

## AC-OPF Based Static Transmission Expansion Planning: A Multi-objective Approach

**Abstract.** This paper presents a methodology for transmission expansion planning using AC optimal power flow. A multi-objective framework has been considered. The objectives are to minimize the investment cost (IC), minimize the operation cost (OC) and also to minimize the power losses. The augmented  $\epsilon$ -constraint method was used so as to solve the multi-objective mathematical programming (MMP) problem. The proposed model has been applied to Garver's six bus test system and also to a real system of northeastern part of the Iranian national 400-kV transmission grid.

**Streszczenie.** W artykule zaproponowano metodologię planowania przesyłania energii przy optymalnym przepływie mocy. Minimalizuje się koszty operacyjne i straty mocy. Wykorzystano narzędzia wieloobiektywego programowania. (Planowanie przesyłu energii z optymalnym przesyłem mocy OPF – metoda wieloobiektywa)

**Keywords:** AC optimal power flow, Multi-objective mathematical programming, Transmission expansion planning  
**Słowa kluczowe:** OPF - optymalny przesył mocy, programowanie wieloobiektywne

### Introduction

Transmission Expansion Planning (TEP) addresses the problem of augmenting an existing transmission network to optimally serve a growing electric load while satisfying a set of economical, technical and reliability constraints [1]. In general, TEP is a stochastic decision making problem that consists of determining the time, the location, and the type of the transmission lines to be built [2].

Based on the solution methods there are three types of algorithms to solve the planning problem: 1) mathematical optimization methods, 2) heuristic methods and, 3) combinatorial methods called meta-heuristic methods. Transmission expansion planning can be categorized to static and dynamic planning. In the dynamic expansion planning the constructing time of lines will be determined in the optimization process, while in the static one there is only a "target year" that the selected optimal lines should be built within that. The planners of the power system will face many uncertainties during the planning. So far many published papers have been considered uncertainties during the planning process [3-5].

The authors of reference [3] presented a bi-level optimization model for TEP. Roh et al. [4] presented a stochastic coordination of generation and transmission expansion planning model in a competitive electricity market. Torre et al. [6] employed a mixed-integer linear programming (LP) formulation for the long-term transmission expansion planning problem in a competitive pool-based electricity market. References [7,8] have analyzed the TEP and Generation Expansion Planning (GEP) problem together. Ref. [9] studied TEP considering the load uncertainty using benders decomposition.

The papers reviewed use a DC approach in order to solve the TEP problem which is not completely suitable due to ignoring the reactive power. This paper proposes an approach for transmission planning based on AC optimal power flow (AC-OPF). Using AC-OPF could provide us a more precise picture of the active and reactive power flows in the expanded power network. Although the new model is more complicated than previous models, however it certainly leads to more precise and optimal plan in the future. The augmented  $\epsilon$ -constraint method is used in order to solve the formulated multi-objective decision making problem. Our novel contributions to this paper are: 1) Using AC-OPF based optimization. 2) A new revisited formulation based on multi-objective mathematical programming considering active power losses.

### Problem formulation

In the  $\epsilon$ -constraint method, one of the objective functions

is selected to be optimized using the other objective functions as constraints [10]:

$$\begin{aligned} & \text{Min } F_1(x) \\ (1) \quad & \text{subject to : } F_2(x) \leq \epsilon_2, \quad F_3(x) \leq \epsilon_3, \\ & F_4(x) \leq \epsilon_4, \quad \dots, \quad F_p(x) \leq \epsilon_p \end{aligned}$$

In order to avoid the weakly Pareto optimal solution we use the method proposed in [11]. The new optimization problem will be as follows:

$$\begin{aligned} & \text{Min } F_1(x) - \delta \times \left( \frac{s_2}{r_2} + \frac{s_3}{r_3} + \dots + \frac{s_p}{r_p} \right) \\ (2) \quad & \text{subject to : } F_2(x) + s_2 = \epsilon_2, \\ & F_3(x) + s_3 = \epsilon_3, \quad \dots, \\ & F_p(x) + s_p = \epsilon_p \end{aligned}$$

where  $\delta$  is a small number (between  $10^{-3}$  and  $10^{-6}$ ) and  $r_i = z_i^{nadir} - z_i^{ideal}$ .  $z_i^{nadir}$  is the upper bounds of  $F_i(x)$  in the feasible region of the problem or "nadir value" of  $i^{th}$  objective function and  $z_i^{ideal}$  is the lower bounds of  $F_i(x)$  or "ideal value" of  $i^{th}$  objective function.

