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Systems of recurrent polyphase signals

***Abstract**. The algorithm of forming and results of research of correlation properties of the system containing polyphase signals formed on a base of recurrent sequences and Frank code are offered. This investigation has shown that the more phase levels signal has, the more accurate is its autocorrelation function and bigger system of signals can be built. The main "users" of this technique are cell phone communication systems, radar systems and some others specific applications.*

Streszczenie.** W artykule opisano tworzenie systemu generującego sygnały wielofazowe z wykorzystaniem nawracających sekwencji i kodu Franka oraz zbadano właściwości korelacji dla takich sygnałów. Wykazano, że wielofazowe sygnały pozwalają lepiej odtworzyć funkcję autokorelacji , dzięki czemu można stworzyć systemy o większych wymiarach.* ***(Systemy generacji nawracających sygnałów wielofazowych)

Keywords: correlation, polyphase signals, phase coded signals
Słowa kluczowe: korelacja sygnały wielofazowe, sygnały kodowanie fazowo.

Introduction

Construction of the radio engineering systems with the use of adaptive algorithms of work requires to search not only the laws of change of structure of the system and principles of its work but also the choice of signal which in every specificl situation would allow (at all identical parameters of the system) to provide the receipt of more high-quality indexes of work.

The wide field for construction of the large systems of signals is opened in the case of the use of polyphase signals with correlations and crosscorrelation descriptions.

The systems of recurrent polyphase signals appear on the basis of sequences of maximal length, formed first-formative polynomials on a base no panders from the Galua fields GF(p), where p - prime numbers (p=3, 5, 7, 11.). The phase quantized on N levels, such signals are attributed to the polyphase systems.

Generation of polyphase sequences

Polyphase sequences are generated based on the shift registers of the number with base N, spanned by feedback, that is built using prime polynomial over field GF(p), when p = N, i.e., all calculations are done with mod N.

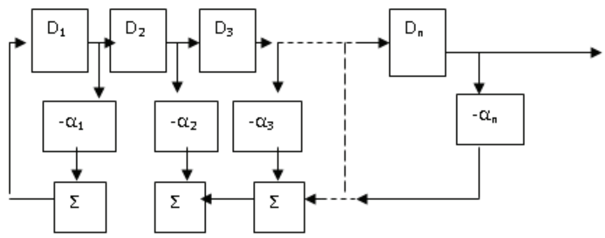

Fig. 1.Block diagram of multiphase sequence generator

For example, for the prime polynomial of $\alpha_0 x^3 + \alpha_1 x^2 + \alpha_2 x + \alpha_0$ with coefficients $\{\alpha_0, \alpha_1, \alpha_2, \alpha_3\}$= {1021} generation scheme of triple sequence of maximum length $M=N^n-1$= 3^3 -1 =27 – 1 = 26 is presented at Fig.2.

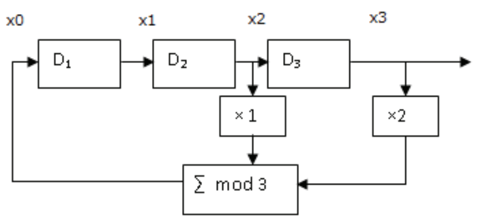

Fig. 2.Block diagram of triple recurrent sequence generator

For any initial combination (except case of 000, when register will always be empty) sequence …**100**20212210222001012112011**100**20212210222001012112011**100**202......... will appear at the shift register output.

Marked initial combination 100 will repeat every 26 clock ticks, i.e., maximum length sequence will be generated.

Turning to the forming of VHF pulses with duration τ_0 and initial phase φ_i, periodic with duration $M\tau_0 = 26\tau_o$ and period $T_п = M\tau_o$ or non-periodic with only duration $T_c = M\tau_o$ polyphase (triple phase) signals can be generated

Main properties of polyphase recurrent sequences.

