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Analytical inverse model with flexible parameters for dynamic hysteresis loops modelling

***Abstract**. Analytical inverse model was proposed to find simple mathematical approximation of dynamic hysteresis loops of soft magnetic materials. It consists of dynamic loop construction from three components by means of mathematical functions. Favourable mimicking coefficients were searched, which depend on excitation amplitude and frequency. The nature of its behaviour was studied in order to be able to predict the loop shape. The model was tested for opened specimen of classic silicon electrical steel at sinusoidal excitation, measured by compensation ferrometer.*

***Streszczenie.** Zaproponowano inwersyjny model analityczny do opisu matematycznego dynamicznej pętli histerezy materiałów magnetycznie miękkich. Model był sprawdzony dla próbki otwartej klasycznej blachy elektrotechnicznej przy sinusoidalnym magnesowaniu. (**Analityczny model inwersyjny pętli histerezy**)*

Keywords: dynamic hysteresis loop, mathematical modelling, inverse model, analytical approximation
Słowa kluczowe: dynamiczna petal histerezy, modelowanie.

Introduction

Mathematical model based on hysteresis loop construction from analytical functions and suitable for AC magnetizing is proposed. It approximates symmetrical dynamic hysteresis loops and performs reasonably in the whole range of commonly used excitation.

It is a part of a more complex task – improving the properties of the measuring device KF9a [1, 2, 3] by means of simple prediction algorithm implementation. Instead of the usual $B_{\pm} = \mathrm{f}(H)$ function its inverse $H_{\pm} = \mathrm{f}(B)$ is needed for calculating magnetic field strength H from magnetic flux density B because KF9a has B as input variable. Knowing the rough estimate of material behaviour, the desired goal is optimum magnetizing parameters setting and measuring process accelerating.

In the first step, work was focused on suggestion of suitable analytical functions, which are able to approximate the shape of the loop successfully. After that, optimum coefficients finding and simulation quality verification follows. Further work consists of studying the coefficients behaviour with regard to excitation and frequency. Finally, the model should predict (for certain class of soft magnetic materials) the shape of dynamic loop and related important parameters.

Methods used for dynamic hysteresis loops modelling and analytical functions utilization

Hysteresis loop modelling represents one of important ways of magnetic materials behaviour description. Hysteresis loops and other curves could be described by different mathematical and experimental models [4, 5, 6].

Frequently used methods for dynamic loop modelling are represented by dynamic Preisach model and various differential models: Jiles-Atherton model, Chua model, Hodgdon model, Duhem model. There are a lot of other dynamic models available. Most of existing models have their inverse variant, too.

Dynamic models usually give good results, however, for the purpose of KF9a prediction algorithm, they seem to be rather complicated or time consuming. Hence a more simple way – analytical approximation – was searched.

Analytical approximation models represent the simplest way of describing magnetization curves by approximating their shape exploiting mathematical functions. Various convenient functions are used [4] – power series, rational fractures, exponential functions, tangent functions. The models of Takacs [7] (hyperbolic tangent) and Wlodarski [8] (Langevin function) should be mentioned as representatives of more recent approaches using analytical functions.

However, most of existing analytical models perform well at slow magnetizing only; with increasing frequency they are able to approximate saturated loops but fail in mimicking rounded smaller loops, or the full loop has to be put together from several segments.

Proposed analytical method for inverse dynamic hysteresis loops modelling

Proposed mathematical approach consists of dynamic loop composition from three components:

$$H_{\pm} = h_1(B) + h_2(B) + h_3(B)\,. \tag{1}$$

The H_+ and H_- terms denote ascending and descending branches of the loop, respectively. The upper sign in $\pm$, $\mp$ symbols used below is valid for H_+ and the lower for H_-.