This type of  $\epsilon$ -constraint method is called augmented  $\epsilon$ -constraint method or AUGMECON method [11].

The lower bounds of the Pareto optimal set are obtained by minimizing each of the objective functions individually subject to the feasible region. Obtaining the upper bounds of the Pareto optimal set is not a trivial task. The nadir and ideal values can be calculated from the "payoff table" that has been demonstrated in reference [12].

Since  $F_1$  is the main objective function in our MMP problem, only the ranges of objective functions:  $F_2$  and  $F_3$  should be calculated. These ranges for  $F_2$  and  $F_3$  are divided by  $q_2$  and  $q_3$ . Considering the minimum and maximum values of the ranges, we have the total of  $(q_2+1)$  and  $(q_3+1)$  grid points for  $F_2$  and  $F_3$ , respectively. Thus, we should solve  $(q_2+1) \times (q_3+1)$  optimization sub-problems where sub-problem  $(i, j)$  has the following form:

$$\begin{aligned} & \text{Min } F_1(x) - \delta \times \left( \frac{s_2}{r_2} + \frac{s_3}{r_3} \right) \\ (3) \quad & \text{subject to : } F_2(x) + s_2 = \epsilon_{2i}, \quad F_3(x) + s_3 = \epsilon_{3j} \\ & \epsilon_{2i} = \text{Max}(F_2) - \left( \frac{\text{Max}(F_2) - \text{Min}(F_2)}{q_2} \right) \times i \quad i = 0, 1, \dots, q_2 \\ & \epsilon_{3j} = \text{Max}(F_3) - \left( \frac{\text{Max}(F_3) - \text{Min}(F_3)}{q_3} \right) \times j \quad j = 0, 1, \dots, q_3 \end{aligned}$$

Three objective functions are considered for the proposed problem: 1) Investment Cost (IC), which is the construction cost of new lines, 2) Operation Cost (OC), which is the cost of generation in the power system and 3) Power Losses (PL), which is the total active power loss in the power system.

Using an AC power flow, the expansion planning problem can be formulated as follows:

$$(4) \quad \text{Min} \begin{cases} F_1 = IC = \underbrace{\sum_{l=1}^{nc} u_l IC_l}_{\text{Investment Cost (IC)}} \\ F_2 = OC = \underbrace{\left( 8760 \times \sum_{g=1}^{ng} C_{1g} P_{G,g} + C_{0g} \right)}_{\text{Operation Cost (OC)}} \\ F_3 = PL = \underbrace{\left( \sum_{l \in EL} P_{Loss,l}^0 + \sum_{l \in CL} P_{Loss,l} \right)}_{\text{Power Loss}} \end{cases}$$

Subject to the following equality and inequality constraints:

Equality constraints are as below:

$$(5) \quad P_{G,i} - P_{D,i} = \sum_{l \in EL_i} P_{L_{i \rightarrow j},li}^0 + \sum_{l \in CL_i} P_{L_{i \rightarrow j},li} \quad \forall i \in B$$

$$(6) \quad Q_{G,i} - Q_{D,i} = \sum_{l \in EL_i} Q_{L_{i \rightarrow j},li}^0 + \sum_{l \in CL_i} Q_{L_{i \rightarrow j},li} \quad \forall i \in B$$

$$(7) \quad \begin{aligned} P_{L_{i \rightarrow j},l}^0 &= |V_i|^2 G_{ij}^0 - |V_i| |V_j| \times \\ &\left( G_{ij}^0 \cos(\theta_i - \theta_j) + B_{ij}^0 \sin(\theta_i - \theta_j) \right) \quad \forall l \in EL, \forall i, j \in B \end{aligned}$$