1. Polyphase recurrent sequence is periodic with period of $M = N^n - 1$, where n is the shift register order.
2. In sequence all symbols are equiprobable, except symbol "0", which appears one time less.
3. Because of logarithmic dependence of n from M

$$n = \frac{\log(M+1)}{\log N}$$

small increasing of registers number causes significant increasing of pulses number in the recurrent sequence period.
4. Cyclic shift of the recurrent sequence (for example, in e way of other initial stage selection) is also the recurrent sequence.
5. Summation of recurrent sequence with shifted recurrent sequence is also recurrent sequence.

Forming of the polyphase sequences system

The systems of polyphase signals are formed with the help of the shift registers of number with a basis N, overcame by the circles, spanned by feedback, built on the basis of polynomials no panders above the field GF(p) (two, necessarily different), at p = N (that is all operations are executed after the module N).

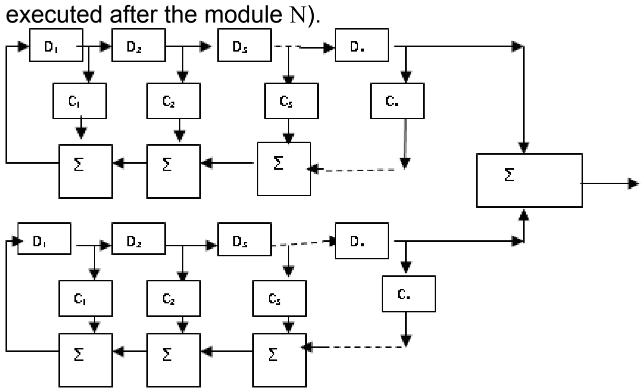

Fig.3.Block diagram of system polyphase signals generator

Table 1. Triple sequences from the system of polyphase recurrent signals

i	0	1	2	3	4	5	6	7	8	9	10	11	12
f-st	1	0	0	2	0	2	1	2	2	1	0	2	2
s-d	0	0	1	1	1	0	2	1	1	2	1	0	1
∑	1	0	1	0	1	2	0	0	0	0	1	2	0
i	13	14	15	16	17	18	19	20	21	22	23	24	25
f-st	2	0	0	1	0	1	2	1	1	2	0	1	1
s-d	0	0	2	2	2	0	1	2	2	1	2	0	2
∑	2	0	2	0	2	1	0	0	0	0	2	1	0

As a result sequences turn of number with the base of N which radio-pulsed signal with initial phases belong in accordance, multiple $\Delta\varphi=2\pi/N$.

We will account for the process of forming of polyphase sequences on the example of forming of the system of triple sequences with $M=26$.

So by means polynomial $C_0x^3+C_1x^2+C_2x+C_3$ no pander, set by coefficients $\{C_0, C_1, C_2, C_3\}=\{1021\}$ by the first shift register is formed first triple to the sequence of maximal length $M=N^n-1=27-1=26$, which for initial combination 100 acquires a kind:

…..10020212210222001012112011**100**2021221022200 1012112011**100**2**…..**

For second polynomial $C_0x^3+C_1x^2+C_2x+C_3$ no pander, set by coefficients $\{C_0, C_1, C_2, C_3\}=\{1201\}$ by the second shift register is formed second triple to the sequence of maximal length $M=N^n-1=27-1=26$, which for initial combination 100 acquires a kind:

…..10022201221202001110211210**100**2220122120200 1110211210**100**2**.....**

An initial triple sequence appears in the process of addition at the mod 3 shift registers of two recurrent triple sequences got on outputs.

Setting be – what initial combination on generator of the second sequence (and such combinations it is possible to set M) on the output of chart we get to M of different triple recurrent sequences which make the basis of the system of three-phase signals.

Forming radiopulsed signal by duration of initial phases, chosen according to an algorithm, the system of discrete polyphase radiopulsed signal is formed.

Basic properties of the system of recurrent polyphase signals

A plenty of no panders original of them polynomials at the same values of p ($p=3, 5, 7$ ..) allows to build the large systems of polyphase recurrent signals with far the best autocorrelation and crosscorrelation functions.