The model components (functions h_1, h_2, h_3) were derived simply from the knowledge of dynamic loop shape, apart from physical nature of dynamic magnetization mechanisms. They are defined as follows:

$$h_1 = a_1 \cdot \mathrm{sign}\left(B \mp b_1\right) \cdot \left(\tan\left(\frac{\pi}{2} \cdot \frac{B \mp b_1}{B_{\max}}\right)\right)^2 \pm a_0\,. \tag{2}$$

The h_1, based on square power of the frequently used tangent function [4], is responsible for the sigmoid shape of strongly saturated loop, whose shape is similar to the static one. This component prevails at high saturation level (Fig.1), at low saturation h_1 is weak but should not be omitted (Fig.2). For better model flexibility, the 'maximum' flux density $B_{\max}$ in (2) is supposed to be variable parameter.

$$h_2 = \pm a_2 \cdot \left(1 - \left(\frac{B}{B_{\mathrm{a}}}\right)^2\right)^k \tag{3}$$

The next h_2 component represents contribution of the 'lagged response' to the excitation signal. It features the rounded as far as elliptical shape of dynamic loop at smaller saturation level (Fig.2). Coefficient k causes the necessary elliptical shape variation; B_{a} is magnetic flux density amplitude.

(4) $h_3 = a_3 \cdot B$

The h_3 component is linear, representing loop declination, remarkable with decreasing saturation.

For credible loop shape approximation only six parameters (a_1, a_2, a_3, b_1, B_{max}, k) are required; a_0 is a constant, resulting from the condition $H_+(B_a) = H_-(B_a)$:

(5) $$a_0 = \frac{a_1}{2}\left(\tan^2\left(\frac{\pi}{2}\cdot\frac{B_a + b_1}{B_{max}}\right) - \tan^2\left(\frac{\pi}{2}\cdot\frac{B_a - b_1}{B_{max}}\right)\right).$$

In addition, in special cases, not all the model components are inevitable. The smallest of them can be neglected, so that the number of necessary parameters is less than six.

The frequency and excitation dependence is hidden in the coefficients which have different values for each loop, so that it would be more correct to write $a_1(B_a, f)$, $a_2(B_a, f)$ etc. However, the problem of variable parameters could be simplified when the coefficients dependence is to be revealed or when some coefficient stays constant.

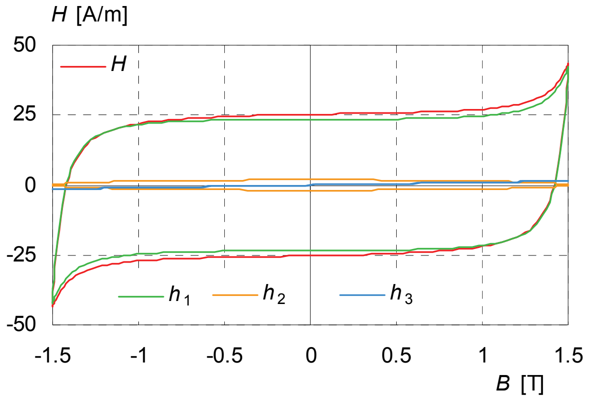

Fig.1. Simulation for f = 50 Hz and B_a = 1.5 T

In Fig.1, Fig.2 examples of simulated hysteresis loops with separate components are depicted. Contribution of individual components depends on magnetic flux density amplitude B_a, final loops are depicted by red line. It is obvious from Fig.1 that both the elliptical h_2 and the linear h_3 component is weak even can be omitted. On the other hand, in Fig.2, the h_1 component is weaker than h_2 but necessary for credible approximation.

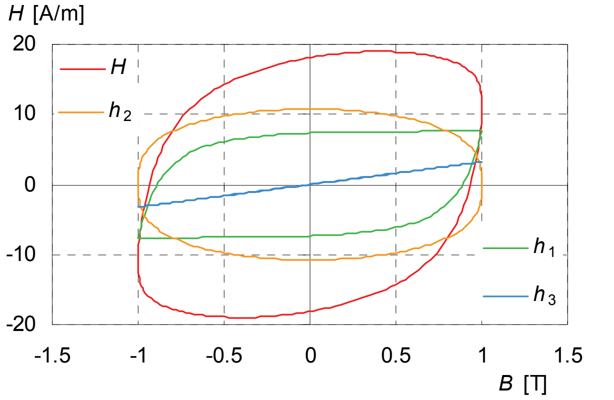

Fig.2. Simulation for f = 50 Hz and B_a = 1.0 T

Newton method was used for the first acceptable parameters' estimation. Then the genetic algorithm was applied by using standard Matlab instruments (function ga, population size 300, 45 generations), with fitness function minimizing the mean squared deviation. Criteria of the least mean absolute deviation and least maximum absolute deviation were tested, too. Searching was limited to non-negative parameter values (with the only exception of a_2 at higher saturation, see Fig.9).

