Bitao ZHANG, Youguo PI

Colleague of Automation Science and Engineering, South China University of Technology, Guangzhou, P.R. China

Design of Fractional Order Sliding Mode Controller Based on Parameters Tuning

***Abstract**. For dealing with the chattering phenomenon existing in conventional first order sliding mode controller, in this paper, a fractional order sliding mode controller (FOSMC) is proposed. Based on the advantage of fractional order differential and integral calculus, the sliding mode surface is designed with fractional order differential but not integral order. Under the stable performance of system, the parameters of sliding mode surface are tuned by given crossover frequency and phase margin. Furthermore, the soft-switching gain is obtained used fuzzy logic inference system. Simulations and experiments demonstrate that the proposed fractional order sliding mode controller not only achieve high control performance, but also is robust with regard to external disturbance.*

***Streszczenie.** W artykule zaproponowano ułamkowy sterownik ślizgowy FOSMC. Przy ustalonych parametrach systemu powierzchnia ślizgowa jest dobierana przez uwzględnianie częstotliwości i marginesu fazy. Przełączane wzmocnienie jest realizowane po zastosowaniu układu logiki rozmytej. Układ jest bardziej odporny na zewnętrzne zakłócenia. (**Projekt sterownika ślizgowego ułamkowego rzędu bazujący na dobieraniu parametrów**)*

Keywords: Fractional order; sliding mode control; chattering; parameter tuning
Słowa kluczowe: sterowanie ślizgowe, rząd ułamkowy.

Introduction

Due to the feature of high accuracy, simplicity and robustness, the siding-mode control (SMC) is widely and successfully applied in practice. However, the chattering phenomenon of the first-order sliding-mode control (FOSMC), originated from the interaction between parasitic dynamics and high-frequency switching control, limits the implement of SMC in industry required high precision [1]. Three main approaches to counteract the chattering phenomenon in SMC systems were proposed in the last two decades: use continuous approximation of the relay or introduce bounder layer near the sliding surface[2], use an asymptotic state-observer to confine chattering in the observer dynamics bypassing the plant[3], use higher-order or second-order sliding mode control (SOSMC)algorithms[4]. However, the main drawbacks of the continuous approximations and of the observed-based approach are the deterioration of accuracy and system robustness, respectively. Meanwhile, even the SOSMC algorithms suffer from chattering if parasitic dynamics are present increasing the system relative degree [5]. Furthermore, it is difficult to design the control law to guarantee the convergence properties of system and short of systemic design approach for higher-order or second-order sliding mode controller [6-8].

Using the differentiation and integration of fractional order or non-integer order in systems control is gaining more and more interests from the systems control community. Due to adding the extra degree of freedom, several fractional order controllers can achieve better control performance than integral order controller [9-12].

It is the motivation of this study to counteract the chattering phenomenon and maintain the robustness advantages of the SMC by used the advantage feature of the fractional order differentiation and integration. In this paper, a fractional order sliding mode surface is designed. The parameters of the sliding manifold are tuned by given gain crossover frequency and phase margin. Furthermore, the switching gain is obtained by used fuzzy logic inference system. The major contributions of this paper are that the FOSMC tuning method proposed is simple, practical, systematic, and can achieve favorable dynamic performance as well as robustness.

The remainder of this paper is organized as follows. First, the design of fractional order sliding mode controller is introduced in detail. Next, analyze the performance of fractional order sliding mode control system and give the tuning methods of parameters for fractional order sliding mode controller. Then, some simulations and experiments are carried out to demonstrate the effectiveness of the proposed method on a PMSM servo drive plant. Finally, the conclusions have been draw.

Design of Fractional order Sliding Mode Controller

In this study, the design method of FOSMC is focused on the following typical plant.

(1) $$\dot{x} = Ax + Bu$$

where $u, x \in R^n, A \in R^{n\times n}, B \in R^{n\times 1}$.

The control objective of the proposed control scheme is to get the state $x(t)$ to track the specific states $x^*(t)$. It is well known that the crucial of SMC design are the construction of the sliding surface $s(x), s(x) \in R$ and design of control law.