$$(8) \quad \begin{aligned} P_{L_{i \rightarrow j},l} &= u_l \left( |V_i|^2 G_{ij} - |V_i| |V_j| \times \right. \\ &\left. \left( G_{ij} \cos(\theta_i - \theta_j) + B_{ij} \sin(\theta_i - \theta_j) \right) \right) \quad \forall l \in CL, \forall i, j \in B \end{aligned}$$

$$(9) \quad \begin{aligned} Q_{L_{i \rightarrow j},l}^0 &= -|V_i|^2 B_{ij} - |V_i| |V_j| \times \\ &\left( G_{ij}^0 \sin(\theta_i - \theta_j) - B_{ij}^0 \cos(\theta_i - \theta_j) \right) \quad \forall l \in EL, \forall i, j \in B \end{aligned}$$

$$(10) \quad \begin{aligned} Q_{L_{i \rightarrow j},l} &= u_l \left( -|V_i|^2 B_{ij} - |V_i| |V_j| \times \right. \\ &\left. \left( G_{ij} \sin(\theta_i - \theta_j) - B_{ij} \cos(\theta_i - \theta_j) \right) \right) \quad \forall l \in CL, \forall i, j \in B \end{aligned}$$

$$(11) \quad \begin{aligned} P_{Loss,l}^0 &= G_{ij}^0 \times \\ &\left( V_i^2 + V_j^2 - 2 \times V_i V_j \cos(\theta_i - \theta_j) \right) \quad \forall l \in EL, \forall i, j \in B \end{aligned}$$

$$(12) \quad \begin{aligned} P_{Loss,l} &= u_l \times G_{ij} \times \\ &\left( V_i^2 + V_j^2 - 2 \times V_i V_j \cos(\theta_i - \theta_j) \right) \quad \forall l \in CL, \forall i, j \in B \end{aligned}$$

Inequality constraints are as below:

$$(13) \quad P_{G,g}^{\min} \leq P_{G,g} \leq P_{G,g}^{\max} \quad \forall g \in G$$

$$(14) \quad Q_{G,g}^{\min} \leq Q_{G,g} \leq Q_{G,g}^{\max} \quad \forall g \in G$$

$$(15) \quad \left( P_{L_{i \rightarrow j},l}^0 \right)^2 + \left( Q_{L_{i \rightarrow j},l}^0 \right)^2 \leq \left( AP_{L,l}^{\max} \right)^2 \quad \forall l \in EL$$

$$(16) \quad \left( P_{L_{i \rightarrow j},l} \right)^2 + \left( Q_{L_{i \rightarrow j},l} \right)^2 \leq u_l \left( AP_{L,l}^{\max} \right)^2 \quad \forall l \in CL$$

