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Evidence Representations in Position Fixing

***Abstract**. Uncertain data are used in navigation, thanks to the nautical knowledge position is then fixed and evaluated. Data processing is rather limited since traditional mathematical apparatus based on probability theory with necessary assumptions is not flexible enough to include knowledge and ignorance into position fixing calculation scheme. Limited possibility is available regarding fix accuracy evaluation. In the paper possibilistic extension of Mathematical Theory of Evidence is explored as new platform enabling modeling and solving problems with uncertainty.*

***Streszczenie.** Niepewne dane są używane w nawigacji, na ich podstawie, dzięki nautycznej wiedzy, pozycja obserwowana jest określana i oceniana. Przetwarzanie danych wejściowych jest raczej ograniczone głównie z powodu ograniczeń tradycyjnie wykorzystywanego aparatu matematycznego. Aparat taki, oparty na podejściu probabilistycznym, narzuca szereg uwarunkowań, a jego elastyczność nie pozwala na uwzględnienie wiedzy jak i ignorancji. W artykule posybilistyczne rozszerzenie Matematycznej Teorii jest przedmiotem eksploracji, jako nowa platforma umożliwiająca modelowanie i rozwiązywania zadań z niepewnością. (**Reprezentacje ewidencji w określaniu pozycji**).*

Keywords: evidence representation, fuzzy reasoning, belief structures, position fixing.
Słowa kluczowe: reprezentacja ewidencji, wnioskowanie rozmyte, struktury przekonań, określanie pozycji.

Introduction

Possibilistic extension of Mathematical Theory of Evidence proved to be flexible enough to enable modeling and solving problems with uncertainty [23, 26, 27]. Concept of using the new platform for position fixing was presented by the author in his previous publications. Herein possibilistic versus probabilistic approaches are depicted in context of solving position fixing problem. The theory exploits belief and plausibility measures and operates on belief structures as converted versions of evidence representations. Usually using theory of evidence one has to normalize engaged data structures. Suggested normalization approaches feature serious drawbacks, therefore new method of data transformation was proposed.

Uncertain evidence and its representation

In Mathematical Theory of Evidence [22] also known as Dempster-Shafer Theory evidence and hypothesis frames are considered. In its possibilistic extension uncertain evidence is represented using fuzzy sets and masses of confidence attributed to these sets. Relations between hypothesis and evidence spaces are encoded into evidence representation. Fuzzy sets embrace grades expressing possibilities of belonging of consecutive hypothesis items to the sets related to each piece of evidence. As already mentioned each of the fuzzy sets has assigned credibility mass. Therefore fuzzy evidence representation consists of "fuzzy set – mass" pairs. The mapping is described by formula (1).

(1) $$\mathrm{m}(e_i) = \{(\mu_{ij}(x_k), f(e_i \rightarrow \mu_{ij}(x_k))\},\ j = 1,...,n$$

where: $\mu_{ij}(x_k)$ – membership function, $f(e_i \rightarrow \mu_{ij}(x_k))$ – support for $\mu_{ij}(x_k)$ embedded in *i*-th evidence.

In presented applications fuzzy evidence mapping consists of pairs: vectors representing locations of a set of points within sets related to each piece of evidence – degrees of confidence assigned to these vectors. Degrees of confidence reflect probability of an isoline being located within given strip area or a position being located inside two belts intersection region. Appropriate imprecise values are at disposal based upon statistical investigations of measurements distributions.

Fuzzy sets are represented by membership functions that reflect relations between the two universes. It is assumed that each piece of evidence is accompanied by a set of areas, ranges, therefore membership functions reflects relations between elements belonging to hypothesis space and sets attributed to elements of the evidence frame. Membership function converts the hypothesis space into power set of [0, 1] interval. Membership functions can be perceived as following mapping: $\mu : \{x_k\} = \Omega_H \rightarrow 2^{[0,1]}$. Membership functions for nautical applications are discussed in the author previous papers [8, 9, 10].

It is quite often when discrete unary intervals are used for evidence representations. Counts of opinions falling within each interval are exploited when fuzzy distance to navigational obstacle is considered [5, 6]. In order to evaluate situation within confined congested regions the same sort of data can be used [7].

In position fixing fuzzy sets are interpreted in different way. Figure 1 shows such specific interpretation. In the figure intersection of two imprecise isolines fragments are presented. Example hypothesis space (Ω_H) is also shown in the illustration. Elements from hypothesis space $\{x_k\}$ are to be located within reference sets $\{o_{ij}\}$ related to each piece of available measurements. Binary or fuzzy valued locations are grades of membership functions: $\mu_{ij}(x_k)$ that define location vectors.