Define of fractional order sliding manifold

In this study, a fractional order sliding mode surface($s(x)$) is defined as:

(2) $$S = {}_0D_t^r x + Cx$$

Where $C \in R^{1\times n}$ and ${}_aD_t^r(.)$ is a fractional order fundamental operators defined by[13]

(3) $${}_aD_t^r f(t) = \begin{cases} \dfrac{1}{\Gamma(n-r)}\displaystyle\int_a^t \dfrac{f^{(n)}(\tau)}{(t-\tau)^{r+1-n}}d\tau, & n-1<r<n \\ \dfrac{d^n}{dt^n} f(t), & r=n \end{cases}$$

where $\Gamma(z)$ is the Gamma function with the definition below

(4) $$\Gamma(z) = \int_0^\infty t^{z-1}e^{-t}dt$$

Design of control law

In this paper, the following control law is adopted

(5) $$\begin{aligned} \dot{S} &= -Ksign(S) \\ K &= diag(k_i)(k_i > 0) \end{aligned}$$

where sign (.) is the sign function defined as

(6) $$sign(S) = \begin{cases} 1, & S>0 \\ 0, & S=0 \\ -1, & S<0 \end{cases}$$

Take the (1) and (2) into (5), yield

(7)
$$\dot{S} = {}_0D_t^{r+1}x + C\dot{x} = {}_0D_t^{r+1}x + C(Ax+Bu) = -Ksign(S)$$

Then one can obtain the following control law

(8)
$$u = -(B)^{-1}[(C)^{-1}{}_0D_t^{r+1}x + Ax + (C)^{-1}Ksign(S)]$$
$$A = diag(a_i), B = diag(b_i), C = diag(c_i)$$

Performance analysis

Convergence performance of the proposed controller ensures the proposed fractional sliding surface is existent. It means that wherever the initial states are, control law drives the initial states converge to sliding manifold.

Without loss of generality, assume initial state is not on the sliding manifold ($S_i(t_0) \neq 0, S_i \in S$).

When $S_i(t_0) > 0$, according to (5), $\dot{S}_i = -k_i$ holds. It means

(9)
$$S_i(t) - S_i(t_0) = -k_i(t - t_0)$$

Then when the following condition

(10)
$$t = t_0 + k_i^{-1}S_i(t_0)$$

is satisfied, $S_i(t)$=0 is obtained.

Similarly, When $S_i(t_0) < 0$, $\dot{S}_i = k_i$ holds. It means

(11)
$$S_i(t) - S_i(t_0) = k_i(t - t_0)$$

Then when the following condition

(12)
$$t = t_0 - k_i^{-1}S_i(t_0)$$

is satisfied, $S_i(t)$=0 is obtained.

Thus, when the following condition

(13)
$$t \geq t_0 + \max\left\{k_i^{-1}\left|S_i(t_0)\right|\right\}$$

is satisfied, the system converges to switching manifold at any initial states.

Then, when the sliding mode occurs (S=0), the proposed FOSMC satisfies the following condition

(14)
$${}_0D_t^r x = -Cx$$

The solution of (14) is[13]

(15)
$$x(t) = E_{r,1}(t)x_0$$

where $E_{r,1}(t) = \sum_{k=0}^{\infty}\frac{A^k t^{rk}}{\Gamma(rk+1)}$ $\quad 0 < r \leq 1$ is states transfer function.

Specially, the states transfer function of integer order system, namely r=1, is

(16)
$$E_{r,1}(t) = E_{1,1}(t) = \sum_{k=0}^{\infty}\frac{A^k t^k}{\Gamma(k+1)} = \sum_{k=0}^{\infty}\frac{A^k t^k}{k!} = \exp(At)$$

Thus, the decaying type of fractional order system is different from integer order system.

Parameters tuning

Parameters tuning for fractional order sliding manifold

According to control law (8), when the system reaches sliding motion(S=0), control law (8) can be represented

(17)
$$u = -(b_i)^{-1}[(c_i)^{-1}{}_0D_t^{r+1}x + a_i x]$$

And the transfer fuction of (17) is

(18)
$$C(s) = k_c + k_d s^{r+1}$$
$$k_c = -a_i / b_i, k_d = -1/b_i c_i$$

One can represent the control law in frequency domain as

(19)
$$C(j\omega) = (k_c - k_d\omega^{r+1}\sin\frac{r\pi}{2}) + jk_d\omega^{r+1}\cos\frac{r\pi}{2}$$

Thus, one has the frequency characteristics

(20)
$$\mathrm{Arg}[C(j\omega)] = \tan^{-1}\frac{k_d\omega^{r+1}\cos\frac{r\pi}{2}}{k_c - k_d\omega^{r+1}\sin\frac{r\pi}{2}}$$

(21)
$$\left|C(j\omega)\right| = \sqrt{(k_d\omega^{r+1}\cos\frac{r\pi}{2})^2 + (k_c - k_d\omega^{r+1}\sin\frac{r\pi}{2})^2}$$