$$(17) \quad |V_i^{\min}| \leq |V_i| \leq |V_i^{\max}| \quad \forall i \in B$$

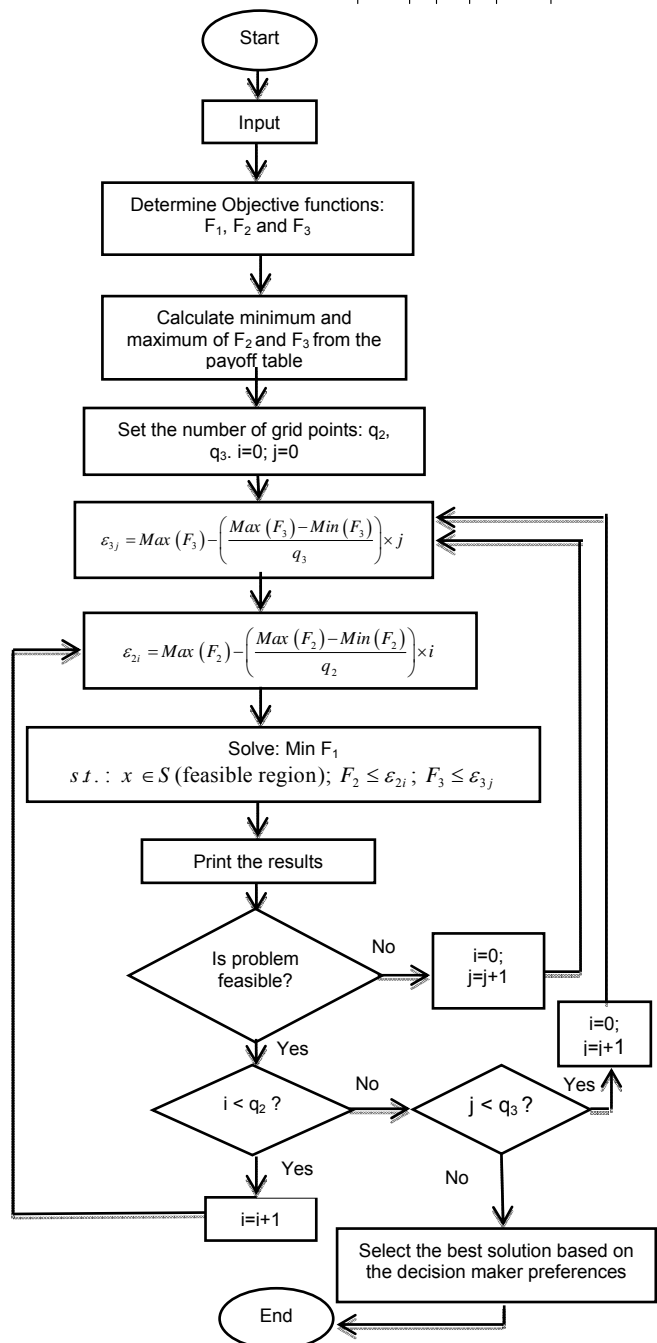


Fig.1. Flowchart of the proposed method

Equation (5) shows the objective functions (i.e. Investment Cost (IC), Operating Cost (OC) and Power Losses (PL), respectively). Equations (6) represent the active power balance for both existing and candidate buses. Equations (7) represent the reactive power balance for both existing and candidate buses. Constraints (8) and (9) indicate the active power flows from existing and candidate lines, respectively. Constraints (10) and (11) indicate the reactive power flows from existing and candidate lines, respectively. Superscript index 0 is used to denote the existing lines. Eqs. (12) and (13) are used in order to calculate the active power losses in line ij for existing and candidate lines, respectively. Active and reactive power generation limits of the generators are represented by Eqs.

(14) and (15). Transmission flow limits are shown by Eqs. (16) and (17) for the existing and candidate lines, respectively. The voltage constraints are shown by Eq. (18). Fig. 1 shows the flowchart of the proposed algorithm.

### Numerical Example

The proposed model has been successfully applied to the Garver six bus test system and also to an actual system as illustrated in case A and case B, respectively. The software used to solve the problem is DICOPT under GAMS (General Algebraic Modelling System) [13].

#### Case A: Garver System:

The Garver test system is depicted in Fig. 2. It has six buses, 15 candidate branches, and a total demand equal to 760 MW. Generators and loads data have been shown in Table 1. Reactive power demand in each bus is assumed to be 10% of the active power demand in that bus. We assume every generator submits its supply offer in the form of a linear function  $C_g P_g$ . Table 2 shows the existing and candidate lines data.

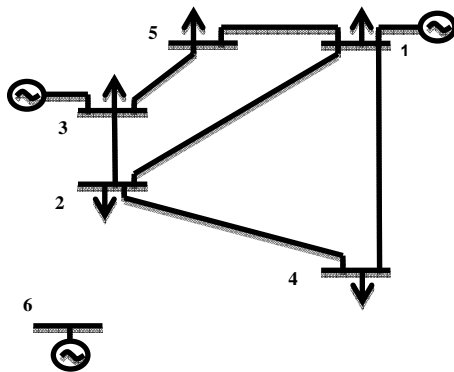


Fig.2. Garver's 6-bus test system

Table 1. Generators and Loads Data

Bus	Generators		Demand	
	Offer coefficients	-		
	$c_g$	$P_G^{\max}$	$P_D$ [MW]	$Q_D$ [MVar]
1	10	150	80	8
2	-	-	240	24
3	20	360	40	4
4	-	-	160	16
5	-	-	240	24
6	30	600	-	-