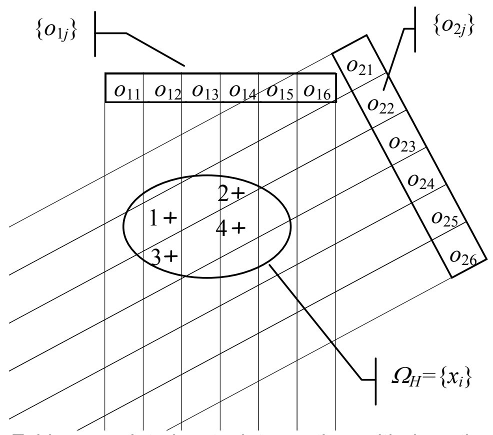

Fig.1. Evidence related sets intersection with hypothesis space elements

Below presented limitations (2) applied to evidence representation exclude empty sets. Item 2 in constraints specification stipulates normality of fuzzy sets. Normal sets should include highest grade equal to one. Apart from these two limitations typical for fuzzy mapping, additional requirements 3 and 4, regarding greater than zero masses and their total value, are also to be observed. Consequently mappings should include all normal fuzzy sets and total sum of their masses is to be one.

$$
(2)\qquad
\begin{aligned}
&1.\quad \mu_{ij}(x_k) = g(\{x_k\} \to o_{ik} \in \Omega_E) \neq \varnothing \\
&2.\quad \max_k \mu_{ij}(x_k) = 1 \\
&3.\quad m_{ij} = f(e_i \to \mu_{ij}(x_k)) \geq 0 \\
&4.\quad \sum_{j=1}^{n} f(e_i \to \mu_{ij}(x_k)) = 1
\end{aligned}
$$

Evidence mapping (1) also called basic probability assignment [2, 3, 16] can be considered as belief structure provided conditions (2) are satisfied.

Hereafter function $f(e_i \to \mu_{ij}(x_k))$ is assumed known for each referential item linked to *i*-th measurement or indication [11, 13]. Let us consider measurement that is a random variable governed by Gaussian distribution with standard deviation σ. Ranges $C_{j-1} \cdot \sigma \leq x < C_j \cdot \sigma$ of abscissa meant as distances from obtained isoline in its gradient direction define confidence intervals. Cumulated probability equal to the area under the bell curve between limits defined by range $[C_{j-1}, C_j)$ can be calculated using formula (3).

$$
(3)\qquad
\begin{aligned}
&P(a \leq x < b) = \frac{1}{\sigma\sqrt{2\pi}} \int_a^b \exp\left(-\frac{(x-x_o)^2}{2\sigma^2}\right) dx \\
&a = C_{j-1} \cdot \sigma \\
&b = C_j \cdot \sigma
\end{aligned}
$$

Calculated probabilities for two sets of consecutive confidence intervals defined by constants C_j are presented in Table 1. It is usual that the range of $[\sigma^-, \sigma^+]$ is given instead of crisp, single value of standard deviation. Thus established ranges have imprecise and expanding limits, consequently they are fuzzy sets, example is shown in figure 2.

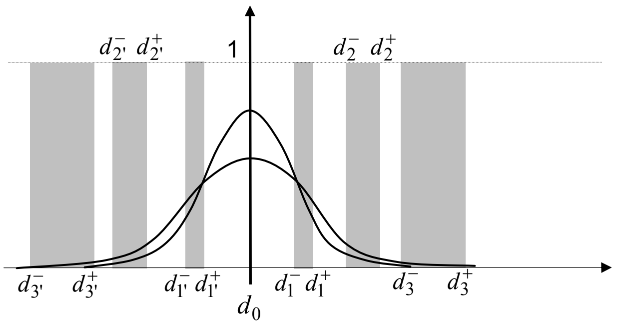

Fig.2. Ranges established around obtained isoline with their imprecise limits

Table 1. Probabilities for two sets of consecutive confidence intervals

C_j	0	0.5	1	1.5	2	2.5	3
$P(..)$	–	0.192	0.150	0.092	0.045	0.017	0.005
C_j	0	1		2		3	
$P(..)$	–	0.342		0.136		0.021	

Figure 1 presents set of six ranges situated in the vicinity of isolines related to two observations. Ranges reflect probabilities of the true isolines locations around the taken ones. It is assumed that measurements are random variables governed by various but known distributions. Moreover estimated parameters of such distributions varies at real scale, consequently borders between ranges are imprecise ones [12].

