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Calculation and analysis of local magnetic forces in ferrofluids

Abstract. *This paper presents a non-exhaustive overview of magnetic force formulations mainly used in ferrohydrodynamics. All of these formulations give the same global result but the local force density is different from each other. We propose to compare these formulations in order to highlight which one is the best representation of the local force density. In order to find a reference of local force distribution, computations have been carried out using the virtual work method on particles surrounded by thin layers of air. The whole particles and air have been homogenized to create a linear uniform material. Finally, force distributions have been computed and have been compared to the reference calculated by virtual work method.*

Streszczenie. *W artykule przedstawiono przegląd metod określania siły magnetycznej w ferrofluidach. W różnych metodach otrzymuje się różne lokalne wartości siły. W celu weryfikacji metod prze[prowadzono obliczenia numeryczne w których modelowano cząstki otoczone cienką warstwą powietrza. (**Obliczenia i analiza lokalnej wartości siły magnetycznej w ferrofluidach**)*

Keywords: Ferrofluid, Ferrohydrodynamics, Magnetic force distribution, Magnetic force analytical formulation
Słowa kluczowe: ferrofluidy, siiła magnetyczna.

Introduction

Ferrofluids are colloidal suspensions of coated magnetic particles. The Brownian movement, due to the natural convection, is enough to consider them as homogeneous liquids. Regarding the magnetic properties of nanoparticles, ferrofluids can be attracted by a magnet. To predict this movement, local magnetic forces should be evaluated precisely. Many physical equivalences and analytical formulas, deriving from magnetostatics, give a good estimation of the global force applied to a given massive body when it is submitted to an external magnetic field [1]. Conversely, each equivalence or formula gives a different distribution of the local forces in the volume or at the surface of the body. In order to evaluate which formulation provides the most representative distribution of the local forces, we propose to compare these different formulations thanks to simulations.

The fist part of this paper will present formulations which are commonly used in hydrodynamics. A second part will show the comparison between formulations with first a global point of view and, in a second time, a local point of view. Finally, we will analyze the difference between each formulation.

Main magnetic force density formulations used in ferrohydrodynamics

The formulations which are mainly used in ferrohydrodynamics come from magnetostatics in massive body. In the following part of the paper, we will present some of them.

A. Equivalent current sources

The Amperian representation is one of the commonly used formulations [2,3]. It introduces two terms in the magnetic force expression. One corresponds to a volume contribution and the other to a surface one. These terms come directly from the equivalent volume and surface current sources $\mathbf{J_V}$ and $\mathbf{J_s}$ associated to any magnetized body and from the force calculation using Lorentz law.

(1) $$\mathbf{J_V} = \nabla \times \mathbf{M}$$

(2) $$\mathbf{J_s} = \mathbf{M} \times \mathbf{n}$$

(3) $$\mathbf{f_v} = \mathbf{J_v} \times \mathbf{B}$$

(4) $$\mathbf{f_s} = \mathbf{J_s} \times \mathbf{B_s}$$

where **M** is the magnetization of the material [A/m], **B** the induction of the material [T], $\mathbf{B_s}$ the induction at the surface of the material [T], **n** the normal vector of the considered surface.

B. Equivalent magnetic charges

The Coulombian representation assumes that the magnetic field created by a magnetized body is similar to the one generated by virtual magnetic charges densities which are distributed inside the volume (ρ_v) and at the surface (ρ_s) of the body [2, 3]. These charges are defined with equations (5) and (6) and can be used to express the volume and the surface magnetic forces, which can be computed using electrostatics equivalent formulas (7) and (8).

(5) $$\rho_v = -\mu_0 . \nabla \cdot \mathbf{M}$$

(6) $$\rho_S = -\mu_0 . \mathbf{n} \cdot \mathbf{M}$$

(7) $$\mathbf{f_v} = \rho_v \cdot \mathbf{H}$$

(8) $$\mathbf{f_s} = \rho_s \cdot \mathbf{H_s}$$

where $\mathbf{H}$ the magnetic field in the material [A/m], $\mathbf{H_s}$ the magnetic field at the surface of the material [A/m].