Table 2. Lines Data

Lines (L)	From	To	Capacity (MW)	Length	Resistance (p.u.)	Reactance (p.u.)	Investment Cost (\$10 <sup>6</sup> US)
EL <sub>2</sub>	1	4	80	60	0.15	0.60	-
EL <sub>3</sub>	1	5	100	20	0.05	0.20	-
EL <sub>4</sub>	2	3	100	20	0.05	0.20	-
EL <sub>5</sub>	2	4	100	40	0.10	0.40	-
EL <sub>6</sub>	3	5	100	20	0.05	0.20	-
CL <sub>1</sub>	1	2	100	40	0.10	0.40	40
CL <sub>2</sub>	1	3	100	38	0.09	0.38	38
CL <sub>3</sub>	1	4	80	60	0.15	0.60	60
CL <sub>4</sub>	1	5	100	20	0.05	0.20	20
CL <sub>5</sub>	1	6	70	68	0.17	0.68	68
CL <sub>6</sub>	2	3	100	20	0.05	0.20	20

CL <sub>7</sub>	2	4	100	40	0.10	0.40	40
CL <sub>8</sub>	2	5	100	31	0.08	0.31	31
CL <sub>9</sub>	2	6	100	30	0.08	0.30	30
CL <sub>10</sub>	3	4	82	59	0.15	0.59	59
CL <sub>11</sub>	3	5	100	20	0.05	0.20	20
CL <sub>12</sub>	3	6	100	48	0.12	0.48	48
CL <sub>13</sub>	4	5	75	63	0.16	0.63	63
CL <sub>14</sub>	4	6	100	30	0.08	0.30	30
CL <sub>15</sub>	5	6	78	61	0.15	0.61	61

The problem has 1762 single equations, 1189 single variables and 15 discrete variables. We have considered  $q_2=q_3=6$ , therefore the total 49 sub-problems must be solved in which the execution time to solve each sub-problem by DICOPT varies from 0.047 to 0.124 seconds. Table 3 is the obtained payoff table. Among all 49 sub-problems, 12 sub-problems are infeasible. Therefore the remaining 37 sub-problems have integer solution or are locally optimal. Please note that only the unique solutions (4 solutions) have been shown in Table 4.

Table 3. "Payoff table" for case A

	IC [\$]	OC [\$]	PL [MW]
IC [\$]	$3.96 \times 10^8$	$1.508663 \times 10^8$	34.073
OC [\$]	$6.28 \times 10^8$	$1.503352 \times 10^8$	32.052
PL [MW]	$6.28 \times 10^8$	$1.503352 \times 10^8$	32.052

Table 4. Objective functions values in each sub-problem for case A

	IC [\$]	OC [\$]	PL [MW]
1	$3.96 \times 10^8$	$1.5087 \times 10^8$	34.073
2	$4.54 \times 10^8$	$1.5050 \times 10^8$	32.683
3	$5.34 \times 10^8$	$1.5036 \times 10^8$	32.138
4	$6.28 \times 10^8$	$1.5034 \times 10^8$	32.052

For IC equals  $3.96 \times 10^8$  the candidate lines 3, 5, 6, 9, 10, 11, 12, 14 and 15 should be built. For IC equals  $4.54 \times 10^8$  all candidate lines but candidate lines 1, 7, 8 and 13 should be built. Also for IC equals  $5.34 \times 10^8$  all candidate lines except candidate lines 8 and 13 should be built. For IC equals  $6.28 \times 10^8$  all candidate lines should be built.

#### Case B: northeastern part of Iranian national 400-kV transmission grid:

Fig. 3 shows the simplified actual power system of northeastern part of Iranian national 400-kV transmission grid. The connection of the system to the Iranian main grid at Aliabad bus is considered as a positive load and the connection to the neighboring country, Turkmenistan grid, is considered as a negative load. As it can be seen a new power plant at bus Shirvan and a new load at bus Kashmar will be added to the network in the planning horizon. The candidate lines are represented in Fig. 3 by dashed lines. Table 5 shows the lines data.

