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The equivalent thermal network for the model of heat exchanger

Abstract. *The elements modelling heat transfer in the heat exchanger were investigated. The method of parametric identification revealed the oscillations in the STEP responses of these elements. The four-terminal network diagram corresponding to the modelling exchanger was prepared using a method of the equivalent thermal network (ETN) based on the electro-thermal analogy. The oscillations in the STEP responses indicate that ETN diagram of the heat exchanger should include a thermal inductivity element. The thermal inductivity idea correspond to potential and kinetic energy exchange during the heat exchange process which induces the oscillations.*

Streszczenie. *Zastosowanie metody identyfikacji parametrycznej do analizy dynamiki wymiany ciepła ujawniło występowanie oscylacji w przebiegach charakterystyk skokowych, wyznaczanych dla różnych elementów modelowanego wymiennika. Metodą zastępczej sieci cieplnej, opartą na analogii termoelektrycznej, opracowano odpowiadający mu schemat czwórnika elektrycznego. Oscylacje pojawiające się w charakterystykach skokowych wskazują na potrzebę wprowadzenia elementu indukcyjnego do schematu sieci cieplnej odwzorowującej wymiennik ciepła. Pojęcie indukcyjności cieplnej należy kojarzyć z wymianą energii potencjalnej i kinetycznej między elementami uczestniczącymi w procesie wymiany ciepła, co jest przyczyną występujących oscylacji.**(Zastępcza sieć cieplna dla modelowego układu wymiennika)***

Keywords: heat transfer, equivalent thermal network, thermal resistance, thermal capacity, thermal inductivity
Słowa kluczowe: wymiana ciepła, zastępcza sieć cieplna, rezystancja cieplna, pojemność cieplna, indukcyjność cieplna

Introduction

The heat transfer in elements modelling the heat exchanger was investigated. The method of parametric identification applied to analysis of these elements dynamic revealed the oscillation in STEP characteristics [1]. There were observed in a lot of analyzed experiments but not always. Theirs courses and parameters indicated the strong dumping and the short time of their appearance in comparison with the all time need to reach the stationary state

The experimental and analytical methods

The heat exchanger transient state analysis showed some ambiguities in their courses. It indicated that the more precise experiments in laboratory conditions should be carried out. The detailed methodology of this experiment has been described in [2]. The heat exchanger was treated as two bodies system: the active body, which transmitted heat in the exchanger input and the passive body receiving heat in the exchanger output. The heat transmission (without contact between both of the elements: active and passive) was realized through separating them medium. In these experiments it was an air. The kanthal or copper bar or spiral were investigated as the active elements. The copper bar was usually taken as the passive element, except the case in which the two copper spiral wound in parallel as a two-start thread (one active and the other passive) were investigated.

The results of experiments were first analyzed using the EXCEL packet [3] and then using the MATLAB packet and System Identification Toolbox [1]. The visualization of the heat transfer process was also carried out with usage of the schlieren [4]

The method of the electro-thermal analogy was used for analyzing and interpretation of the results. This method describes thermal and electrical phenomena corresponding each other, using the similarities between them. For example, the temperature corresponds to the electric potential, the difference between temperatures corresponds to the voltage (difference between potentials), the heat flux corresponds to the current and the heat resistance and the heat capacitance correspond to the resistance and capacitance [5]. The electrical diagram corresponds to heat element systems are built using this analogy. These diagrams have corresponding properties and dependencies between all elements to the original heat system. The heat system modeling as the electric diagram enables using the interpretation of the thermal and electrical phenomena corresponding ones to the others.

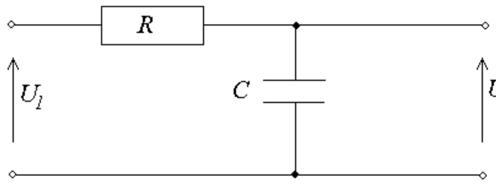

Fig. 1. The electric network diagram for the heat transfer process in the constant temperature body