Example of evidence mappings are presented in table 2. Presented assignments refer to the scheme shown in figure 1. In this case hypothesis space embraces four points, therefore: $\Omega_H = \{x_1, x_2, x_3, x_4\}$. Each considered point can be treated as potential fixed position. The truth of the pproposition on representing the fix is to be proved based on selected criteria that are metrics exploited in theory of evidence, they are plausibility and belief measures [1, 4].

Due to particular allocation of the hypothesis frame points, sets related to each piece of evidence can be reduced to the following items: $e_1 \to \{o_{12}, o_{14}\}$ and $e_2 \to \{o_{22}, o_{23}\}$. Thus membership function grades, for the first piece of evidence, takes the form of expression: $\mu_{ij}(\{x_1, x_2, x_3, x_4\}) = g(\{x_1, x_2, x_3, x_4\} \to \{o_{12}, o_{14}\})$. It can be read that membership grades are degrees of inclusion of hypothesis points within evidence frame. Considering single grade $\mu_{ij}(x_k)$ one can use formula (4) to obtain its value. In the formula $C = 1$ for binary location, $C \in [0, 1]$ in fuzzy approach.

$$
(4)\qquad \mu_{ij}(x_k) = \begin{cases} C & \text{if } x_k \in o_{ij} \\ 0 & \text{otherwise} \end{cases}
$$

Table 2. Evidence of two evidence representations

	μ_{oij}	x_1	x_2	x_3	x_4	m_{oij}
m(e_1)	μ_{o12}=	{1	0	1	0}	m_{o12}=0.2
	μ_{o14}=	{0	1	0	1}	m_{o14}=0.8
m(e_2)	μ_{o22}=	{1	0.8	0	0}	m_{o22}=0.3
	μ_{o23}=	{0	0.2	1	1}	m_{o23}=0.7

Data presented in table 2 indicate that points x_1 and x_2 are situated within area o_{12}. Points x_3 and x_4 are located outside this area. Moreover type of belonging is binary one, particular points are situated within or outside given range. This type of membership is justified for any location close to the middle section of a considered strip. Membership of the second point inside area o_{22} is to be treated in quite different, fuzzy way [11]. It is situated close to the border of an area. Subsequently its location should be partial within adjacent ranges. To some extent it is located within range o_{22} and partly inside the next one. From table 2 it is seen that this point membership grade for area o_{22} is 0.8, and for range o_{23} is equal to 0.2.

In probabilistic approach membership grades can be seen as Bayesian conditional probability [18, 19]. Value of $p(x_l|o_{ik})$ can be interpreted as probability of point x_l being located inside range o_k provided *i*-th piece of evidence is considered. At the same time probability of location of the true isoline within respective area: $p(o_{ik})$ is assumed known [14]. The interpretation enables encoding evidence [19] in a way that is in line with Bayesian representation. It should be stressed that the approach features serious drawbacks and limitations. Probability distribution stipulates sum of one regarding locations within all considered ranges. It can be hardly observed in selected cases presumably related to small discrepancies in parameters estimation [10] in general this requirement cannot be fulfilled. Additionally in probabilistic approach there is no simple way of modelling uncertainty. Despite these obvious shortcomings references to this approach will be made for a few reasons. For the first possibilistic scheme of reasoning take its origin in probability theory. For the second common conclusions, the same important formula can be derived using these two approaches.

In traditional approach probability theory is exploited to cope with uncertainty. It can be successful whenever samples counts satisfy imposed requirements. It many practical cases this condition cannot be observed. Therefore statistical conclusions cannot be drawn and the approach is not justified, in such cases possibilistic models can be used instead [27]. Possibility theory is an emerging, useful tool to be exploited to model and cope with uncertainty, which is ubiquitous in navigation.

Combining evidence and position fixing

Single belief structure containing encoded data regarding isoline referring to particular measurement is not sufficient to make a fix. In order to achieve the goal one has to engage at least two lines of position [14]. In Mathematical Theory of Evidence structures combination is carried out [3], [4]. During combination all pairs of location vectors are associated and product of involved masses is assigned to the result set. Obtained assignment is supposed to increase informative context of the initial structures. Combination of structures embracing measurements data is assumed to result in position fixing. The goal can be achieved provided association of sets enables selection of common points. In navigation points located within intersection of introduced ranges are to be selected. Selection is done thanks to T-form operations [21] used during fuzzy sets association [24]. The simplest T-form results in smaller values being taken from consecutive pairs of elements in associated vectors (see formula (6)). This operation is used in numerical examples further presented in the paper.