Table 5. Lines (Existing and Candidate) Data

	Capacity (MVA)	Length (km)	Investment Cost (\$/MVA-km)
L <sub>1</sub>	800	183	N.A.
L <sub>2</sub>	800	180	200
L <sub>3</sub>	800	175	200
L <sub>4</sub>	1100	110	N.A.
L <sub>5</sub>	800	240	N.A.
L <sub>6</sub>	1100	44	N.A.
L <sub>7</sub>	800	181	N.A.
L <sub>8</sub>	1200	132	N.A.
L <sub>9</sub>	1100	80	200
L <sub>10</sub>	1100	110	N.A.
L <sub>11</sub>	800	170	N.A.
L <sub>12</sub>	1200	90	N.A.
L <sub>13</sub>	1100	120	200
L <sub>14</sub>	1200	60	200

L <sub>15</sub>	800	198	200
L <sub>16</sub>	1100	105	200
L <sub>17</sub>	800	230	200
L <sub>18,19</sub>	800	350	N.A.
L <sub>20,21</sub>	800	265	N.A.

Table 6 shows the generators data (coefficient of GENCO's bid and maximum active power) and loads data (active and reactive power) for the end of the planning horizon. A linear offer function for GENCOs in the form of  $C_{1g}P_{G,g}+C_{0g}$  has been considered. Upper and lower reactive power generation limit of power plant is assumed to be 50% and -40% of the upper active power generation limit. Tous bus is selected to be reference bus.

The problem has 3960 single equations, 2638 single variables and 8 discrete variables. We have considered  $q_2=q_3=6$ , therefore the total 49 sub-problems must be solved in which the execution time to solve each sub-problem by DICOPT varies from 0.1 to 1.1 seconds. Among all these sub-problems, 10 sub-problems are infeasible.

Table 7 is the obtained payoff table. Payoff table clearly shows the conflicts among the considered objectives and demonstrates that a multi-objective approach is essential in the power system planning. Multiplying the objectives by a number will multiply its values in payoff table in the same number meaning that the weighting factors for objectives have no impact in the results and therefore we will face no scaling problem in the optimization process.

Table 6. Generators and Loads Data

Bus	Generators			Demand	
	Offer coefficients		-	P <sub>D</sub> [MW]	Q <sub>D</sub> [MVar]
	c <sub>1g</sub>	c <sub>0g</sub>	P <sub>G</sub> <sup>max</sup>		
Tous	0.0113	12.14	1650	506	100
Torbat	-	-	-	810	162
Dolat	-	-	-	823	165
Ghaen	0.0113	15.14	800	434	87
Shadmehr	-	-	-	1250	250
Neyshabour	0.0222	17.92	1000	531	106
Sabzevar	-	-	-	530	106
Esfarayen	0.03	20.02	1200	362	72
Shirvan	0.01	12.44	1500	-	-
Kashmar	-	-	-	520	104
Aliabad	-	-	-	300	60
Turkmenistan	-	-	-	-300	-60

Table 8 shows only the obtained unique solutions for the remaining sub-problems that have integer solutions.

Table 7. "Payoff table" for case B

	IC [\$]	OC [\$]	PL [MW]
IC [\$]	9.7980×10 <sup>8</sup>	1503810.04	154.76
OC [\$]	1.7878×10 <sup>8</sup>	1498384.44	134.12
PL [MW]	1.7878×10 <sup>8</sup>	1541666.41	128.43