For credible loop approximation, the fitness function could be improved by setting the condition of precise agreement in most important points of the loop.

Testing of proposed model and results

Features of the proposed model were tested for classic soft magnetic material – oriented silicon electrical steel Eo10 (sheet 500 mm length, 500 mm width, thickness 0.35 mm) using sinusoidal magnetization at different frequencies f and magnetic flux densities B_a.

Material samples were measured in 200 points per loop by means of compensation ferrometer KF9a, PC controlled single sheet/strip tester [1, 2, 3] at following conditions:

f = 50 Hz, 0.1 T≤ B_a ≤ 1.7 T (step 0.1 T);
40 Hz ≤ f ≤ 400 Hz, B_a = 0.5 T–1.0 T–1.5 T–1.7 T.

In the course of optimum parameters' searching, the initial premise that model parameters are variables, dependent on saturation and frequency, proved true. Nevertheless, the parameters often stay rather invariable in certain range of saturation and frequency, or it is possible to anticipate their behaviour.

Experimental results demonstrated in Fig.3 – Fig.4 show rather good similarity with measured values (black line – measured loops, red line – simulated loops). Only in the rounded loop 'tips' at higher frequency appears remarkable dissimilarity (Fig.4, 400 Hz). Smaller difference is visible in the loop flexure (Fig.3, 1.5 T and 1.7 T). Parameter values relating to Fig.3 – Fig.4 are presented in Table 1, Table 2.

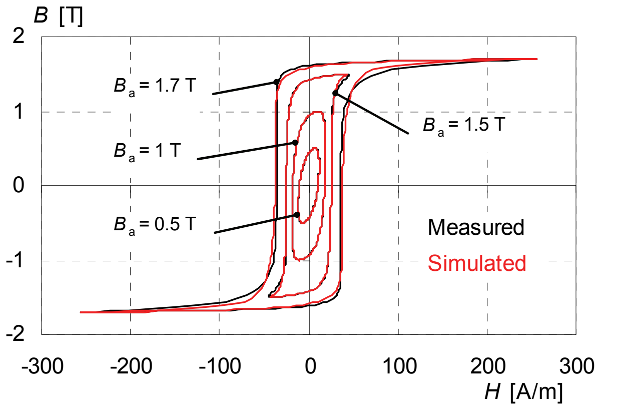

Fig.3. Measured and simulated hysteresis loops at B_a = 0.5 T – 1.0 T – 1.5 T – 1.7 T, f = 50 Hz

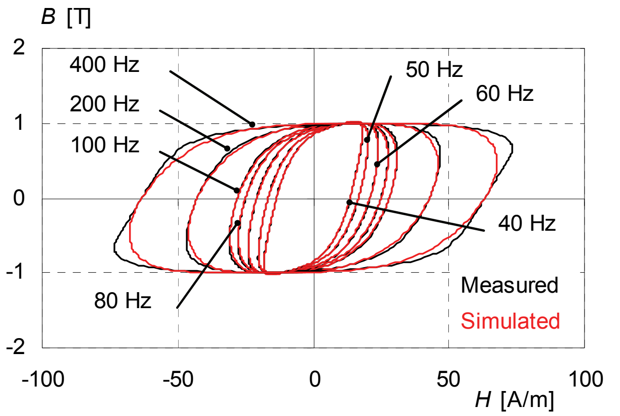

Fig.4. Measured and simulated hysteresis loops at B_a = 1.0 T, f = 40 – 50 – 60 – 80 – 100 – 200 – 400 Hz