C. The Kelvin force

Many authors [4,5] affirm that the force density can be calculated thanks to the following Kelvin law (9) while others authors [6] propose the equation (10).

(9) $$\mathbf{f} = \mu_0 (\mathbf{M} \cdot \nabla)\mathbf{H}$$

(10) $$\mathbf{f} = \mu_0 (\mathbf{M} \cdot \nabla)\mathbf{H}_0$$

where μ_0 the vacuum permeability [H/m], $\mathbf{H_0}$ is the external magnetic field in which the body is placed.

However, It has been shown that the formulation (9) is not enough to describe the total force density. It represents only the volume forces and it is necessary to add a surface contribution as made in (11) to obtain the right expression [5].

(11) $$F = \int_V \mu_0 (\mathbf{M} \cdot \nabla)\mathbf{H} dV + \oint_S \frac{\mu_0}{2} (\mathbf{M_n})^2 dS$$

where $\mathbf{M_n}$ is the normal magnetization vector at the surface of the body.

An application of the Green-Ostrogradski theorem on this formulation gives a new expression of the global force density:

(12) $$F = \int_V \left(\mu_0 (\mathbf{M} \cdot \nabla)\mathbf{H} + \frac{\mu_0}{2} \nabla (\mathbf{M}^2) \right) dV$$

Thus a local force expression can be deduced:

(13) $$f = \mu_0 (\mathbf{M} \cdot \nabla)\mathbf{H} + \frac{\mu_0}{2} \nabla (\mathbf{M}^2)$$

The global force calculated by (11) is equal to the one computed by (12). However, the distribution of local forces is different. This last formula was used by Riccetti [7] during his PhD on magnetocaloric pumps.

The driving force which moves the ferrofluid consists essentially of volume forces. The surface ones are not very important and give the pressure constraint.

Global and local force comparison

A. Global force comparison

We consider a ferrofluid bar, with a magnetic linear permeability μ_r, which is placed in an external magnetic field as shown in Fig.1. This field is oriented perpendicularly to the ferrofluid bar and is created by a permanent magnet having a magnetic polarization of 1T. Different simulations have been done with different magnetic permeability values varying from 1 to 20. This permeability range is close the commercial ferrofluids permeability.

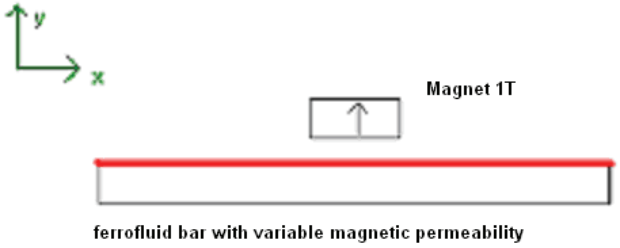

Fig.1. Geometry used for the simulation

The simulations are performed in 2 dimensions with the finite element Flux© software which is able to give not only the magnetic field H(x,y) and flux density B(x,y) distributions but also the global force components Fx and Fy. These forces are computed by the virtual work method and are considered as a reference in this study. Knowing B and H values at each node of the geometry, we have also computed the global force by integrating each formulation, we have presented before, over the entire volume of ferrofluid. The results obtained are then compared to the ones given by the Flux© software as shown in Fig. 2.

In this figure, only the evolution of the vertical component of the global force exerted on the full ferrofluid bar is displayed. In fact, the horizontal one is zero regarding the symmetries conditions. These results show that formulations (10), (11) and (13) are quite similar and are very close to the virtual work reference. However, the formulation (9) is insufficient to have the good global force value. The force calculated in that case tends to 0 when the permeability increases. This can be explained by the fact that the magnetic field H gradient is used and as H decreases when the permeability increases, the global force decreases and become a not realistic formulation. Furthermore, when the surface contribution is added as done in formulation (11), a result very close to the reference is obtained.