Table 8. Objective functions values in each sub-problem for case B

	IC [\$]	OC [\$]	PL [MW]
1	9.7980×10 <sup>8</sup>	1.5038×10 <sup>8</sup>	154.760
2	1.7878×10 <sup>8</sup>	1.4984×10 <sup>8</sup>	134.115
3	9.7980×10 <sup>8</sup>	1.5417×10 <sup>8</sup>	150.114
4	1.1558×10 <sup>8</sup>	1.5015×10 <sup>8</sup>	146.093
5	1.1558×10 <sup>8</sup>	1.5272×10 <sup>8</sup>	142.806
6	1.1558×10 <sup>8</sup>	1.5128×10 <sup>8</sup>	144.587
7	1.1558×10 <sup>8</sup>	1.5056×10 <sup>8</sup>	145.539
8	1.1558×10 <sup>8</sup>	1.5417×10 <sup>8</sup>	141.192
9	1.1558×10 <sup>8</sup>	1.5023×10 <sup>8</sup>	145.984
10	1.1558×10 <sup>8</sup>	1.5200×10 <sup>8</sup>	143.675

11	1.1558×10 <sup>8</sup>	1.5056×10 <sup>8</sup>	145.539
12	1.4198×10 <sup>8</sup>	1.4986×10 <sup>8</sup>	134.840
13	1.4198×10 <sup>8</sup>	1.5345×10 <sup>8</sup>	130.090
14	1.4198×10 <sup>8</sup>	1.5128×10 <sup>8</sup>	132.836
15	1.4198×10 <sup>8</sup>	1.5056×10 <sup>8</sup>	133.832
16	1.4198×10 <sup>8</sup>	1.5129×10 <sup>8</sup>	132.820
17	1.7878×10 <sup>8</sup>	1.5128×10 <sup>8</sup>	132.060
18	1.7878×10 <sup>8</sup>	1.5417×10 <sup>8</sup>	128.432

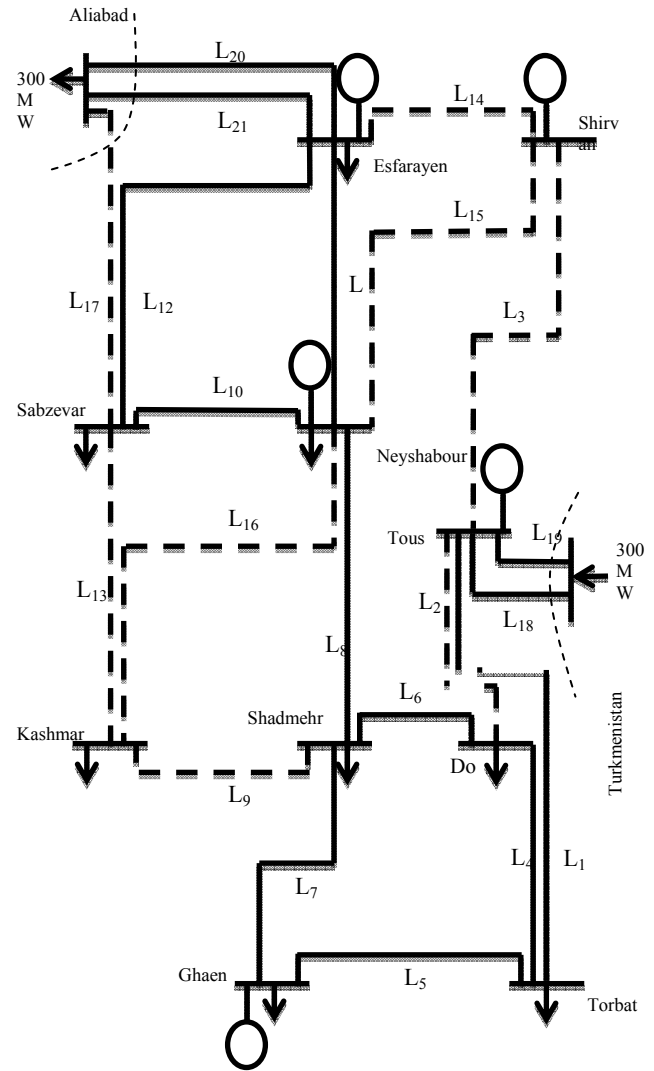


Fig.3. Northeastern part of Iranian national 400-kV transmission network

For IC equals 9.7980×10<sup>8</sup> the candidate lines 2, 14, 15 and 16 should be built. For IC equals 1.7878×10<sup>8</sup> all candidate lines but candidate line 3 should be built. Also for IC equals 1.1558×10<sup>8</sup> candidate lines 2, 14, 9, 15 and 16 should be built while for IC equals 1.4198×10<sup>8</sup> candidate lines 2, 9, 13, 14, 15 and 16 should be built. Also as it can be seen from the payoff table the power loss when OC is the main objective, is less than when IC is the main objective. This can be justified as follow: the more losses in the system means the more power should be produced by generators in order to meet the demand that will lead to more production cost.