Two assignments that satisfy conditions (2) can be combined within two dimension structure. Simplified result of combination takes the form of assignment presented by formula (5).

(5) $$\mathrm{m}_c(e_c) = \{(\mu_{c1}, m_{c1}(\mu_{c1})), \cdots, (\mu_{cl}, m_{cl}(\mu_{cl}))\}$$

where: $\mathrm{m}_c(e_c)$ – combined assignment, μ_{c1} – result membership function, $m_{c1}(\mu_{c1})$ – result mass assigned to combined evidence.

Result grades of membership functions μ_{cij} are selected using expression (6)

(6) $$\mu_{cij}(x_k) = \mu_{1i}(x_k) \wedge \mu_{2j}(x_k) = \min(\mu_{1i}(x_k), \mu_{2j}(x_k))$$

where: $\wedge$ – is the applied T-norm operator.

Result of association with T-norm operator may be empty or subnormal. Therefore certain amount of mass may be assigned to null set that means conflicting situation also called as inconsistency. It causes that achieved assignment (4) is pseudo belief structure, which is to be subject of transformation in order to obtain normal mapping.

There are two main methods of normalization named after their original inventors: one can use Dempster [1] and Yager [25] transformation procedures. It should be stressed that the original concepts intended for binary evidence mutated being adjusted to emerging new requirements. Introducing fuzzy evidence then accepting imprecise masses both resulted in modification of the initial propositions.

In Dempster concept masses assigned to non empty vectors are increased depending on total inconsistency mass. This might lead to unacceptable conclusion that final solution (fixed position) can be corrupted by normalization process. Fix must solely depend on probabilities of an isoline being located within selected strips.

Alternative normalization method was proposed by Yager. In the approach subnormal fuzzy set is made normal by increment of all its grades by complement of the set height. Note that grades equal to zero indicate that given point is outside particular area. Thus the transformation contributes to certain amount of belonging to all considered points. It should be stressed that values of calculated metrics are increased but relations among their relative values are maintained. Unfortunately this leads to disability in detecting of all inconsistency cases. Consequently final uncertainty measure is corrupted. In this approach non empty sets masses remain unchanged, masses assigned to empty sets increase uncertainty. In order to get rid of above mentioned drawbacks herein new transformation method is proposed.

Proposed normalization procedure uses height of a fuzzy set: $h_i = \max_k(\mu_{ci}(x_k))$. During normalization masses attributed to location vectors are reduced by the height of particular set. Sets are normalized through grades division by their heights (first and second expressions in formula (7)). It should be noted that product of modified masses and grades remain unchanged. It is to be stressed that these products contribute to final values of plausibility on representing a fix.

(7)
$$\begin{aligned}
&1.\quad m_{ci}^F = m_{ci} \cdot h_i \\
&2.\quad \mu_{ci}^F(x_k) = \frac{\mu_{ci}(x_k)}{h_i},\ h_i > 0 \\
&3.\quad m(\mu_{ci}^F(\Omega)) = m(\mu_{ci}(\Omega)) + m(\mu_{ci}(\varnothing)) \\
&4.\quad m_d = \sum_i (m_{ci} - m_{ci} \cdot h_i)
\end{aligned}$$

Last expression in formula (7) specifies that in this proposal reduction of masses increase additional factor that should be recorded and then analyzed when the fix is evaluated.

Table 3. Results of combination of two belief structures

area		x_1	x_2	x_3	x_4	$m_{cij}(..)$
$o_{12}\cap o_{22}$	μ_{c11}=	{1	0	0	0}	0.06
$o_{12}\cap o_{23}$	μ_{c12}=	{0	0	1	0}	0.14
$o_{14}\cap o_{22}$	μ_{c21}=	{0	0.8	0	0}	0.24
$o_{12}\cap o_{23}$	μ_{c22}=	{0	0.2	0	1}	0.56
	$p(x_l)$ =	0.06	0.304	0.14	0.56	

