both different load torque $M_m(t)$.

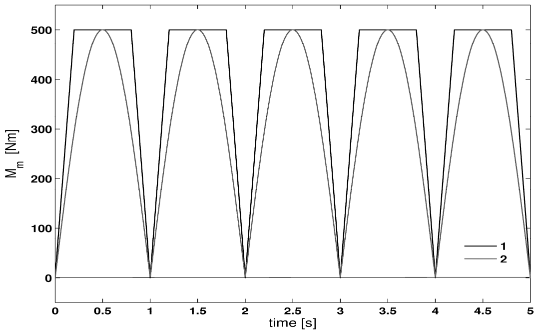

Fig.2. Load torque $M_m(t)$: 1-trapezoidal, 2-sinusoidal

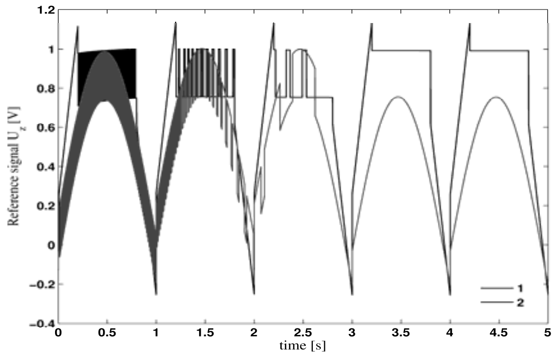

Fig.3. Reference voltage signal $U_z(t)$

Conclusions

It can be said that some expected features of the higher order sliding mode control has been confirmed. These are particular, robust sliding dynamics to the external disturbances, smooth 'sliding trajectory' $S(x)=0$ of the first order and chattering phenomena on the second order sliding dynamics. The variable load torque does not impair sliding dynamics. Second order dynamics is realised with accuracy of order $O(T_p^r)$. Based on simulation results, continued research has been taken towards the implementation of sliding algorithm introducing observers of the load torque and velocity and differentiating subsystems. Parameter sensitivity not satisfied matching conditions should be analysed. These problems will be presented in the next article which is a continuation of this work.

APPENDIX

Motor parameters

ω_n=60.7 [rad/s] ψ_e =3.452 [Vs/rad]
Pn=46 [kW] λn=2.2
Un=220 [V] p=70
In=231 [A] $p*In$= 16170 [A/s]
ω_0=63.73 [rad/s] Jt=5.5 [$kg*m^2$]
R_t=44.6 [$m\Omega$] $Jmech$=5.5 [$kg*m^2$]
L_t=0.36 [mH] $J=Js+Jmech$

Power converter parameters

Lu=2.94 [mH] Ru=0.1331[Ω] Kp=51.3 [V/V]

Current control system parameters

Y=0.004 - *feedback gain* $T=L/R$=0.0221 [s]
$B=J*R/(\psi_e)^2$ =0.1229 [s]
$T1$=0.5*B*(1-sqrt(1-4*T/B)) =0.0289 [s]
$B1=B*T/T1$ =0.0940 [s] $T_z=\lambda n/p$=0.0314 [s]
$T_R=T_z*Y*Kp*B/(R*(B1-T_z))$ =0.0952 [s] $K_R=T1/T_R$
$K_z=Kp*B/(T_R*R+Kp*B*Y)$ = 166.4054 [A/V]
$U_{zmax}=\lambda n*In*Y*B1/(B1-T_z)$ = 3.054 [V]

Sliding regulator c=1 c_0 =2 k1=5 k2=5 d=20÷500

T_P = 0.0001 [s]

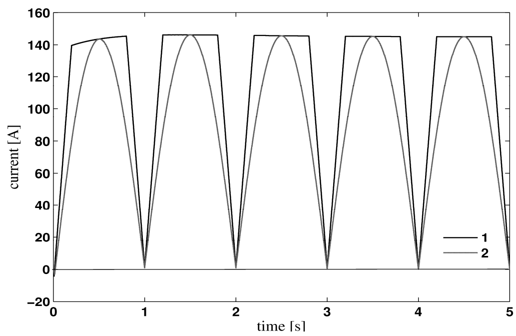

Fig.4. Armature current $i(t)$

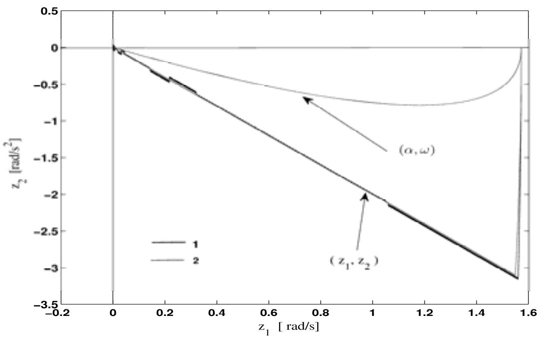

Fig.5. (α,ω) graph and second order sliding plane (z_1 z_2)

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