Usually the equivalent diagram as the corresponding four–terminal network RC type is assumed for thermal systems (Fig. 1) [5, 6, 7]. It is possible for the simple thermal systems in which the temperature inside the body is constant. It denotes that the thermal conduction resistance R_λ inside the body is little in comparison to the thermal convection resistance R_α outside the body. The Biot coefficient N_B is introduced: $N_B = \dfrac{R_\lambda}{R_\alpha}$ and it is assumed that their value must not be greater than 0,1 if the diagram presented in Figure 1 can be used [8]. The homogenous body with the constant temperature T is then represented by the node of four – terminal network. When the Biot coefficient for the heat system is too great then the system must be divided into subsystems. Their number is determined from the condition that $N_B \leq$ 0,1 for all the subsystems. Then the diagram of such a complex system is developed into a form presented in Figure 2. The nodes, one by one, represent the subsystems with the temperature values jumped variable modeling the continuous temperature variable using the stepped approximation. For each subsystem described in this way the temperature is assumed as constant and equal to temperature of the corresponding node [8].

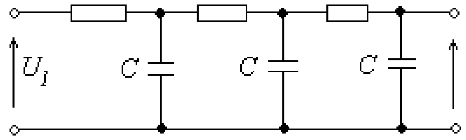

Fig. 2. The electric network diagram for the heat transfer process in the variable temperature body

The constituent elements of this network may be treated as: R – thermal resistance (the sum of conduction, convection and radiation) and C – thermal capacitance of each of subsystems

The method of equivalent thermal network (ETN)

The method of the equivalent thermal network (ETN) is used for the approximate presentation of three–dimensional distribution of temperature and propagation of heat flux. This approximation is the consequence of substitution of real thermal system of many non–homogenous bodies with parameters distributed spatially by the system of homogenous elements with concentrated parameters with one – direction heat transfer. The computations carry out with medium values of surfaces and volumes temperatures and medium of heat flux [6, 9, 10]. This thermal network consists of

- nodes, corresponding to homogenous bodies, in which the real system is divided
- arms including thermal resistances and capacitances, connecting the nodes
- point of heat losses flowing–out (e.g. surrounding)

Between the nodes i,j the heat flux P_{ij} flows. The energy may be emitted in the nodes (active nodes) or not (passive nodes).

The method of ETN is used for solving very different problems. Some of the scientists prepared the model of solar collectors [11], hybrid systems of renewable energy [6], solar installation for water heating [12], thermal computation of electrical machines [9, 10], unsteady magneto hydrodynamic flow [13], gas bubble dynamics [14], transient radiative transfer process between the thick walls [15], non – Newtonian fluid flows [16] and the others

The ETN for the heat exchanger

The real thermal system in the ETN method is substituted by a simplified system represented by the electric circuit. The same way is used in presentation of the diagram of long line with dissipated parameters as a four – terminal network with concentrated parameters (Fig. 4) [17]. The ETN diagram for the heat exchanger was proposed in [6] as the diagram of four–terminal network of T type (Fig. 3).

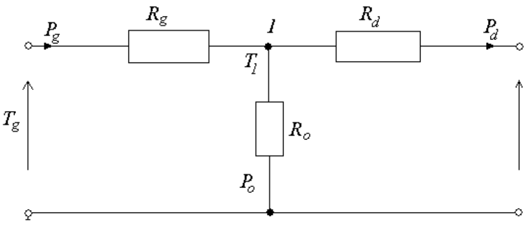

Fig. 3. The heat exchanger equivalent thermal network according to [6]

This diagram in the electro-thermal analogy corresponds to equivalent scheme of transformer. The primary side represents the input of the heat exchanger (with medium temperature of hot side T_g) and the secondary side represents the output from the heat exchanger (with medium temperature of cold side T_d). The heat exchanger concentrated parameters are assumed. But in the real heat exchanger the surface of heat exchange is maximized (dissipated in surface). Then the heat exchange is observed often in the long distances and it corresponds to the long line in the electricity.

There is analogy between the heat system in which the heat exchange performs in the considerable distance (as it takes place e.g. in the heat exchange) and the fragment of long line. The detailed description of this analogy is presented in [5] .

The fragment of long line is characterized by the parameters:
elementary: resistance $R_l = R_c / l$, capacitance $C_l = C_c / l$, inductance $L_l = L_c / l$ and conductance $G_l = G_c / l$, where l is the length of the line. The line is characterized by the total values of R, C, L, G. If the fragment of long line is assumed as the equivalent thermal network diagram for the heat exchanger then the separate parameters (except the inductivity) correspond to the analogous heat exchanger parameters in their elementary fragment.

