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Simulation of static stability of three phase induction motor

Streszczenie. *Zaproponowany algorytm analizy statycznej stabilności ustalonych stanów urządzeń elektrotechnicznych na przykładzie trójfazowego silnika asynchronicznego z kondensatorem w trzeciej fazie. Podane rezultaty symulacji komputerowej. (**Symulacja stabilności statycznej trójfazowego silnika asynchronicznego**)*

Abstract. *In the paper is proposed the algorithm of computation of static stability of steady-state of electric devices on example of three phase induction motor by capacitor in third phase. The results of computation are given..*

Slowa kluczowe: statyczna stabilnosc, stan ustalony, urzadzenie elektrotechniczne.
Keywords: static stability, steady-state, electrical devices.

Introduction

The calculation of steady-state static stability of some electric device is actual problem which belong to the one of four fundamental grades of analysis as calculation transitional and steady-state processes, and parametric sensitivity. We shell demonstrate as to search for it on example of three-phase asynchronous induction motor with capacitor in one of three phases. For successful solution of given problem was necessarily to solve at first important theoretical problem of building of substituted model of parametric sensitivity at first [1,2]. On this base was being built the monodromy matrixes of systems of differential equations

Mathematical model of motor

The winding of rotor we reduce to quantitatively of windings coils to winding of stator. The rotor currents are reducing to frequency of stator currents too. In this case The equations of electromagnetic state of motor we can to write down in the form [1]

(1) $$\frac{di}{dt} = A(u - \Omega'\Psi - Ri),$$

where

(2) $$\begin{array}{|c|}\hline \lambda_S \\ \hline \lambda_R \\ \hline \end{array}, \ \lambda = u, \Psi, i; \quad \mathrm{A} = \begin{array}{|c|c|}\hline A_S & A_{SR} \\ \hline A_{RS} & A_R \\ \hline \end{array};$$
$$\Omega' = \begin{array}{|c|c|}\hline 1 & \\ \hline & \Omega \\ \hline \end{array}; \quad R = \begin{array}{|c|c|}\hline R_S & \\ \hline & R_R \\ \hline \end{array}.$$

Here: $i_k = (i_{kA}, i_{kB})_t$, $k = S, R$ are columns of phase currents of stator winding and transformed currents of rotor winding; $u_k = (u_{kA}, u_{kB})_t$, $k = S, R$ are columns of phase voltages of stator winding; A_S, A_{SR}, A_{RS}, A_R are matrixes

(3) $$A_S = \alpha_S(1 - \alpha_S G); \ A_{SR} = A_{RS} = -\alpha_S \alpha_R G;$$
$$A_R = \alpha_R(1 - \alpha_R G),$$

where: *G*, Ω are matrixes

(4) $$G = \begin{array}{|c|c|}\hline T + b_A i_A & b_B i_A \\ \hline b_A i_B & T + b_B i_B \\ \hline \end{array}, \quad \Omega = \frac{\omega}{\sqrt{3}} \begin{array}{|c|c|}\hline -1 & -2 \\ \hline 2 & 1 \\ \hline \end{array}.$$

besides that

(5) $$b_A = b(2i_A + i_B); \quad b_B = b(i_A + 2i_B); \quad b = \frac{2}{3}\frac{R - T}{i_m^2};$$

(5) $$R = \frac{1}{\alpha_S + \alpha_R + \rho}; \qquad T = \frac{1}{\alpha_S + \alpha_R + \tau}.$$

Here: τ, ρ are inverse static and differential inductances. Theirs we find according to no-load characteristic of motor as:

(6) $$\tau = \left[\frac{\psi_m(i_m)}{i_m}\right]^{-1}; \quad \rho = \left[\frac{d\psi_m(i_m)}{di_m}\right]^{-1},$$

where i_m is modulus of spatial vector of magnetizing currents

(7) $$i_m = 2\sqrt{(i_A^2 + i_A i_B + i_B^2)/3}; \ \ i_A = i_{SA} + i_{RA}; \ i_B = i_{SB} + i_{RB}.$$

In case of saturation absence magnetization characteristic is degenerated in straight line $i_m = \alpha_m \psi_m$, where α_m is inverse main inductance, but matrix (4), according to (6) is transformed in diagonal form (scalar)

