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On modeling thermal steady state control of thermal systems

Abstract. *A new modeling approach for steady-state fluid flow analysis of networked thermal systems with temperature control devices is proposed in this paper. The problem of temperature control is crucial for the design and analysis of such thermal systems as air conditioning systems of data centers, for their control as well as providing the best data center performance and reliability. It is shown that the approach consists in formulating and solving nonlinear programming problem with positive simple constraints. Mathematical and physical aspects of the approach studied and its appropriateness are demonstrated using example thermal systems.*

Streszczenie.*Proponuje się nowe podejscíe do analizy statycznej przepływów cieczy w sieciach systemów cieplnych z regulatorami temperatur. Jest to zagadanie krytyczne dla analizy i projektowania systemów klimatyzacji centrum danych oraz regulowania tymi systemami w celu uzyskania jak nawiększej ich efektywności oraz niezawodności. Zademonstrowano że to podejście polega na formułowaniu i rozwiązaniu zadania programowania nieliniowego przy dodatnich wartościach ograniczen prostych. Kwestie matematyczne i fizyczne badanego podejścia oraz jego adekwátnosc są poparte przykładami systemów cieplnych. (****Modelowanie systemów cieplnych z regulatorami temperatur****)*

Keywords: data center, thermal system, steady state flow, thermal control, optimization approach
Słowa kluczowe: centrum danych, system cieplny, przepływ statyczny, regulowanie temperatury, podejście optymalizacyjne

Introduction

A thermal system (TS) is a network of interacting components. Its task is to transform the parameters of a working fluid from one state into another by transforming its energy and matter. The transformation is performed in the TS components, with each type defining its unique transformation process. Thus, the TS type is characterized by the totality of such processes. TS is a product of design, i.e., on the one hand, its structure should be defined – a graph of components types and the sequence of the transformation processes in TS components. On the other hand, each component characteristics should be specified, defining qualitatively intensity of a transformation process in each TS component. The latter is provided by design of a component, which is often reduced to its sizing. Normally, TS is designed for a typical mode of its functioning. The simulation models should be used to determine that state parameters are still within allowable ranges of TS operation. Otherwise, the state parameters must be adjusted. Control devices are important components of any TS. They provide design values of temperatures, flows and pressures in certain parts of a system when TS boundary conditions change. One of the basic physical mechanisms of control is throttling, meaning the change of valve resistances. It changes the amount of working fluids with different temperatures mixed in the temperature control devices in that way redistributing temperatures in the whole TS as well as flows and pressures. In engineering, control devices should maintain the operational state parameters and flow rates close to the ones obtained at the design stage adjusting resistance magnitudes in a way to minimize the difference between the parameters of working fluid being controlled at the given point and control devices settings. It suggests formulating and solving an optimization problem. Valve resistances calculated for various control devices and boundary conditions may characterize steady state temperature, pressure, humidity or flow rate controllability of TS. This may support decision-making at TS design in specifying composition and location of TS control devices.

The above general description concerns any thermal system type, including air conditioning systems of data centers. It is well-known how important it is to keep data center, servers or computer room at an optimum temperature for the best data center performance and reliability. For example, if the humidity drops below some relative humidity in a data center, static electrical charges can build-up causing sparks that damage servers and IT equipment. Data centers evolve. They are flexible architectures that may change, even drastically, their powers and emitting heat. That is, a lifecycle of servers does not match the lifecycle of air-conditioning systems of data centers. For the cost savings sake the components of a data center are integrated into a building management system that complicates essentially their design, analysis and control because its each component should be designed or analyzed as an integral part of the totality of interacting components. The detail analysis and design of such thermal systems become possible to perform if they are computer-aided. However, it is equally important to design the TS performing the desired task in emergency or changing environmental conditions. The range of these conditions constitutes the TS envelope within which the TS is considered to be statically controllable. Hence, the envelope specification accompanies TS design. It is impossible to do without the model which is capable of analyzing different TS configurations invoked by the changed valve positions of various types of control devices. Thus, the robust method for analysis of TS static controllability must result in a solution at any TS control configuration.

The state-of-art of modeling and simulation of flow and pressure control is presented in [1], where the optimization approach is developed based on the content model [2, 3, 4, 5]. It is stated in [5] that in case of locally controlled devices the TS has unique solution, whereas the inclusion of distributed feedback devices can lead to multiple solutions. As distinct from flow or pressure control devices temperature control devices belong to the distributed feedback devices in nature, since their control nodes (sensor devices) are not directly connected to the devices (valves, which may be located at arbitrary TS points).