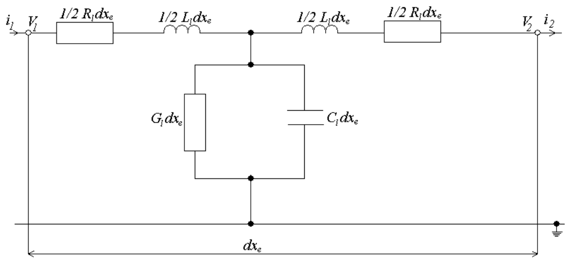

Figure 4. The long line fragment correspond to heat transfer in a long distance

If the line is homogenous in all their length then the difference between potentials V_1 and V_2 and the difference between values of currents i_1 and i_2 are described by dependencies:

$$V - \left(V - \frac{\partial V}{\partial x_e} dx_e \right) = i R_l dx_e + L_l dx_e \frac{\partial i}{\partial t_e} \tag{1}$$

$$i - \left(i - \frac{\partial i}{\partial x_e} dx_e \right) - G_l dx_e V - C_l dx_e \frac{\partial V}{\partial t_e} = 0 \tag{2}$$

After some transformations of both of the equation one can obtain:

$$\frac{\partial^2 V}{\partial x_e^2} = R_l \frac{\partial i}{\partial x_e} + L_l \frac{\partial^2 i}{\partial t_e \partial x_e} \tag{3}$$

$$\frac{\partial^2 i}{\partial x_e \partial t_e} = C_l \frac{\partial^2 V}{\partial t_e^2} + G_l \frac{\partial V}{\partial t_e} \tag{4}$$

The first range partial derivative of i (computing from equation (2)) and the second range partial derivative of i (computing from equation (4)) are then inserted in equation (3). For the line without conductance (G_l = 0) one can obtain:

$$\frac{\partial V}{\partial t_e} = \frac{1}{R_l C_l} \frac{\partial^2 V}{\partial x_e^2} - \frac{L_l}{R_l} \frac{\partial^2 V}{\partial t_e^2} \tag{5}$$

The velocity of propagation of the potential electric disturbance can be denoted by w_e and taking into consideration that for the long line

$$w_e = \frac{1}{\sqrt{L_l C_l}} \tag{6}$$

The transformed expression (5) can be presented in the form:

$$\frac{\partial V}{\partial t_e} = \frac{1}{R_l C_l} \frac{\partial^2 V}{\partial x_e^2} - \frac{1}{R_l C_l w_e^2} \frac{\partial^2 V}{\partial t_e^2} \tag{7}$$

The dependencies defining the resistances and capacitances:

$$R_l = \frac{R}{l} = \frac{1}{\gamma F_e},\ C_l = \frac{C_e}{l} = \frac{c_e \rho_e V_e}{l},\ V_e = F_e l$$

where γ – conductivity [$\Omega^{-1} \cdot m^{-1}$]; V_e – volume of line [m^3] and if one denotes

(8) $$\frac{\gamma}{c_e \rho_e} = a_e$$

Than the equation (7) can be expressed in the form:

(9) $$\frac{\partial V}{\partial t_e} = a_e \frac{\partial^2 V}{\partial x_e^2} - \frac{a_e}{w_e^2} \frac{\partial^2 V}{\partial t_e^2}$$

This equation can be generalized for the three–dimensional (3-D) bodies:

(10) $$\frac{\partial V}{\partial t_e} = a_e \nabla^2 V - \frac{a_e}{w_e^2} \frac{\partial^2 V}{\partial t_e^2}$$

If in the presented equations the inductance is assumed as equal to zero, L_l = 0 then one can obtain the equations:

(11) $$\frac{\partial V}{\partial t_e} = \frac{1}{R_l C_l} \frac{\partial^2 V}{\partial x_e^2}$$

(12) $$\frac{\partial V}{\partial t_e} = a_e \frac{\partial^2 V}{\partial x_e^2}$$