(8) $$G = 1/(\alpha_S + \alpha_R + \alpha_m),$$

what considerably simplifies the equation (1). In this case we receive the simplest from all known mathematical model of asynchronous motor; R_S, R_R are resistance matrixes

(9) $$R_s = \begin{array}{|c|c|}\hline r_S & \\ \hline & r_S \\ \hline \end{array}; \ R_R = \begin{array}{|c|c|}\hline r_R & \\ \hline & r_R \\ \hline \end{array},$$

besides that: α_S, α_R, are inverse dissipation inductance of stator and rotor windings; r_S is resistance of stator phase; r_R is transformed resistance of rotor winding; ω is angular velocity.

The components of columns of mine full magnetic flux of stator and rotor windings we find as

(10) $$\Psi_{kj} = \frac{1}{\tau}(i_{Sj} + i_{Rj}) + \frac{1}{\alpha_R} i_{kj}, \quad k = S, R; \ j = A, B.$$

The elements of columns stator and rotor voltages are

(11) $$u_S = U_m \sin(\omega_0 t) + u_C/3, \ U_m \sin(\omega_0 t - 120^0)_t +$$
$$+ u_C/3; \quad u_R = 0,$$

where U_m, ω_0 are amplitude and angular frequency of line supply: u_C is condenser voltage.

Naturally, that differential equation (1) it is necessary to add to differential equation of capacitor

$$\frac{du_C}{dt}=-\frac{i_{SA}+i_{SB}}{C}, \tag{12}$$

where C is capacitor capacitance.

The equation of mechanical movement we receive according to Lagrange's equation having neglected of stiffness and dissipation of mechanical links

$$\frac{d\omega}{dt}=\frac{p_0}{J}(M_E-M(\omega)),\ M_E=\sqrt{3}p_0(\Psi_{SA}i_{SB}-\Psi_{SB}i_{SA}), \tag{13}$$

where: $M(\omega)$ *is* torque; p_0 is number of pair of magnet poles; *J* is rotor moment of inertia; M_E is electromagnetic moment.

The system of differential equations (1), (12), (13) is mathematical model of induction motor with capacitor in phase C. It serves for analyze of transitional and steady-state processes. For practical using of it we must know such input data: resistance and inverse dissipation induc–tance of stator and rotor, no-load characteristic of motor (in case of saturation absence inverse main inductance), capacitor capacitance, number of pair of magnet poles and moment of inertia of rotor. As input signals are: line supply voltage of stator, torque of drive shaft.

The solution of Cauchy problem.

The system of ordinary differential equations (1), (12), (13) we write down in common form

$$\frac{dx}{dt}=f(x,t),\qquad x=(i,u_C,\omega)_t. \tag{14}$$

The integration of differential equations (14) under the initial condition that

$$x(t)_{|t=+0}=x(0) \tag{15}$$

and represents Cauchy problem for given system of differential equations, which represents problem of calculation of transitional electromechanical processes of motor.

The solution of two-point boundary problem.

The equation of periodicity by given period of input signals *T* for all unknown function (14) of under investigation device we write down in usual form

$$f\left(x(0)\right)=x(0)-x(x(0),T)=0, \tag{16}$$

where $x(0)$ is column of unknown initial condition which include transitional response.

Having integrated the equation (1), (12), (13) by given initial conditions $x(0)$, we receive a possibility to enter in periodic steady-state, having gone transitional process.

The equation (14), (16) are two-point boundary problem for differential equations of electromechanical state of three phase induction motor by capacitor in third phase. Transcendental equation (16) we solve by iteration Newton's method

$$x(0)^{(s+1)}=x(0)^s-f'\left(x(0)^{(s)}\right)^{-1}f\left(x(0)^{(s)}\right). \tag{17}$$