In the paper we focus mainly on temperature control, thereby assuming that flow and pressure control problems are somehow solved [1, 3, 4, 5]. It is shown that the optimization problem modeling distributed feedback devices does not necessarily involve, as the literature suggests [1], multiple solutions as there exists an additional condition inherent to the very nature of the problem that eliminates the multiplicity. To the author knowledge such a problem solution is not reported in the literature.

Problem nature

The source of solution multiplicity is hidden in the vary essence of temperature control. For instance, every time we have our hands washed with warm water we play the role of a temperature regulating device (TRD), with our hands serving as a sensor and taps (denoted by 1 and 2, respectively, in Fig. 1) as valves. Changing valves resistances (throttling), i.e.

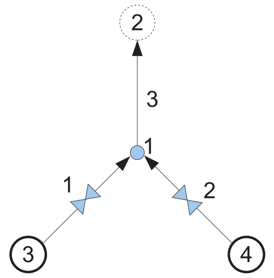

Fig. 1. Simple TS thermal control

turning hot or cold taps, step-by-step we achieve a necessary temperature (a temperature setting), $\mathbf{T_s}$, at the sensor (our hands) location point (e.g. at node 2 in Fig. 1). Throttling is a method to minimize the difference between an available temperature and sensor set temperature. This, mathematically, can be formulated as an optimization problem that minimizes the square of the differences between a predicted temperature, $\mathbf{T_{TRD}}$, and $\mathbf{T_s}$

$$f = (\mathbf{T_{TRD}} - \mathbf{T_s})\,(\mathbf{T_{TRD}} - \mathbf{T_s})^T \,. \tag{1}$$

To formulate a TS model we use the graph theory as a framework to relate process models described by fundamental laws of physics. TS graph, $G = (V, E)$ is a number set of nodes $V \in \mathbb{N}$ and edges $E \subseteq V \times V \in \mathbb{N}$, with $n_t = |V|$, $e = |E|$ being the number of TS graph nodes and edges, respectively. Besides, $V = V_i \cup V_b$ such that $V_i = \{i \in \mathbb{N} : 1 \le i \le n_i\}$, $V_b = \{i \in \mathbb{N} : n_i < i \le n, \, \mathrm{d_G}(i) = 1\}$, where $n_i = |V_i|$, $n_b = |V_b|$, $n = n_i + n_b$, and $\mathrm{d_G}$ stands for a graph node degree. Nodes from V_b are called boundary nodes, where some process parameters are given values. Edges may also be specified by some process parameter, variable or fixed (e.g. by nodal flow demands). We specify a TS graph with an incidence matrix, $\mathbf{A}_{n_i \times e}$.

Three basic conservation physical principles are needed to determine the available temperature: continuity, momentum and energy. To grasp the nature of the multiplicity behind a mathematical model we needn't the complex models to attack the problem as simple models are sufficient to comprehend its essence. Hence, we assume that processes that occur in TS are steady-state and isothermal, and that given discharges from the TS interior nodes are absent. It is easy to obtain relations in this case expressing the conservation principles (e.g. see [6] and references herein)

$$\mathbf{h}_1 = \mathbf{A}\begin{pmatrix} \dot{\mathbf{m}} \\ \dot{\mathbf{M}} \end{pmatrix} = \left(\tilde{\mathbf{A}}_{n \times e_m} | \mathbf{0}\right)\dot{\mathbf{m}} \tag{2}$$

$$+ \left(\mathbf{0} | \bar{\mathbf{A}}_{n \times e_{TRD}}\right)\dot{\mathbf{M}} = \mathbf{A_m}\dot{\mathbf{m}} + \mathbf{A_M}\dot{\mathbf{M}} = \mathbf{0} \,,$$

$$\mathbf{h}_2 = \mathbf{A}\mathbf{D_m}\mathbf{T} = \mathbf{0} \,, \tag{3}$$

$$\mathbf{h}_3 = \mathbf{A}^T\mathbf{P} + \mathbf{D_r}\begin{pmatrix} \dot{\mathbf{m}} \\ \dot{\mathbf{M}} \end{pmatrix} = \mathbf{0} \,, \tag{4}$$