Table 1. Parameters at f = 50 Hz for different B_a

B_a [T]	B_{max} [T]	a_1 [A/m]	b_1 [T]	a_2 [A/m]	k [–]	a_3 [m/H]
0.5	1.25	0.619	0.5	7.878	0.439	7.96
1.0	1.763	0.966	0.487	10.76	0.339	3.212
1.5	1.698	0.966	0.07	-1.756	0.707	0.944
1.7	1.776	1.105	0.0068	-8	0.84	1.364

Table 2. Parameters at B_a = 1.0 T for different f

f [Hz]	B_{max} [T]	a_1 [A/m]	b_1 [T]	a_2 [A/m]	k [–]	a_3 [m/H]
40	1.465	1.299	0.183	7.820	0.319	7.137
50	1.547	1.323	0.250	10.457	0.329	7.273
60	1.795	0,919	0.508	13.743	0.330	7.706
80	2.219	0.450	0.975	17.999	0.343	8.111
100	2.274	0.446	0.998	22.019	0.330	8.912
200	2.202	0.174	1.002	38.709	0.313	12.528
400	2.061	0.075	0.955	57.718	0.247	16,743

Deviations between simulated and measured loops for both examples mentioned above are depicted in Fig.5 and Fig.6. Results are expressed in %, related to measured H_a.

(6) $$\delta = \frac{H_{\text{simulated}} - H_{\text{measured}}}{H_{a\ \text{measured}}} \cdot 100\,\%$$

In several points (Fig.5) deviation seems rather high – especially in the loop flexure. However, the mean absolute deviation related to H_a stays less than 3 % for all the simulated loops.

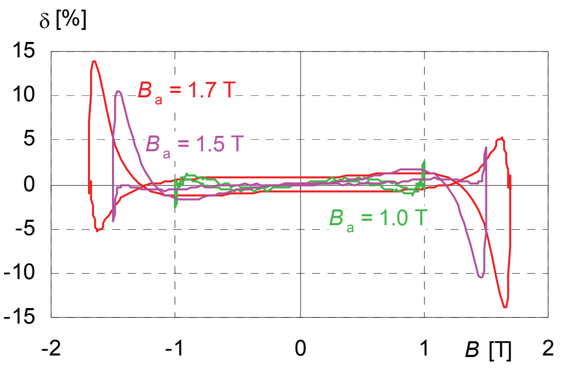

Fig.5. Deviation between simulated and measured loops at f = 50 Hz, B_a = 1.0 T – 1.5 T – 1.7 T

Results in Fig.6 are better (with the exception of f = 400 Hz), because respective loops have similar shapes (Fig.4). The mean absolute deviation related to H_a stays less than 5 % for all the simulated loops.

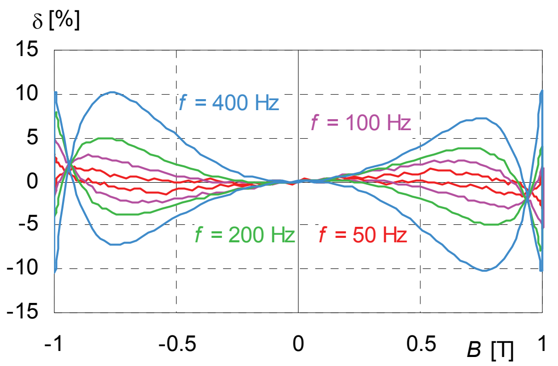

Fig.6. Deviation between simulated and measured loops at f = 50 Hz – 100 Hz – 200 Hz – 400 Hz, B_a = 1.0 T

Parameter variations (for all the parameters a_1, a_2, a_3, b_1, B_{max}, k) at f = 50 Hz for increasing B_a are illustrated in following Fig.7 – Fig.9, with related RMS deviation in Fig.10.

The RMS deviation was calculated according to equation (7) for N = 200 measured points.