The Jacobi's matrix we receive by integration of purpose function (16) over $x(0)$

$$f'\left(x(0)\right)=E-\Sigma(T), \tag{18}$$

where $\Sigma(T)$ is monodromy matrix

$$\Sigma(T)=\frac{\partial x(x(0),t)}{\partial x(0)}\Big|_{t=T}. \tag{19}$$

In order to receive the matrix (19), we must build the substituted model of sensitivity to initial condition [1, 2]. Wherefore we create substituted column of unknown *y*

$$y=(\Psi,u_C,\omega)_t. \tag{20}$$

Proper to (20) differential equation (1) will be

$$\frac{d\Psi}{dt}=u-\Omega'\Psi-Ri, \tag{21}$$

Monodromy matrix (19) we write down as [1]

$$\Sigma=(\mathrm{A}z,q,w)_t, \tag{22}$$

where:

$$z=\frac{\partial\Psi}{\partial x(0)};\quad q=\frac{\partial u_C}{\partial x(0)};\quad w=\frac{\partial\omega}{\partial x(0)}. \tag{23}$$

The variation equations for calculation of submatrixes (23) we receive by differentiation of equations of electromechanical state (12), (13), (21) over x(0).

Having differentiated (21) over x(0), we receive

$$\frac{dz}{dt}=\frac{\partial u}{\partial x(0)}+(\Omega'-R\mathrm{A})z+\frac{\partial\Omega'}{\partial\omega}w\Psi. \tag{24}$$

The first derivative over x(0) in (24) according to (2), (11) will be

$$\frac{\partial u}{\partial x(0)}=\frac{1}{3}(q,q,0,0). \tag{25}$$

Having differentiated (12) over x(0), we receive

$$\frac{dw}{dt}=-\frac{1}{C}\left(\frac{\partial i_{SA}}{\partial x(0)}+\frac{\partial i_{SB}}{\partial x(0)}\right). \tag{26}$$

Having differentiated (13) over x(0), we receive

$$\frac{dw}{dt}=\frac{p_0}{J}\left(\sqrt{3}p_0(\frac{\partial\Psi_{SA}}{\partial x(0)}i_{SB}+\Psi_{SA}\frac{\partial i_{SB}}{\partial x(0)}-\frac{\partial\Psi_{SB}}{\partial x(0)}i_{SA}+\Psi_{SB}\frac{\partial i_{SA}}{\partial x(0)})-\frac{\partial M(\omega)}{\partial\omega}w\right). \tag{27}$$

The $\partial\Psi_{SA}/\partial x(0),\ \partial\Psi_{SB}/\partial x(0),\ \partial i_{SA}/\partial x(0),\ \partial i_{SB}/\partial x(0)$ are elements of matrixes *z*, *Az*, therefore they are known.

So the creating of monodromy matrix needs of integration of equations of first variation (24), (26), (27).

On *s*-th iteration of Newton's formula (17) the linear variation equations (24), (26), (27) liable to integration with nonlinear equations of electromechanical state (1), (12), (13) on time interval [0,*T*]. As a result we find purpose function (16) and necessary Jacobi's matrix (18), that determines the right part of iteration formula (17) and its left part $x(0)^{(s+1)}$. Process of iteration is finished when will be achieved given precision ε of entering in periodic state

$$\left|f\left(x(0)^s\right)\right|\le\varepsilon. \tag{28}$$

Computation of static stability

Let a initial condition $t = t_0$, $x = x_0$ correspond the solution of differential equation $x = x(t)$, $(0 \le t \le \infty)$ which we shall call as no disturb. The solution by other initial condition $t = t_0$, $\tilde{x} = \tilde{x}_0$ we shall call as disturb. According to Lyapunov no disturb solution is called as stable if by infinite small changing of initial condition disturb solution is retained in infinite nearness from no disturb during all further time

(29) $\max\left(\tilde{x}(t) - x(t)\right)\Big|_{t_0 \le t \le \infty} \to 0$, *if* $\left|\tilde{x}_0 - x_0\right| \to 0$.

If the condition (29) is not satisfied then no disturb. solution is called as no stable according to Lyapunov.

The solution is called as asymptotic stable if in addition to (29) by enough small $\left|\tilde{x}_0 - x_0\right|$ will be certainly satisfied condition

(30) $$\left|\tilde{x}(t) - x(t)\right|_{t \to \infty} \to 0.$$

The stability of periodic solution we shall be determine according to equations of first variation (24), (26), (27). The system of this differential equations is linear therefore for stability its zero solution necessarily and sufficiently, in order to all its solution was limited by $t \to \infty$. For asymptotic stability is necessarily and sufficiently, in order to all its solution was approached to zero by $t \to \infty$, what follows from determination of both stabilities.