$$\mathbf{h}_4 = \mathbf{B}_{n_T \times e}\mathbf{T} = \mathbf{0} \,, \tag{5}$$

where $\mathbf{A} = \mathbf{A_m} + \mathbf{A_M}$; $\dot{\mathbf{m}}_{e_m \times 1}$, $\dot{\mathbf{M}}_{e_{TRD} \times 1}$, $\mathbf{T}_{e \times 1}$, $\mathbf{P}_{n \times 1}$, $\mathbf{K}_{e \times 1}$ are vectors of flow rate variables, fixed flow rates, temperatures, pressures, and resistances, respectively; $e_m = e - e_{TRD}$; $\mathbf{D_m} = \mathrm{diag}(\dot{\mathbf{m}}, \dot{\mathbf{M}})$, $\mathbf{D_r} = \mathrm{diag}(\mathbf{K}\,|\dot{\mathbf{m}}|\,, \mathbf{R}\,|\dot{\mathbf{M}}|)$, where $\mathbf{R}_{e_{TRD} \times 1}$, in turn, denotes a vector of TRD minor resistances. Encoding of matrix $\mathbf{B}_{n_T \times e}$ is specific to the TS model in question. The peculiarity of the isothermal model of TS is a possibility to treat a temperature variable as an edge variable instead of being a node variable as it takes place in the general case. To encode the isothermal model of TS we introduce a set $V_T = \{k \in V : \Theta_k > 1\}$, where $\Theta_k = \mathrm{od_G}(k)$ denotes the out-degree of k. We also define a set $\Pi \supseteq \Pi_k = \Theta_k(1) \times (\Theta_k - \Theta_k(1))$ such that $\Pi = \bigcap_k \Pi_k = \oslash$, $k \in V_T$. Notice that the content of Π can be determined from $\mathbf{A}$. Then, n_T and $\mathbf{B}_{n_T \times e}$ in Eq.(5) are defined as follows, respectively $n_T = \sum_{k=1}^{|V_T|} |\Pi_k|$

(6)

$$B_{l,*} = \begin{cases} B_{l,i} = 1, \; B_{l,j} = -1, & \exists (i,j) \in \Pi_l \\ B_{l,k} = 0 & \forall k \notin (i,j)\,, l = \overline{1, n_T}. \end{cases}$$

Applied to the TS shown in Fig. 1 constraints Eqs.(2)-(4) give the following system of nonlinear equations

$$(R_1 + K_1)\,\dot{m}_1^2 + K_3\dot{m}_3^2 - (P_3 - P_2) = 0 \,, \tag{7}$$

$$(R_1 + K_1)\,\dot{m}_1^2 - (K_2 + R_2)\,\dot{m}_2^2 - (P_3 - P_4) = 0 \,, \tag{8}$$

$$\dot{m}_1 + \dot{m}_2 - \dot{m}_3 = 0 \,, \tag{9}$$

$$\dot{m}_1 T_1 + \dot{m}_2 T_2 - \dot{m}_3 T_3 = 0 \,, \tag{10}$$

which is a valid description provided that boundary pressures prescribed guarantee assumed flow directions. Equations (7)-(10) represent the loop form of Eqs.(2)-(4) and suggest that edges 1 and 2 contain two components (or types of resistances). If temperature control device setting $T_{3,s}$ is such that $T_{3,s} = T_1$ and the cold valve is closed ($\alpha_2 = 90^0$, i.e. $\dot{m}_2 = 0$ or $R_2 = \infty$), then any hot valve turn results in $T_3 = T_1$. Indeed, in this case we have from Eqs.(7)-(10) the following equations

$$R_1 = \frac{P_3 - P_2}{\dot{m}_1^2} - (K_1 + K_3) \,, \tag{11}$$

$$T_3 = T_1 = T_{3,s} \,, \tag{12}$$

which justify the fact that though temperature is controllable (see Eq.(12)), nevertheless R_1 can not be uniquely determined as it may take on as many values as $\dot{m}_1$ does. The unique solution is possible to obtain if $\dot{m}_1$ is specified. However, temperature control as an optimization problem is not needed in this case. Controllability loses if T_1 changes for some reasons, i.e. if Eq.(12) does not hold true. To provide the controllability, valve 2 should be at least open, ($\alpha_2 = 0$, i.e. $R_2 = 0$, where α stands for valve deflection angle). In this case of TS configuration the resistance of valve 1 can be derived from Eqs.(7)-(10) as follows

$$R_1 = K_3\left(1 + \frac{T_1 - T_3}{T_3 - T_2}\right)^2 \frac{P_3 - P_4}{P_4 - P_2} \tag{13}$$

$$+ K_2\left(\frac{T_1 - T_3}{T_3 - T_2}\right)^2 \frac{P_3 - P_2}{P_4 - P_2} - K_1 \,.$$