The Amperian and Coulombian formulas (3, 4, 7, 8) are not represented on this graph but they give the same results. The calculated forces correspond however to the global surface forces, the volume forces being zero due to the linearity of the material.

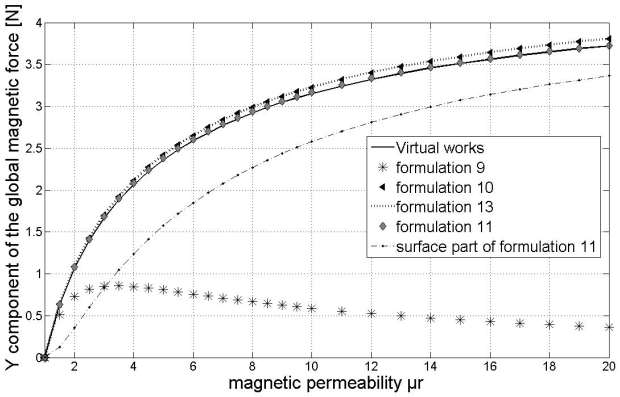

Fig.2. Evolution of the global force on a ferrofluid bar as a function of the permeability.

B. Local force distribution comparison

Consideration of the global force is not relevant to effectively compare the different formulations and find those that can calculate fairly local forces. In fact the local force density distribution is strongly dependent on the used formulation. The next figures compare, for example, the surface force density computed by formulations (3, 4) and (7, 8). There are large differences between the two computations, while they lead to the same solution when they are used to determine the global force. The Amperian and the Coulombian representations give any force distributions in volume due to the linearity of the material.

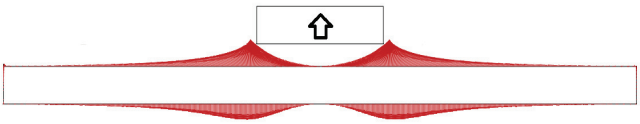

Fig.3. Local force density distribution obtained with formulation (3) and (4) (Amperian equivalent representation)

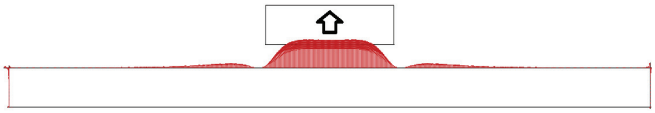

Fig.4. Local force density repartition of the local force with formulations (7) and (8) (Coulombian equivalent representation)

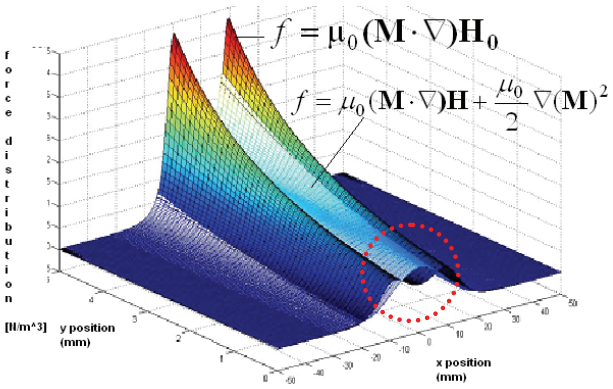

Fig.5. Local force density distribution obtained with formulations (10) and (13)

The same observations are made for the volume force densities when formulations (10) and (13) are applied (Fig.5). While the global result is the same, the local force distributions are very different.

Analysis of the local force distribution

In order to bring out the best solution which gives the closest physical result, we need to compare formulations (10) and (13) to the reference method based on the virtual work calculation. However, Flux2D software is only able to compute a global force on a body. So to face this difficulty, the ferrofluid bar is subdivided into square particles which are surrounded by thin air layers. The global force exerted on each particle and computed by Flux2D is considered as a reference local force. To calculate the local forces using the analytical formulations (10) and (13), the B(x,y) and H(x,y) maps are required on an homogenous body. So a homogenization of the particles is achieved thanks to an equivalent permeability model and finally new Flux2D simulations are performed.