(13) $$\frac{\partial V}{\partial t_e} = a_e \nabla^2 V$$

The equation (13) jest is analogical for the Fourier – Kirchhoff equation. But the dependencies (10) for the 3–D body and (9) for one-dimensional (1–D) body have not equivalents in the thermo – kinetics equations. The Fourier – Kirchhoff equation described the heat exchange in the body was based on the assumption that $\mathbf{q} = -\lambda \nabla T$ that is equal to assume the infinity value of heat propagation velocity. That is not correct assumption, because this velocity is considerable but not as high as the velocity of the electric propagation. This assumption is equivalent to assume the zero value of inductivity in electric disturbance propagation. But in real conditions the heat flux must be lagged behind the temperature gradient alike the current in the line behind the electrical potential gradient. The correctly formulated expression for the heat flux density should include the component showing the density changing ratio. In origin it is equal to zero and increases in time. This expression has a form formulated by Vernotte [18]:

(14) $$\mathbf{q} = -\lambda \nabla T - \frac{a_t}{w_t^2} \frac{\partial \mathbf{q}}{\partial t_t}$$

Taking into account this added component in the equation of heat conduction, one can obtain, after a several transformation, the following equation for 3-D body for a solid body without internal heat sources:

(15) $$\frac{\partial T}{\partial t_t} = a_t \nabla^2 T - \frac{a_t}{w_t^2} \frac{\partial^2 T}{\partial t_t^2}$$

where $a_t = \lambda c_t^{-1} \rho_t^{-1}$ is the thermal diffusivity of the medium in which the thermal field disturbance is propagated. The dependence for 1–D body has a form:

(16) $$\frac{\partial T}{\partial t_t} = a_t \frac{\partial^2 T}{\partial x^2} - \frac{a_t}{w_t^2} \frac{\partial^2 T}{\partial t_t^2}$$

These equations (15) and (16) are analogical for the equations for the electric fields. The equation (15) is analogical for the (10) which describes the transient electric potential field in the surface with some conductivity, capacity and inductivity. The equation (16) represents analogical situation for the 1–D body

Thermal inductivity

If one makes an assumption about the infinite value of the heat disturbance propagation then the zero value of the inductivity in the electro-thermal analogy must be assumed. But the oscillations revealed in the STEP responses of the heat exchanger modeling elements dynamics indicate uniquely for the some special heat parameter existence. It causes the analogical phenomena in the thermal system as the inductance in the electric circuit.

The analysis of modelling elements was carried out using parametric identification and the STEP response [1]. In courses of them the oscillations revealed and it corresponded for phenomena observed in the electric circuit and in the mechanics, e.g. the spring vibration. In the case of the spring and in the LC circuit the alternate potential and kinetic energy transfer takes place (with dumping by resistance, if exists). By the maximum of the capacitor charge or the maximum of the spring pressure there is the maximum of the accumulated potential energy. Than the energy transfer causes their gradual transformation into kinetic energy of the coil or the spring. The kinetic energy reaches maximum and potential minimum and from this point the energy transfer starts to transform in the opposite direction. This process theoretically can be infinitely continued if the dumping do not exist.

The phenomenon of the oscillation in the thermal system was observed by the some others scientists. It takes place usually during the heat exchanger operation investigation [19, 20, 21, 22, 23] but also during the investigation of the complex systems of the heat exchangers [24, 25] and e.g. the room ventilation [26] and the resistance heating [27].

The phenomena revealing in the experiments lead to the conclusion that in the thermal transfer process there is also the mutual changing of the potential and kinetic energy but with strong dumping. In the ETN method the electric capacity C, being the potential energy store, corresponds to the thermal capacity. The oscillations occurring in the thermal processes have showed that the thermal inductivity corresponding to the kinetic energy of the thermal field is the notion advisable to introduce.

The existence of such a quantity was assumed in [28], but the experiments did not acknowledge it. These experiments were performed as a resistance heating of different elements in the oil. The analysis of its results had been carried out using EXCEL packet. The less advanced experiment methods and informatics techniques probably caused the less precise results of the investigation. The numerical methods used in [3] also do not result the observation of oscillation. The more so where if the MATLAB packet was used and in particular the System Identification Toolbox were given the methods of parametric identification and the possibility of the keen investigation of observed phenomena.

the thermal inductivity notion was also used in [29] and corresponded to the heat exchanger thermal inertia. It caused reducing the driving force in the heat exchanger and in order to correct computation of the heat exchange, requires using the corrective coefficient defined in [30]. In [29] the heat exchanger ETN was proposed with inductive thermal resistance which was presented using the coefficient enlarging the equivalent resistance.