If we additionally assume that $K = K_1 = K_2 = K_3$, $P_3 = P_4$, and $P_3 - P_2 = 1$, then

$$R_1 = K\left[\left(\frac{T_1 - T_3}{T_3 - T_2}\right)^2 - 1\right] \,. \tag{14}$$

The objective to control TS is associated with providing its functioning, with TS state variables being maximally close to the state parameters determined at the TS design stage while some TS parameters change. These parameters could be, e.g., boundary conditions, fouling, emergency conditions, etc. For TS's, depending on their type, design values of state variables at proper points of a TS may be obtained under the assumptions that valves are open or closed, and be assumed as settings of the corresponding regulator types. If we suppose that the design conditions correspond to $\alpha_1 = \alpha_2 = 0$ (open valves) or $R_1 = R_2 = 0$, then mean temperature $T_3 = (T_1 + T_2)/2$ results from Eq.(14), which may serve as a temperature setting $T_{3,s}$ of a temperature control device for this very example network under boundary conditions and values of parameters assumed above. Let us assume that only T_3 changes for some reason. If $T_3 \to T_1$, then at some T_3 one can obtain $R_1 < 0$, meaning that it is impossible to open the valve more than $\alpha_1 = 0$ and thus to increase temperature in edge 3. With this valve we can only decrease the temperature (closing the valve) to the temperature not exceeding T_2 (indeed, at $T_3 \to T_2$, $R_1 \to \infty$ or $\alpha_1 \to 90^0$). Hence, if $T_3 \le T_{3,s}$, then R_1 can be uniquely determined from Eq.(13) or (14) provided that $T_3 > T_2$. The latter inequality is valid at $\alpha_1 \le 90^0$. At $\alpha_1 = 90^0$ (i.e. at $T_3 = T_2$) the TS model is not degenerating because the function Eq.(14) behaves asymptotically. So, to solve the TS control problem we are to add constraints

$$0^0 \le \boldsymbol{\alpha} \le 90^0 \quad \text{or} \quad 0 \le \mathbf{R} < \infty \tag{15}$$

to the model Eqs.(1)-(4). From the analysis made it follows that the required control, the value of α_1, at $T_3 = T_{3,s}$, in a certain range of TS parameters model values can be immediately determined from the constraints Eqs.(7)-(10) even without the optimization approach, provided that it is guaranteed that the solution for α_1 is within the range given by Eq.(15). However, the equation-based approach does not guarantee this, whereas the optimization approach does. Then the optimization approach differs from the equation-based one with the possibility of treating constraints. Is this the only difference ? We now show that there is no other difference. To this end we find analytical solution for T_3 from Eqs.(7)-(10). It is easy to obtain by eliminating variables and solving a quadratic equation under the condition of positiveness of both resistance and temperature

$$T_3 = \frac{T_1 + ST_2}{1+S}, \tag{16}$$

where

$$S = \frac{-2K_3(P_3 - P_4) + \sqrt{D}}{2\left[K_3(P_3 - P_4) + (K_2 + R_2)(P_3 - P_2)\right]}, \tag{17}$$

$$\begin{aligned} D = {} & 4K_3^2(P_3 - P_4)^2 - 4\left[K_3(P_3 - P_4)\right. \\ & \left.+ (K_2 + R_2)(P_3 - P_2)\right] \times \left[K_3(P_3 - P_4) + (K_1 + R_1)\right. \\ & \left.\times (P_3 - P_4) - (K_1 + R_1)(P_3 - P_2)\right] \ge 0 \end{aligned} \tag{18}$$

Assuming that R_k in Eqs.(17)-(18) are functions of α_k, the objective function looks like that shown in Fig. 2. The function is a surface that is formed by a set of curves with all their minimums equal to zero. At first glance, it may seem that the above supports the fact of multiple solutions in modeling distributed feedback devices. However, we show below that additional conditions being imposed on the model Eqs. (1)-(4) resolve the problem of solution multiplicity.