A. Homogenization of the heterogeneous body

Let's consider a regular distribution of square shaped particles, which are spaced by a constant airgap and which have the same size and the same constant permeability, as shown in Fig. 6.

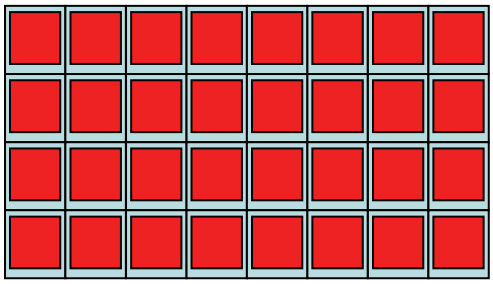

Fig.6. Considered heterogeneous ferrofluid.

The homogenization principle we propose is to represent each magnetic particle and the air surrounded by a mono-particle made of an equivalent material with an apparent permeability. This mono-particle should be the same wherever in the inhomogeneous material and should have a square shape. So the homogenized material is an isotropic one. In order to find the equivalent permeability, an external magnetic field H is first applied to the heterogeneous particle made of material and air and the energy stored is computed using Flux2D software. In a second time, a similar simulation is carried out on the equivalent mono-particle. The equivalent magnetic permeability is adjusted to have the same energy stored in the two cases. This principle of this homogenization is presented in Fig.7.

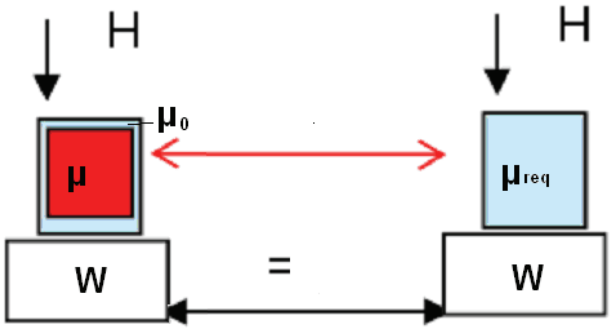

Fig.7. Considered heterogeneous material.

The size and the equivalent permeability, we will use in this paper, are given in the following figure.

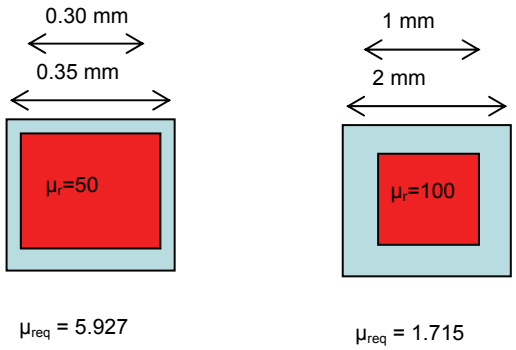

Fig.8. Influence of the size and the magnetic permeability of the heterogeneous particle on the equivalent permeability of homogenized material

Now, we can compute analytically the local forces on the homogeneous material and compare them to the ones given on the heterogeneous material by the Flux2D numerical simulations and the virtual work method.

B. Local force distribution

Two geometries have been tested. A first one is made of 36 particles of 1mm*1mm surrounded by a 0.5mm air layer. As it will be shown later, this geometry gives not accurate results because of the big size and the very low number of particles. A second one with much more particles with lower size is then chosen.

Fig.9 shows the first used geometry. This geometry is composed by a square body of 6*6 particles set near a 1T permanent magnet. The real and the equivalent relative permeabilities are 50 and 1.715 respectively. Regarding to this configuration, the vertical component of the global force must be positive and its horizontal one must be negative. The next Fig.10 and Fig.11 give the local force distribution computed by the virtual work method. The estimated global force is -3.8N and 2N respectively for the horizontal and the vertical components.

These results highlight that only magnetostatic volume forces exist. So the equivalence using current or charges sources are not appropriate. In addition, no discontinuity is observed at the surface of the magnetic body. This allows removing formulation (11).