It is discovered that all solutions of variation equations are gone to zero by $t \to \infty$, is necessarily and sufficiently that modulus of all eigenvalues of monodromy matrix Σ (22) was smaller as unity. Eigenvalues of monodromy matrix are called as multiplicators.

All numerical methods of calculation of eigenvalues we can separate on two groups. To first group belong a methods which need of building of characteristic equations, having solved such as we receive necessary eigenvalues To second group belong iterative methods. Here eigenvalues we receive as limit of some numerical execution sequences. Both groups are developed sufficiently. Nowadays every computer has necessary interface software.

Results of computation

For example we shall be given the results of calculation some electromechanical processes of squire cage induction motor (P = 320 kW, U = 6 kV, I = 39 A, n = 740 rev. per min.), loaded by torque $M(\omega)$ = 2900 Nm. The steady-state on $\omega = 311$ el. rad. per s. is received for three iteration (the fourth, properly speaking is steady-state process) of Newton's formula. Input data:

$$U_m = 4900;\ \omega_0 = 314;\ \alpha_S = 38,9;\ \alpha_R = 70;\ r_{SA} = r_{SB} = r_{SC} = $$
$$= 1,27;\ r_R = 0,11;\ J = 67,5;\ p_0 = 4;\ C = 300 \cdot 10^{-6};$$

no-load characteristic is

$$\psi_m = 9 + 0.508(i_m - 11) + 0.0064(i_m - 11)^2 + 0.000147(i_m - 11)^3.$$

On figures are shown the results of computation of transient (fig. 1) and stable steady-state (fig. 2) processes.

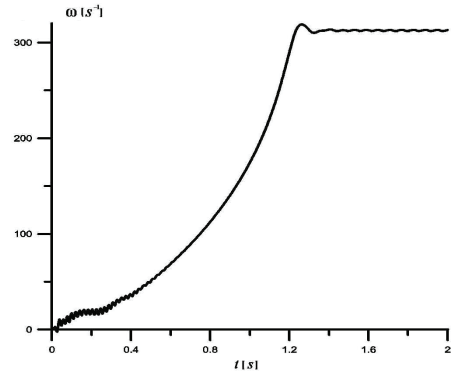

Fig. 1. Setting in motion: time curve of angular velocity

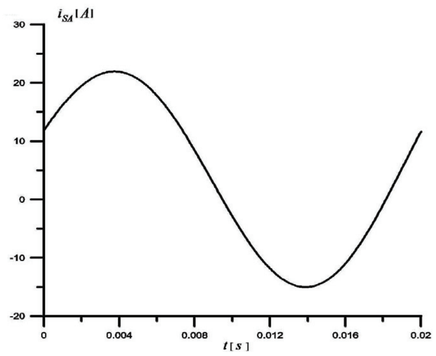

Fig. 2. Periodical current of stator on period of line supply voltage

The multiplicators of monodromy matrix in this stable process are such:

0,6599 ± j0,779; 0,004; -0,012; -0,028 ± j0,004.

As we see theirs modulus considerably smaller as unity. Therefore we draw a conclusion that received solution is asymptotic stable. It is received by zero approaching $x(0)^{(0)} = (0,0,0,0,0,314)$ (zero currents, zero voltages and synchronous velocity of rotor rotation) of Newton's formula (17).

The proposed method received detailed testing in complicated problems of electro mechanics. It is very effective.

Conclusion

The simplest analysis of periodic steady-state processes of electric devices is realized on base of solving of two-point boundary problem for differential equations of their electromechanical state. Herat simultaneously we receive the possibility of analysis of transient and steady-state processes.

REFERENCES

[1]. Tchaban V. Mathematic modeling of electromechanical processes (in Ukrainian). – Lviv, 1997, 344 c.

[2]. Tchaban V. Mathematic modeling in electric engineering(in Ukrainian). – Lviv*: T. Soroka publisher house*, 2010, 508 c.

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