Model

We suggest the following form of the additional conditions in this paper to tackle the problem of modeling temperature control

$$\mathbf{h_4} = \mathbf{A_1}\dot{\mathbf{m}} - \mathbf{A_2}\mathbf{D}_\beta\dot{\mathbf{m}} = \mathbf{0}, \tag{19}$$

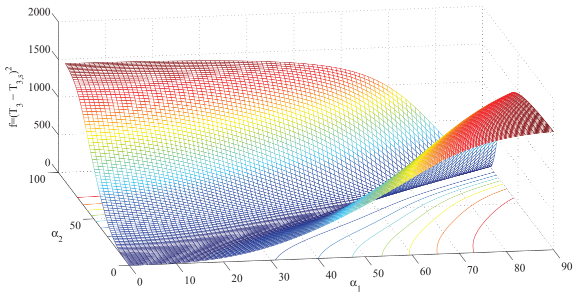

Fig. 2. The function of the model Eqs.(1), (16)-(18)

where $\mathbf{A_1}$, $\mathbf{A_2}$ are submatrices relating edges constituting temperature control devices (see the example considered below), $\mathbf{D}_\beta = \mathrm{diag}(\beta_1, \beta_2, \ldots, \beta_{e_m})$. Equation (19) has a clear physical meaning of the fact that the same temperature can be obtained at different hot and cold fluid (water or air) flow rates but at the unique hot to cold fluid flow rate ratio ($\boldsymbol{\beta}$). Equations (7)-(10) has been used to illustrate the impossibility of straightforward approach to modeling temperature control. Now we demonstrate the result of adding Eq.(19) to Eqs.(7)-(10) and applying the condition that $\dot{m}_3 \equiv \dot{M}_3$ (meaning that flow rate has already been adjusted by a flow control device)

$$T_3 = \frac{(\beta T_1 + T_2)}{(1+\beta)}, \tag{20}$$

$$R_1 = \frac{(1+\beta)^2}{\beta^2}\left[\frac{(P_3 - P_2)}{\dot{M}_3^2} - K_3\right] - K_1, \tag{21}$$

$$R_2 = (1+\beta)^2\left[\frac{(P_3 - P_2)}{\dot{M}_3^2}\frac{(P_4 - P_3)}{\dot{M}_3^2} - K_3\right] - K_2. \tag{22}$$

One can observe the direct relation between $\boldsymbol{\beta}$ and $\mathbf{R}$. To show that the function Eq.(1) possesses minimum we consider the TS shown in Fig. 3 which mathematical model consists of the objective function

$$f = (T_7 - T_{7,s})^2 + (T_8 - T_{8,s})^2 \tag{23}$$

subjected to the constraints Eqs. (2)-(4),(19), where

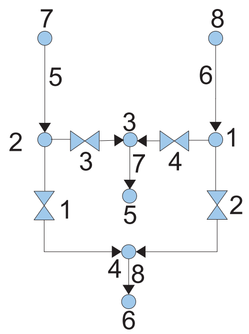

Fig. 3. TS with two temperature control devices

$$\mathbf{A}_{n\times e} = \left(\begin{array}{cccccc|cc} 0 & -1 & 0 & -1 & 0 & 1 & 0 & 0 \\ -1 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & -1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & -1 \end{array}\right),$$

$$\mathbf{A_m} = \left(\begin{array}{cccccc|cc} 0 & -1 & 0 & -1 & 0 & 1 & 0 & 0 \\ -1 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right),$$

$$\mathbf{A_M} = \left(\begin{array}{cccccc|cc} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&-1&0\\ 0&0&0&0&0&0&0&-1 \end{array}\right), \tag{24}$$

$$\mathbf{A_1}_{e_{TRD}\times e_m} = \begin{pmatrix} 0&0&1&0&0&0\\ 1&0&0&0&0&0 \end{pmatrix},$$

$$\mathbf{A_2}_{e_{TRD}\times e_m} = \begin{pmatrix} 0&0&0&1&0&0\\ 0&1&0&0&0&0 \end{pmatrix}.$$

Notice also that for a fixed $\boldsymbol{\beta}$ the constraints form a well-determined system of nonlinear equations (one may verify this using the example network model constraints, Eq.(24)), with the advantage of not requiring the usage of Eq.(4) (or Eq.(7)-Eq.(8) if we liked to perform the similar analysis for the TS shown in Fig.1) as their model. Evidently, it concerns only distribution thermal networks of a special configuration (when both a number of outlet boundary nodes and of combining flow nodes are equal to e_{TRD}), which has been intentionally chosen to simplify the study of the temperature control model presented in this paper. We can easily obtain a feasible initial point from the constraints as $\boldsymbol{\beta} \geq \sqrt{e_M}$ (e_M, machine precision) must always hold. The question that still remains to be answered is whether $\boldsymbol{\beta}$ is a design parameter minimizing the function Eq.(1). Figure 4 is the answer. It has been obtained by evaluating Eq.(23) for the temperatures T_7, T_8 or $\mathbf{T}_{TRD}$ ($\mathbf{T}_{TRD} \in \mathbf{T}$) solved from the constraints for the range of $\boldsymbol{\beta}$ illustrated by Fig. 4.