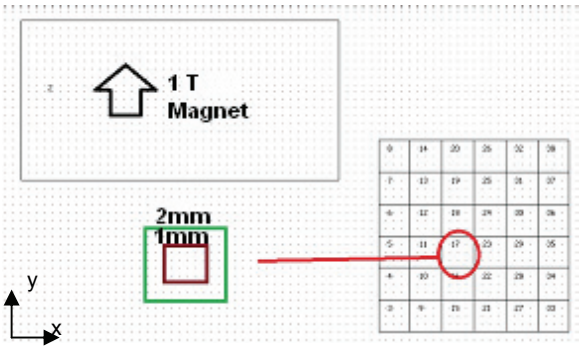

Fig.9. First geometry

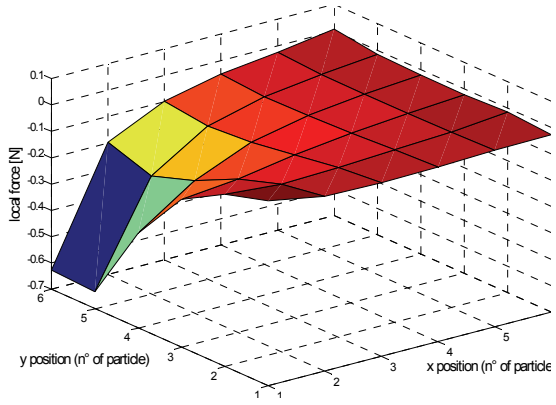

Fig.10. First geometry: horizontal force component Fx (x,y)

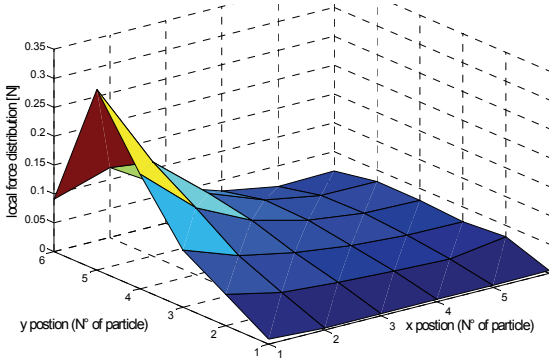

Fig.11. First geometry: vertical force component Fy(x,y)

The local forces computed with the two remaining formulations (10) and (13) are compared to the results. The formulation (10) gives an error of 25% on Fy(x,y) and 45% on Fx(x,y). Formulation (13) fits better but the observed differences cannot allow to decide between the two methods. To obtain more accurate results, we also test the second geometry presented in Fig. 12. The considered particles are smaller that in the first case: 0.3mm*0.3mm size and 0.025mm air layer. Symmetry and anti-symmetry conditions are used in order to multiply artificially, the number of particles. Thus 2592 particles are simulated. Their real and equivalent magnetic relative permeabilities are 100 and 5.927.

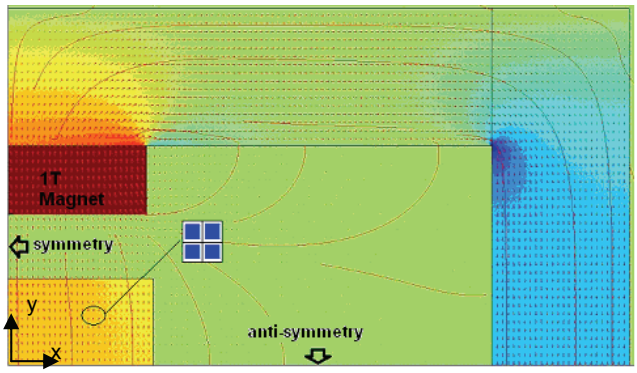

Fig.12. Second geometry

Fig. 13 presents the horizontal component of the local force distribution computed by the virtual work method.