In this study, consider the system(1) as second-type plant

(22)
$$P(s) = \frac{k}{s(Ts+1)}$$
$$k = -b_i / a_i, T = -1/a_i$$

And the frequency charateristics of plant are as follow

(23)
$$\mathrm{Arg}[P(j\omega)] = -\tan^{-1}(T\omega) - \frac{\pi}{2}$$

(24)
$$\left|P(j\omega)\right| = \frac{k}{\omega\sqrt{(T\omega)^2+1}}$$

Thus, the frequency charateristics of open-loop transfer function $G(j\omega) = C(j\omega)P(j\omega)$ are

(25)
$$\left|G(j\omega)\right| = \frac{k\sqrt{(k_d\omega^{r+1}\cos\frac{r\pi}{2})^2 + (k_c - k_d\omega^{r+1}\sin\frac{r\pi}{2})^2}}{\omega\sqrt{(T\omega)^2+1}}$$

(26)
$$\mathrm{Arg}[G(j\omega)] = \tan^{-1}\frac{k_d\omega^{r+1}\cos\frac{r\pi}{2}}{k_c - k_d\omega^{r+1}\sin\frac{r\pi}{2}} - \tan^{-1}(\omega T) - \frac{\pi}{2}$$

Then one can obtain the parameters by given crossover frequency ω_c and phase margin γ .

According to (25), one has

(27)
$$(\omega_c^{r+1})^2 k_d^2 - 2k_c\omega_c^{r+1}\sin\frac{r\pi}{2}k_d + k_c^2 - \frac{(T\omega_c)^2+1}{(k/\omega_c)^2} = 0$$

Then one can deduces

(28)
$$k_d = \frac{2k_c\omega_c^{r+1}\sin\frac{r\pi}{2} \pm \sqrt{(2k_c\omega_c^{r+1}\sin\frac{r\pi}{2})^2 - 4(\omega_c^{r+1})^2(k_c^2 - \frac{\omega_c^2((\omega_c T)^2+1)}{k^2})}}{2(\omega_c^{r+1})^2}$$

According to (26), one has

(29)
$$\mathrm{Arg}[G(j\omega_c)] = \tan^{-1}\frac{k_d\omega_c^{r+1}\cos\frac{r\pi}{2}}{k_c - k_d\omega_c^{r+1}\sin\frac{r\pi}{2}} - \tan^{-1}(\omega_c T) = \gamma + \frac{\pi}{2}$$

Then one can infers

(30)
$$k_d = \frac{k_c\tan[\gamma + \frac{\pi}{2} + \tan^{-1}(\omega_c T)]}{\omega_c^{r+1}[\cos\frac{r\pi}{2} + \sin\frac{r\pi}{2}\tan(\gamma + \frac{\pi}{2} + \tan^{-1}(\omega_c T))]}$$

Clearly, one can solve Eq.(28) and (30) to get r and k_d.

Switching gain adjusting based on fuzzy logic inference algorithm

The inputs of fuzzy logic inference system are fractional order sliding surface($s(x)$) and its derivative($\dot{s}(x)$). The output of fuzzy system is the gain of switching (dk_i).The membership functions of the linguistic terms assigned to the inputs and output are as follow

$$s = \{\mathrm{NB, NM, NS, ZE, PS, PM, PB}\}$$
$$\dot{s} = \{\mathrm{NB, NM, NS, ZE, PS, PM, PB}\}$$
$$dk_i = \{\mathrm{NB, NM, NS, ZE, PS, PM, PB}\}$$

And the universes of discourses are

$$s = \{-3,-2,-1,0,1,2,3\}$$
$$\dot{s} = \{-3,-2,-1,0,1,2,3\}$$
$$dk_i = \{-3,-2,-1,0,1,2,3\}$$

The membership functions for the inputs and output are triangle type shown in Fig. 1.

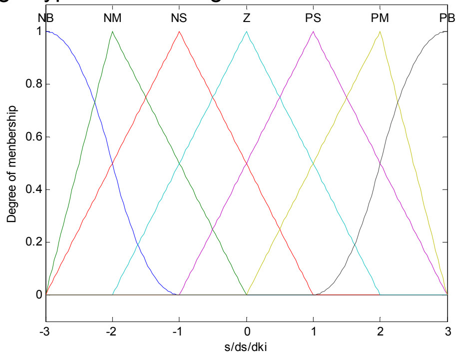

Fig. 1. Fuzzy sets assigned to inputs *s*, *ds* and output dk_i

The fuzzy rules are shown in table.1 and the output of fuzzy inference system is computed by a mechanism of If-Then rules as following form:

IF x_i is $F(x_i)$ and $\dot{x}_i$ is $F(\dot{x}_i)$ **THEN** y_i is $F(y_i)$

Table 1 Rules-base of the fuzzy inference system

du		s						
		PB	PM	PS	ZE	NS	NM	NB
ds	NB	ZE	NS	NM	NB	NB	NB	NB
	NM	PS	ZE	NS	NM	NB	NB	NB
	NS	PM	PS	ZE	NS	NM	NB	NB
	ZE	PB	PM	PS	ZE	NS	NM	NB
	PS	PB	PB	PM	PS	ZE	NS	NM
	PM	PB	PB	PB	PM	PS	ZE	NS
	PB	PB	PB	PB	PB	PM	PS	ZE

Here, a center average defuzzification, a Mamdani implication in the rule base and a product inference engine are used in designing the defuzzification module. Fuzzy controller output(dk_i) can be calculated by the center of area defuzzification as:

(31) $$dk_i = \frac{\sum_{j=1}^{n} k_j u_j(s,\dot{s})}{\sum_{j=1}^{n} u_j(s,\dot{s})}$$

where k_j is the vector containing the output fuzzy center of the membership function of output (dk_i). μ_j represents the membership value of the output to output fuzzy set *j*.

Simulations and experiments

Simulations and results discussion

A step response and pulse response are carried out using proposed fractional order sliding manifold on a second class of system, respectively. Simulates are based on"matlab7.1".

Considering the servo control system modeled by

(32) $$\dot{x}_1 = x_2$$ $$\dot{x}_2 = ax_1 + bu$$

where $a = -20, b = 25$.

The gain of crossover frequency and phase margin are set as $\omega_c = 60, \gamma = 3\frac{\pi}{2}$, respectively. According to the tuning methods proposed in this paper, one can obtain k_d=-1.7e-3 and r =1.8002.

The unit step responses and pulse response of the proposed fractional order control scheme are shown in Fig. 2 and Fig.3 respectively. From Fig.2, it can be seen that, the rise time of FOSMC is fast and there is almost no chattering on sliding motion. Furthermore, it shows from Fig.3 that FOSMC achieves favorable tracking performance.

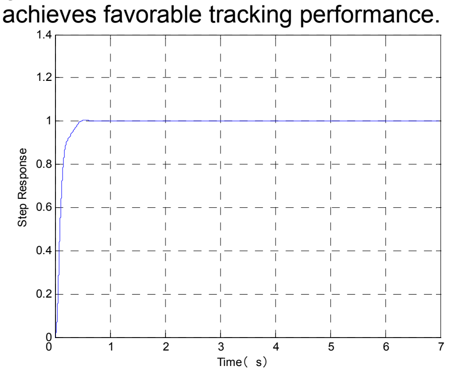

Fig. 2. The unit step response of fractional order sliding controller

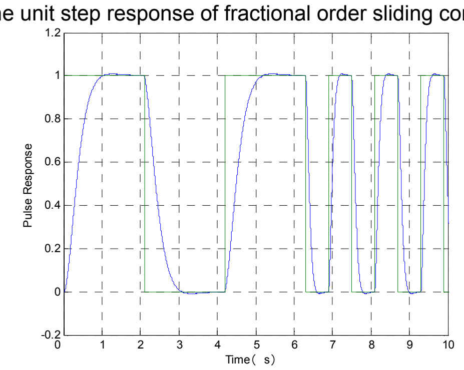

Fig.3. Pulse response of fractional order sliding controller

Experiment and results discussion

A comparison of control performance experiment between integer order SMC and fractional order SMC is designed to illustrate the effectiveness of the proposed control scheme. This experiment is based on the permanent magnet synchronous motor(PMSM) servo drive system. Then, the experiments of sine tracking performance and robustness rejecting the external load disturbance are provided. The experiments are carried out using the Ti Code Composer Studio (CCS) software in PC. The data is sampled, used this software, from encoder to control board, and then load these data into "matlab" package to analyze the results. And the control board is based on"TMSC320F2812" DSP. The configuration of the PMSM drive plant is shown in Fig.4.