(7) $$\text{RMS dev.} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(H_{i\ \text{simulated}} - H_{i\ \text{measured}}\right)^2}$$

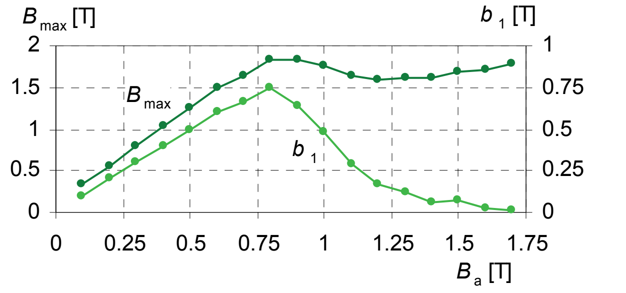

Fig.7. Dependences of parameters B_{max} [T] and b_1 [T] at f = 50 Hz and increasing B_a

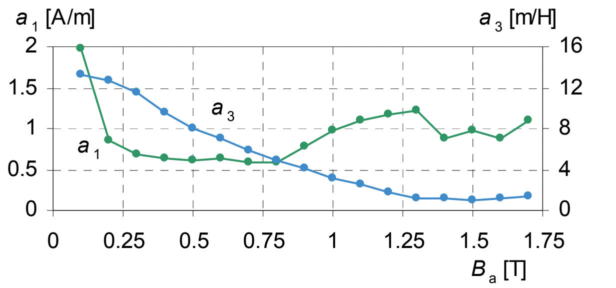

Fig.8. Dependences of parameters a_1 [A/m] and a_3 [m/H] at f = 50 Hz and increasing B_a

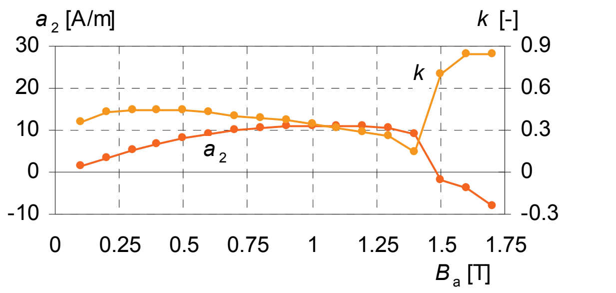

Fig.9. Dependences of parameters a_2 [A/m] and k [–] at f = 50 Hz and increasing B_a

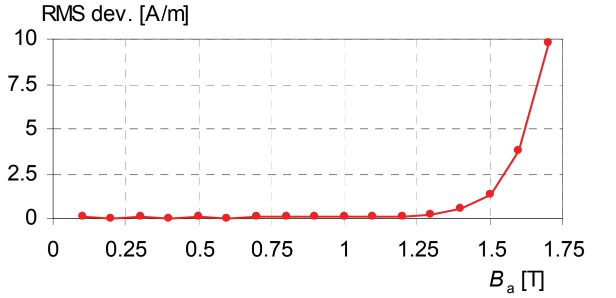

Fig.10. RMS deviation [A/m] at f = 50 Hz and increasing B_a

Parameter variations at B_a = 1.0 T for increasing frequency are illustrated in Fig.11 – Fig.13, with related RMS deviation (7) in Fig.14. In this case, all the parameters have positive values.

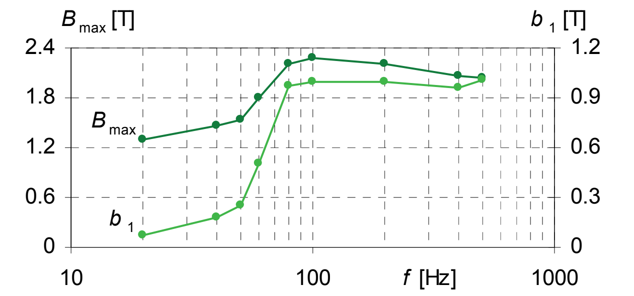

Fig.11. Dependences of parameters B_{max} [T] and b_1 [T] at B_a = 1.0 T and increasing f

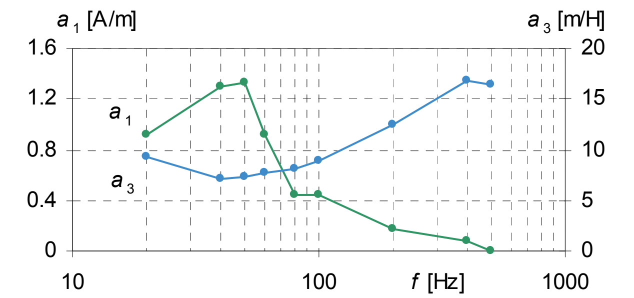

Fig.12. Dependences of parameters a_1 [A/m] and a_3 [m/H] at B_a = 1.0 T and increasing f