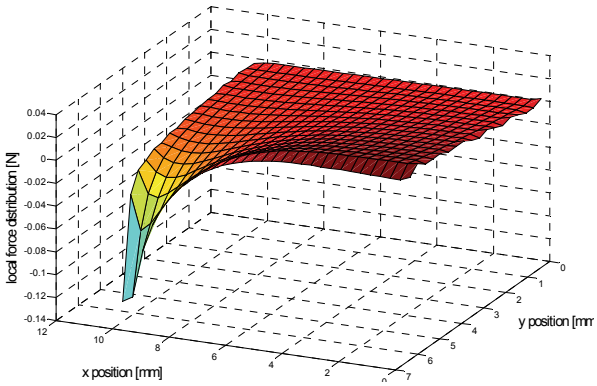

Fig.13. Horizontal component of the local force distribution Fx(x,y)

This distribution is compared first to the one calculated with formulation (10) in Fig. 14. The average error reaches 200%. This value is much higher than the one obtained for the first geometry (45%) but in that case the calculation is not very accurate. Thus, we can conclude that this formulation is not valid to estimate the force density.

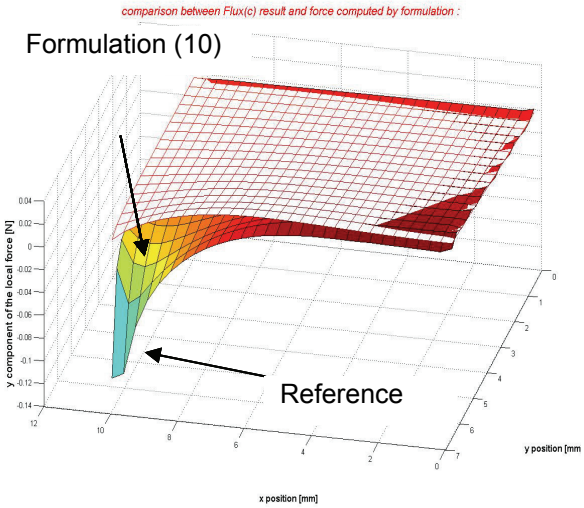

Fig.14. Comparison of the local force distribution computed by virtual work method (Flux) and formulation 10

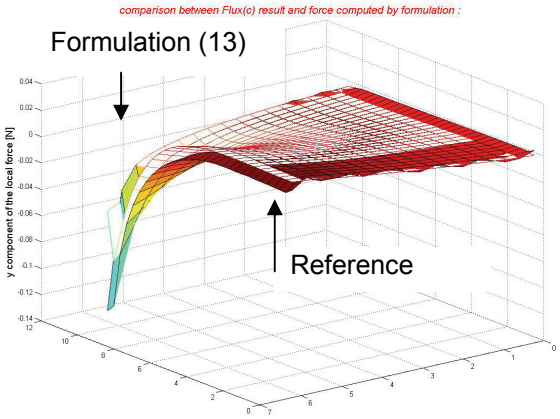

Fig.15. Comparison of the local force distribution computed by virtual work method (lower surface) and formulation (13) (upper surface)

Secondly, the last formulation (13) is tested. Fig.15. shows that the results fit well with the reference calculation. The average error is 12% and is a little bit lower that the

one found with the first geometry. This validates this analytical approach to the prediction on the local force distribution.

Conclusion and future work

Many formulations exist in literature to determine the global forces induced by the interaction of a magnetic body and an external field. FEM simulations confirm that they give the same results except for the Kelvin force. This last force only corresponds to the volume part and the surface part should be added to obtain the real total force.

However the situation is not true when the local force calculation is investigated. In order to find which formulation gives the best result, a heterogeneous body made of square particles is simulated with Flux2D software and the local forces densities are determined thanks to the virtual work method. In parallel the body is homogenized, simulated and the field distribution is used to calculate the forces with the different formulations. The comparison highlight than only one among the five studied formulation is able to determine correctly the local forces.

A next step of this work can be the test of formulation (13) considering other particles shapes, other geometries and in non linear magnetic material conditions. It will be also relevant to investigate an experimental verification even though it is difficult to achieve. But, the analytical formulation may already be exploited to calculate the local ferrofluid speed and flow ratio and to design a pump.

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