In order to verify the hypothesis of a thermal inductivity existence that is the parameter causing the possibility of energy storage in the transient states, the research work was realized in the two-track way. On the one hand the best model describing the experiments was found and their transmittance $G(s)$ was computed. On the other hand the ETN was built using the physical parameters of examined elements in order to find the best approximation of the physical properties of the investigated system.

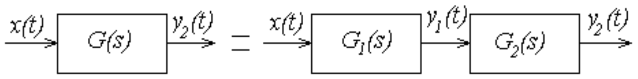

Fig. 5. The scheme of investigated object transmittance: $x(t)$ – heat flux; $y_1(t)$ – the active element temperature, $y_2(t)$ – the passive element temperature

The dynamics of the system heating was analyzed. First the active element from heat flux (the primary side of the heat exchanger) with input $x(t)$ (heat flux [W]) and output $y_1(t)$ (active element temperature [°C]) - transmittance of it was $G_1(s)$. Next the passive element from the active element (the secondary side of the heat exchanger) with input $y_1(t)$ and output $y_2(t)$ (passive element temperature [°C]) - transmittance of it was $G_2(s)$. In order to compute the transmittance $G(s)$ of the system it was divided into two subsystems with the transmittances $G_1(s)$ and $G_2(s)$ and presented as their connection in series. This system has one input $x(t)$ (heat flux [W]) and one output $y_2(t)$ (passive element temperature [°C]) (Fig. 5)

The parametric and non-parametric models from System Identification Toolbox were chosen for the best approximation of the experimental data. From different model structures (ARX, ARMAX, BJ, OE and Process Models, describing in [1]) the models OE (Output Error) structure was finally chosen. Their range was defined, maximizing the FIT parameter and minimizing the FPE parameter in order to find the best model approximating the experimental data. Next the STEP response for this model was examined. The creation of the IDPOLY models divides into two stages: first the discrete-time IDPOLY model is created and then (after choosing the way of transform discrete dates to continuous the *zoh* - zero order hold was chosen) the continuous-time IDPOLY model with transmittance $G(s)$ is created. For the first element of the system the voltage-current transmittance was described:

(17) $$G_1(s) = \frac{Y_1(s)}{X(s)} = \frac{U_2(s)}{I_1(s)}$$

and for the second element of the system the voltage-voltage transmittance was described:

(18) $$G_2(s) = \frac{Y_2(s)}{Y_1(s)} = \frac{U_3(s)}{U_2(s)}$$

The character of models was described from the form of polynomials of the numerator and of the denominator and on the course of the elements STEP response. The parts of models indicated on the existence of the oscillation but the other part was inertial. In addition the models indicated the existence of oscillation (having the complex, conjugate poles) had sometimes the form of STEP response indicated to inertial character. The STEP response for passive element heating, having oscillatory character is presented in Figure 6 and the STEP response for active element having inertial character is presented in Figure 7.

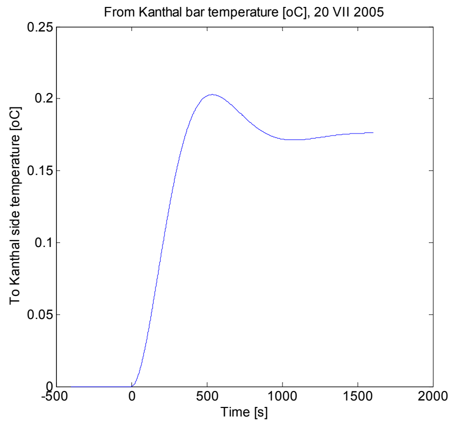

Fig. 6. The exemplified course of STEP response for the passive element heating (OE 131 model)