### Conclusion

In this paper a new expansion planning model for transmission based on AC-OPF was provided and applied to the Garver's six bus test system and to an actual power system of Iranian national 400-kV transmission grid. A new

revisited formulation based on multi-objective optimization was presented and the augmented  $\epsilon$ -constraint method was used in order to solve the formulated mathematical multi-objective programming (MMP). Creating only Pareto optimal solutions and capability of producing all of the Pareto solutions are some of the advantages of the augmented  $\epsilon$ -constraint method. Therefore the proposed method provides more flexibility for the planner of the transmission in order to select the best solution among the non-dominated solutions obtained by AUGMECON method. The results of the case study were shown and analyzed.

#### NOMENCLATURE:

The following notation is used throughout the paper

Indices:

$g$  Index of generators.  
 $i, j$  Indices of buses.  
 $l$  Index of lines.

Sets:

$B$  Set of all buses.  
 $CL$  Set of all candidate lines.  
 $EL$  Set of all existing lines.  
 $G$  Set of all generators.

Constants:

$AD_{L,l}^{\max}$  Maximum apparent power flow of line  $l$ .  
 $C_{1g}, C_{0g}$  Bid coefficients of generator  $g$ .  
 $IC_l$  Investment cost of candidate line  $l$ .  
 $nc$  Number of candidate lines.  
 $ng$  Number of generators.  
 $P_{D,i}$  Active power demand at bus  $i$ .  
 $P_{G,g}^{\min}$  Minimum active power of generator  $g$ .  
 $P_{G,g}^{\max}$  Maximum active power of generator  $g$ .  
 $Q_{D,i}$  Reactive power demand at bus  $i$ .  
 $Q_{G,g}^{\min}$  Minimum reactive power of generator  $g$ .  
 $Q_{G,g}^{\max}$  Maximum reactive power of generator  $g$ .  
 $|V_i^{\min}|$  Minimum voltage magnitude at bus  $i$ .  
 $|V_i^{\max}|$  Maximum voltage magnitude at bus  $i$ .  
 $G_{ij}^0, B_{ij}^0$  Conductance and Susceptance of line  $ij$  for existing and candidate lines, respectively.  
 $G_{ij}, B_{ij}$  Conductance and Susceptance of line  $ij$  for existing and candidate lines, respectively.  
 $\theta_{ref}$  Voltage phase for the slack bus ( $\theta_{ref} = 0$ ).

Variables:

$IC$  Investment cost.  
 $OC$  Operation cost.  
 $PL$  Power losses.  
 $P_{G,g}$  Active power of generator  $g$ .  
 $P_{L_{i \rightarrow j}^0}^0$  Active power flow of existing line  $l$  from bus  $i$  to bus  $j$ .  
 $P_{L_{i \rightarrow j}^l}$  Active power flow of candidate line  $l$  from bus  $i$  to bus  $j$ .  
 $P_{Loss,l}^0$  Active power loss in existing line  $l$ .  
 $P_{Loss,l}$  Active power loss in candidate line  $l$ .  
 $Q_{G,g}$  Reactive power of generator  $g$ .  
 $Q_{L_{i \rightarrow j}^0}^0$  Reactive power flow of existing line  $l$  from bus  $i$  to bus  $j$ .  
 $Q_{L_{i \rightarrow j}^l}$  Reactive power flow of candidate line  $l$  from bus  $i$  to bus  $j$ .  
 $u_l$  Binary variable related to the candidate lines: equals 1 if candidate line  $l$  is constructed, 0 otherwise.  
 $|V_i|$  Voltage magnitude at bus  $i$ .  
 $\theta_i$  Voltage angle at bus  $i$ .

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