Results and discussion

Such a procedure suggests that $\mathbf{T}_{TRD}$ should be treated as an implicit function of $\boldsymbol{\beta}$ that, together with the convexity of the objective function (see Fig. 4), should also encourage us to treat the nonlinear solver for constraints as a black-box procedure or simulator. It is just the method the data presented in the table have been obtained, where $T_{7,s}$, $T_{8,s}$, and $\dot{M}_7$, $\dot{M}_6$ are temperature and flow rate settings in edges 7 and 8, respectively, whereas T_5, T_6 are source temperatures, and β_4, β_2 are the parameters modeling the first and second TRD. All the numerical experiments have been per-

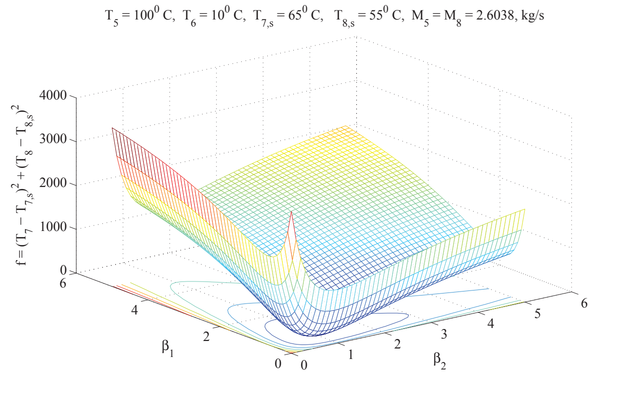

Fig. 4. The function of the model Eqs.(23)-(24) for the TS from Fig. 3

formed in the MATLAB environment. The temperature setting $T_{8,s}$ was intensionally assumed to be the only varied parameter to ease model and results verification. The results in the table reflect properly the effects of thermal control as they are observed in the reality. As expected, β_4 does not change while changing $T_{8,s}$ that is a correct result provided that arbitrary values of source pressures or flow rates are available as it is assumed in the case study example.

$T_5 = 100^oC, T_6 = 10^oC, T_{7,s} = 75^oC, \dot{M}_7 = \dot{M}_8 = 2.6038$							
	$T_{8,s}$						
	75	65	55	45	35	20	11
$\dot{m}_1$	1.8805	1.5912	1.3019	1.0126	0.7233	0.2893	0.0289
$\dot{m}_2$	0.7233	1.0126	1.3019	1.5912	1.8805	2.3145	2.5749
$\dot{m}_3$	1.8805	1.8805	1.8805	1.8805	1.8805	1.8805	1.8805
$\dot{m}_4$	0.7233	0.7233	0.7233	0.7233	0.7233	0.7233	0.7233
$\dot{m}_5$	3.7610	3.4717	2.0252	2.8931	2.6038	2.1698	1.9095
$\dot{m}_6$	1.4466	1.7359	3.1824	2.3145	2.6038	3.0378	3.2981
β_4	0.3846	0.3846	0.3846	0.3846	0.3846	0.3846	0.3846
β_2	0.3846	0.6364	1.0000	1.5714	2.6000	8.0000	89.000

On the other hand, each value of $T_{8,s}$ requires its own unique ratio of mixed flows to be achieved as a result of operation of a temperature regulating device. However, if source pressures are finite for any reason (including costs), then Eq. (4) must be added to the the simulator and solved [6]. This implies multilevel software architecture whose performance is a subject of the near future research.

Conclusion

A seeming solution multiplicity entailed by inclusion of distributed feedback devices does not necessarily hold as the literature suggests. Each TS model having such types of control devices must undergo particular analysis. This paper supports this fact studying the modeling of thermally controlled isothermal pipe networks, which nature itself hints the solution. It consists in taking into account the fact that temperature measured by a thermal regulating device is a function of a unique mixing flows ratio. The introduction of the ratio as a model variable eliminates the multiplicity. Besides, it results in the objective function convexity. It is also shown that the model may be decomposed into submodels each of which mathematically is more simple than the initial. Such a model structure may serve as a basis for future studies on more detailed computer analysis of thermal systems (including air conditioning systems of data centers) accounting for humidity analysis and its control extended to cover the presence of flow and pressure regulating devices as well.

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