Table 4. Normalized results of combination of two belief structures

area		x_1	x_2	x_3	x_4	$m^F_{cij}(..)$
$o_{12}\cap o_{22}$	μ^F_{c11}=	{1	0	0	0}	0.06
$o_{12}\cap o_{23}$	μ^F_{c12}=	{0	0	1	0}	0.14
$o_{14}\cap o_{22}$	μ^F_{c21}=	{0	1	0	0}	0.192
$o_{12}\cap o_{23}$	μ^F_{c22}=	{0	0.2	0	1}	0.56
	Ω	{1	1	1	1}	0.048
	$pl'(x_l) =$	0.06	0.304	0.14	0.56	
	$pl(x_l) =$	0.108	0.352	0.188	0.608	

Table 3 presents results of combination of two belief structures shown in table 2. Collected data contains vectors which grades mean hypothesis points fuzzy locations within region of intersection of two ranges specified in the first column. Masses assigned to these vectors are products of probabilities attributed to involved ranges. It should be noted these masses represent credibility of a fix being located inside the intersection. In probabilistic approach obtained result is a Bayesian representation of combined evidence. From possibilistic standpoint the result appears as pseudo belief structure.

(8) $$p(x_l) = \sum_{k=1}^{j \cdot n} p(x_l | o_k) \cdot p(o_k)$$

where: $o_k = o_{1j} \cap o_{2n}$, *j*, *n* – number of items in first and second assignment respectively.

Set of probabilities calculated with formula (8) are shown in row with header: $p(x_l)$. The greatest value receives point 4 as located at the intersection $o_{14} \cap o_{23}$ of ranges with the highest initial probabilities regarding containing the true isoline.

As it was mentioned above obtained result seen from possibilistic perspective appears as pseudo belief structure since set defined by membership function μ_{c21} is subnormal. Therefore obtained assignment is to be normalized. Result of transformation with proposed method is presented in table 4. All location vectors are normal and one set containing all one grades is added. Set with all one grades represents uncertainty. It expresses common sense opinion that everything is possible. In other words each hypothesis item should be considered likely with the same degree of confidence. Apart from family of modified vectors selected masses were also changed.

Obtained belief structure was further examined in order to calculate plausibility measures of representing the fix. Examination was carried out using formula (9). In the formula component $m(\mu_k(x_i))$ is a credibility mass attributed to *k*-th intersection of ranges. Factor $\mu_k(x_l)$ reflects fuzzy locations of hypothesis points within ranges intersections.

(9) $$pl(x_l) = \sum_{k=1}^{n} m(\mu_k(x_i)) \cdot \mu_k(x_l)$$

Formula (9) calculates plausibility measure of support for a certain fuzzy set embraced in family of related items, it was derived in [10]. Plausibility and belief are basic measures used in MTE they represents interval valued probability of support for selected hypothesis [17]. It should be stressed that formulas (8) and (9) are the same although derived for different approaches, respectively probabilistic and possibilistic ones.

Set of plausibility measures calculated with formula (9) are shown in right hand side of table 3 in row titled with $pl(x_l)$. Note that the set of data in row $pl^{'}(x_l)$, obtained in condition of disregarding uncertainty, is the same as this acquired with formula (8) for Bayesian approach. It should be also emphasized that the same sets of final solution resulted from proposed conversion method that eliminates drawbacks of Dempster and Yager transformations.

Conclusions

Belief structures in nautical applications contain encoded evidence related to taken measurements. Result of structures combination is two-dimensional table that embraces enriched data enabling reasoning on the fix. From possibilistic viewpoint this result is a belief structure that is distribution of possibilities regarding representing the fix by each point out of hypothesis frame. Mechanisms and methods available in the theory of evidence can be exploited in order to derive formulas for calculating interval valued probability of representing fixed position by each of considered points. Interval value limits are equal to belief and plausibility measures. Difference between plausibility and belief expresses uncertainty, this simple rule is true in case of dealing with simple belief structures. Simple structure contains data regarding single event, they are used while frame of discernment contains a single element. Structure used in nautical applications contains data referring to a single observation.

Alternatively, from probabilistic standpoint obtained result of combination can be perceived as Bayesian evidence representation. It should be stressed that this standpoint is justified in selected cases, in general final structure does not fulfil probability requirements. Remaining unresolved dilemma whether stipulated conditions are observed or not one can use Bayesian methods to derive formula for calculating support probability for "being a fix" by any point out of the hypothesis universe. Surprisingly two approaches yield virtually the same formula.

It should be noted that possibilistic approach itself is an extension for probabilistic, Bayesian concept. Extension is much more flexible in respect of modelling and ability to process uncertainty.

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