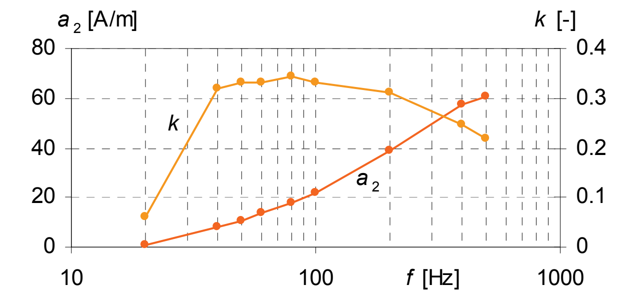

Fig.13. Dependences of parameters a_2 [A/m] and k [–] at B_a = 1.0 T and increasing f

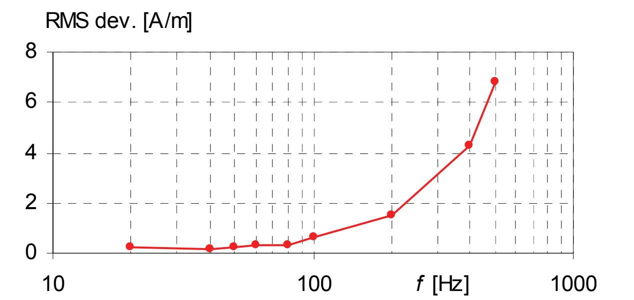

Fig.14. RMS deviation [A/m] at B_a = 1.0 T and increasing f

It is obvious from Fig.7 – Fig.9, Fig.11 – Fig.13 that all six parameters (a_1, a_2, a_3, b_1, B_{max}, k) have expectable behaviour – i.e. their value can be predicted either from functional relation or from look-up table.

Different ways of model simplification were tested, namely if some of the parameters could stay constant. It was found that in the case of constant frequency, parameter a_1 can be invariable (e.g. a_1 = 1 A/m at f = 50 Hz) without significant increase of deviation. Satisfactory approximation could be reached with the aid of five remaining parameters.

In the case of different frequency and constant excitation amplitude, B_{max} can stay constant (e.g. B_{max} = 2.27 T at B = 1.0 T) but yet the approximation quality is not affected.

The uncertainty δ_p of specific power losses evaluation was tested, too.

$$\delta_p = \frac{p_{\text{simulated}} - p_{\text{measured}}}{p_{\text{measured}}} \cdot 100\,\% \tag{8}$$

In the examined range of f and B_a it does not exceed ±3 %.

Conclusions

New analytical three-component mathematical model (with no significant relationship to physical phenomena) with flexible parameters for inverse dynamic hysteresis loops simulation was presented. The model is able to mimic varied shapes of dynamic loops at different frequency and saturation level with acceptable deviation. It was verified for classic electrical silicon steel at sinusoidal magnetization.

The use of proposed model is limited to symmetric scalar excitation, since the compensation ferrometer KF9a is intended for 1-dimensional symmetric applications.

The simulation quality of saturated loops seems to be weaker in the loop flexure. This difficulty can be solved by adding convenient correcting function. Attention should be paid to the ‘tips’ of rounded loops at higher frequency, where obvious difference appears.

Further work will be focused on parameters’ behaviour and analytical description of their dependence on f and B_a. Proposed model will be implemented for amorphous materials and checked out for non-sinusoidal excitation.

Besides dynamic hysteresis loops modelling, the proposed model should be used especially for modelling and prediction of the magnetization characteristic H_a = f(B_a) and specific power losses p = f(B_a). It could represent a way to accelerate the measuring algorithm of KF9a because it has magnetic flux density B as input variable and different operating points for required magnetic field strength amplitude H_a must be set.

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