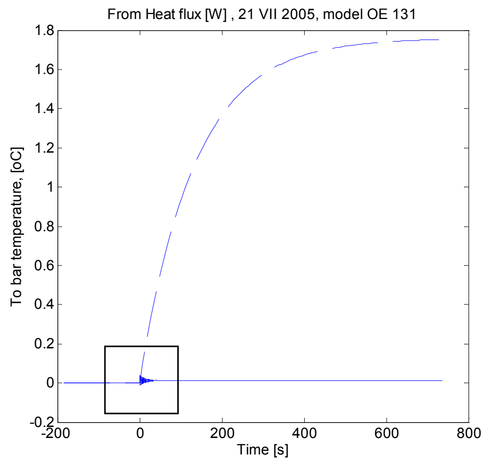

Fig. 7. The STEP response for partial models

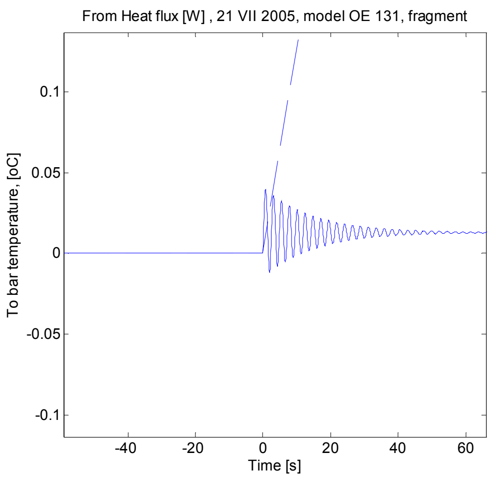

Fig. 8. The STEP response for partial models, fragment

The way to explain the ambiguous situations occurred in the transmittance decomposition for the partial fractions and the examination of behaving of each part separately and then their comparison and summation. This procedure will be demonstrated for the model OE 131 defined for the active heating element from heat flux. Their transmittance $G_1(s)$ has a form:

(19) $$G_1(s) = \frac{L(s)}{M(s)} = \frac{A_1 s^2 + B_1 s + C_1}{s^3 + D_1 s^2 + E_1 s + F_1}$$

where $L(s)$ - numerator, $M(s)$ – denominator, A_1, B_1, C_1, D_1, E_1, F_1 - polynomials coefficients.

The graphical presentation of the STEP response for this model has form indicative its inertial character. The poles (roots of the denominator) and the residua (roots of the numerator) of the $G_1(s)$ transmittance were calculated. In both of them the two complex, conjugated values occurred and the one real value. The inertial STEP response is not acceptable in such a situation. After a transmittance was decomposed for the partial fraction the models for real and complex part of the model were created. Then their courses were investigated by the STEP characteristic examination. Next, for both of the parts, the STEP courses are presented on the common diagram (the solid line presents complex part and the dashed line presents real part) presented on the Figure 8.

The analysis of the STEP response courses for both of the parts explains the seemingly inertial STEP character for the entire model. It is actually the composition of the inertial course and the oscillatory course. In the last the oscillation has a strong dumping and a short lasting. The oscillation occurs only for about one minute from the start of heating.

The phenomenon of the observed oscillation can be in this case so insignificant with regard to small dimensions and connected with them the thermal capacitance of the investigated elements. The oscillation occurring in real heat exchangers have different parameters because of the greater heat exchanger thermal capacitance.

The computation of the oscillation parameters and their exact relation to the heat exchanger parameters will be the matter of the next investigation. It is important matter for the heat exchanger dynamics because the vibrations occurring during transient states can reduce the strength of heat exchanger and it will be reasonable to eliminate them partially or completely.

The ETN for the heat exchanger model

The STEP courses presented for model partial elements help to examine most precisely the course of the entire model. The appearing oscillations have a little amplitude and strong dumping, but they have a significant influence on the course of phenomenon, especially in the origin, soon after acting of the step function. It is advisable in the ETN for the heat exchanger to take into account the element corresponding to the appearing oscillation, exchanging energy with the thermal capacitance and named the thermal inductance.

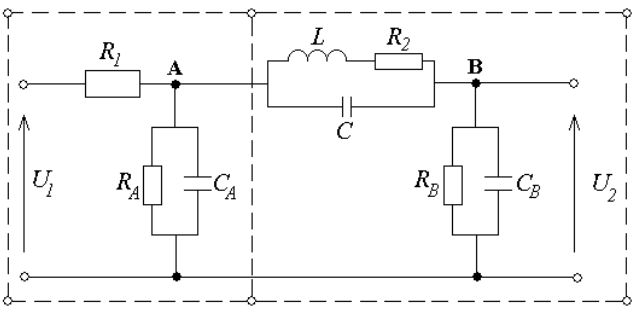

Fig. 9. The equivalent thermal network (ETN) for elements with the transmittance $G(s)$

The proposed structure of this ETN is presented on the Figure 9. It is taken into consideration that the heat exchanger is the system of two homogenous bodies. Between them and between them and the surrounding (as the points of the loss flowing down) the heat exchange takes place. The ETN scheme interpretation is the following:

1. The input of the heat exchanger is represented by the node **A** with the medium temperature of the active element (for traditional heat exchanger the medium temperature in the hot side)
2. The output of the heat exchanger is represented by the node **B** with the medium temperature of the passive element (for traditional heat exchanger the medium temperature in the cold side)
3. The working medium is represented by an arm between nodes **A** and **B.** In this case it is an air between active and passive elements.