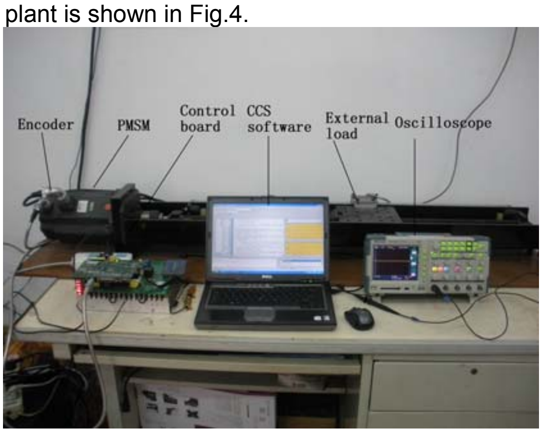

Fig.4. Plant of the PMSM servo drive

The experiments results of ramp response and sine response are shown in Fig.5 and Fig.6 respectively. From Fig.5, it can be seen that the stable state of ramp response based on FOSMC is smooth. But there are some chattering on integer order SMC. Moreover, the sine response shown in Fig.6 illustrates that the FOSMC achieves favorable tracking performance. The experiments results show that the proposed control scheme based on fractional calculus achieves high control performance.

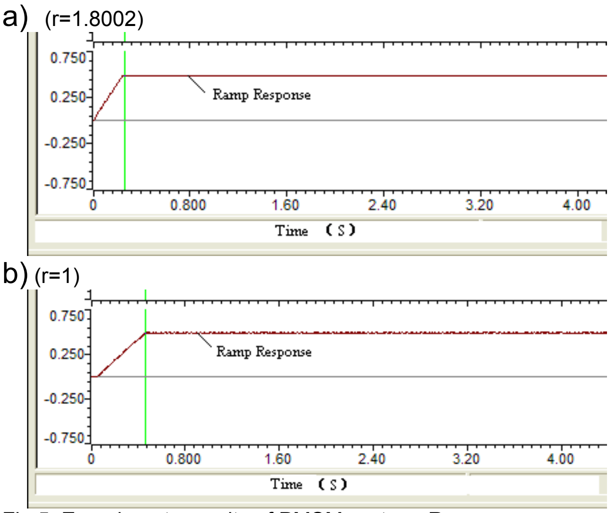

Fig.5. Experiments results of PMSM system. Ramp response: (a) fractional order (b) integer order

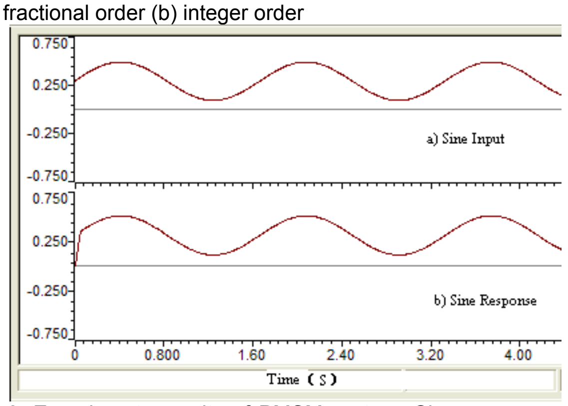

Fig.6. Experiments results of PMSM system. Sine response with FOSMC

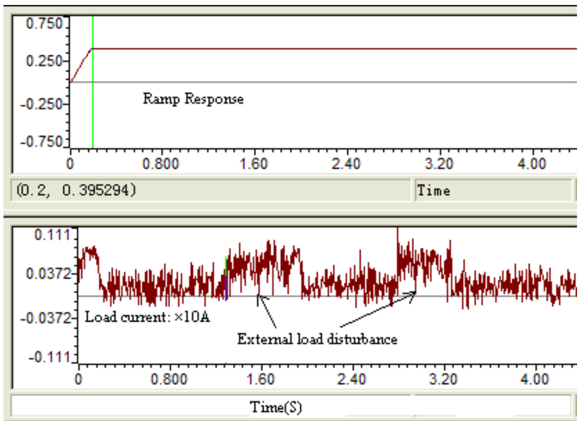

Fig.7. Experiment result. Robustness rejecting external load disturbance

The experiment result of robustness rejecting the external load disturbance is shown in Fig.7. When the system reaches sliding motion, external load disturbance is added at 1s and deleted at 2s. Then when the running time is 3s, the external load disturbance is added again. It can be seen that the ramp response maintains a constant value. It demonstrates the proposed control scheme is robust rejecting the external load disturbance.

Conclusions

A FOSMC scheme based on fuzzy logic inference algorithm has been proposed in this paper. The parameters tuning methods are simple and effective. The simulations and experiments results based on PMSM servo drive system show that the proposed FOSMC achieves better control performance compared with the conventional integer order SMC. And the proposed control scheme is robust rejecting the external disturbance.

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***Authors**: dr Bitao.Zhang ,Colleague of Automation Science and Engineering, South China University of Technology, Guangzhou, P.R. China, E-mail: z.bitao@mail.scut.edu.cn; prof. Youguo.Pi*