Between the nodes of network and between nodes and the surroundings (the point of the loss flowing down) the heat fluxes are flowing by the thermal resistances. The thermal capacitance and inductance are also taken into account.

The quantities on this scheme have the following physical sense:

R_1 - the conductance through the active element
R_A, R_B - the resistances correspond to energy loss through convection and radiation between elements **A** and **B** and surroundings (the loss resistances)
C_A, C_B - the thermal capacitance of the elements **A** and **B**
R_2 - the thermal convection and radiation resistance of medium between elements
C, L - the thermal capacitance and the thermal inductance connected with the accumulation and transferring potential and kinetic energy between the elements through the working medium.

The presented ETN for the heat exchanger model may be treated as a connection in series of two elements (divided in the way presented in Figure 9 by a dashed line). Then the system transmittance may be presented as a product of two parts: $G(s) = G_1(s) \cdot G_2(s)$ which correspond to transmittances presented in the Figure 5. Their computation for the scheme proposed in the Figure 9 has given such a structure:

(20) $$G_1(s) = \frac{A_2}{1 + B_2 s}$$

(21) $$G_2(s) = K \cdot \frac{A_3 s^2 + B_3 s + 1}{s^2 + C_3 s + D_3}$$

where A_2, B_2, A_3, B_3, C_3, D_3 - polynomials coefficients

The structure of the transmittances $G_1(s)$ and $G_2(s)$ computed from the ETN scheme from Figure 9 and presented in the dependencies (20) and (21) corresponding to the structure of the transmittances of the heat exchanger partial elements (Fig. 5) which were presented in (17) and (18) and were computed from the parametric identification. In order to compare the close correspondence each to the other it is necessary comparing the values of transmittances - from the experiments (17, 18) and from the proposed ETN (20, 21). The polynomials of the numerators $L(s)$ and denominators $M(s)$ have the same range. Their coefficients are computed by the numerical approximation of the experimental data (17, 18) or depend on the heat parameters (resistance, capacitance, inductance) of the modelling elements (20, 21). It is necessary to compare these values, computing by the way of the thermal inductance.

Summary

The analysis of the transmittances of the models describing the heat exchange in the heat exchanger modelling system was carried out. The complex poles appearing in these transmittances indicated to the oscillatory course of the process. The next experiments and analysis confirmed this phenomenon and indicated the strong dumping of their courses. For the examined system the ETN was proposed in which the thermal inductivity notion is taken into account. It corresponds to the oscillation observed in the investigated phenomena. It is in accordance with the considerations presented for the electro-thermal analogy in the thermal system with the dissipated parameters and the long line. The omitting of inductance (or assuming their zero value) is equal to assuming the infinite value of heat propagation. This assumption is not correct.

The special experiment in which the oscillation parameters would be greater is worth to carry out. Then it may be verified if proposed ETN represents the investigated phenomenon in right way. For this purpose will be used the heat exchangers with the possibly greatest capacitance.

The existence of the oscillation in the thermal phenomena by electro-thermal analogy methods may be associated with the existence of the inductivity in the corresponding to them electric circuits. For this reason it is proposed to name this phenomenon as the thermal inductivity because of the role in the thermal phenomenon analogical with the role of the electrical inductivity in the electrical phenomena.

Nomenclature:

$\mathbf{j}$, $\mathbf{q}$ – density of current, of heat flux,
l, δ – length of current circuit, of heat circuit,
F_e, F_t – section of current circuit, of heat circuit,
t_e, t_t – time of electrical, thermal phenomenon;
c_e, c_t – specific electric capacity, specific thermal capacity,
ρ_e, ρ_t – mass density of electric system elements, of thermo kinetic system elements,
t - time [s],
T - temperature [°C].

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