The periodic functions of autocorrelation of polyphase recurrent signals are three levels with the standard deviation value of level of lateral petals to even

$$\sigma_\delta=\frac{(1+\frac{4}{M})}{M}\approx\frac{1}{M}$$

Lateral petals of functions of autocorrelation of the limited polyphase recurrent sequences the axes of τ and their standard deviation value more evenly distributed along is evened

$$\sigma_\tau\approx\frac{1}{0.6M}$$

The values of lateral petals function of crosscorrelation some grow and in the first are close to their standard deviation value can be appraised with the help of balance.

Correlation property of such system signals by the illustrate graphs represented on figures 4, 5, 6, 7

$$\sigma_{твз}=\frac{1+\sqrt{M}}{M}$$

Does not the maximal level of lateral petals functions of crosscorrelation exceed the size of $d_{max}=0.28$.

These data are already confirmed that the systems of signals, formed polyphase recurrent sequences on a base N, provide the best correlation properties is comparative with the systems of signals Golda. The increase of number of levels of quantum of phase in signals and increase of size of base conduces to reduction as a relative maximal size of lateral petals so to more even distributing of lateral petals along the axis of τ.

The functions of autocorrelation and functions of crosscorrelation for the three-phase signals from such system of signals acquire the kind represented on figures 4, 5, 6, 7.

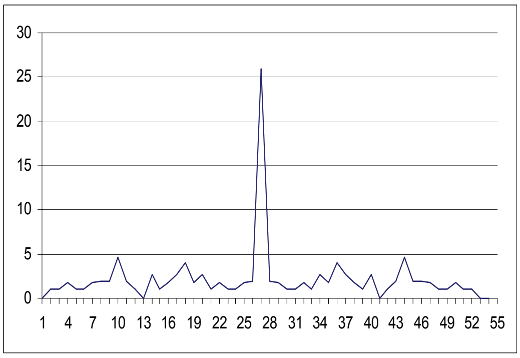

Fig.4. Function of autocorrelation of first formative three-phase sequence of polyphase recurrent signals for M=26

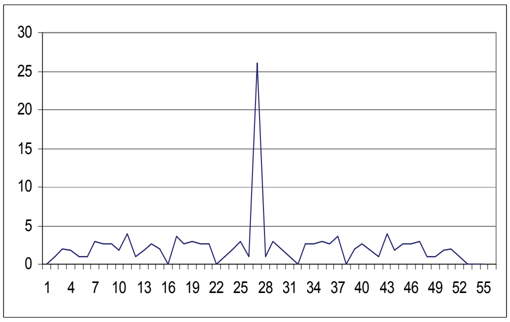

Fig.5. Function of autocorrelation of second formative three-phase sequence of polyphase recurrent signals for M=26

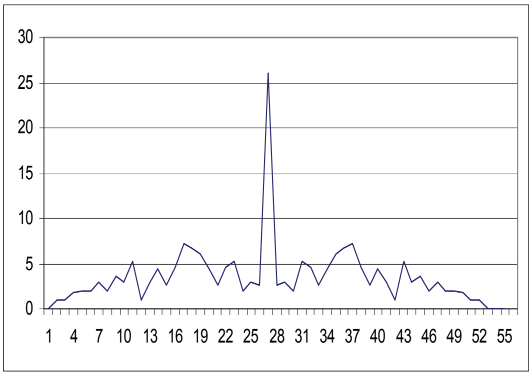

Fig.6. Function of autocorrelation of total signal from the system polyphase (three-phase) recurrent signals for M=26

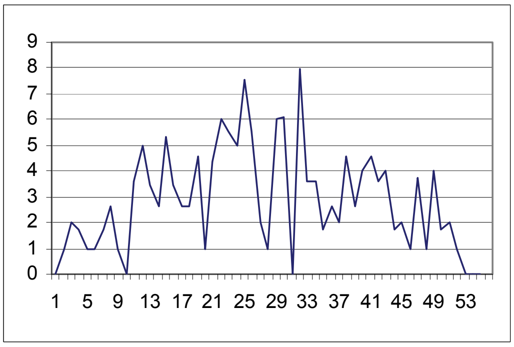

Fig.7. Function of crosscorrelation between by a signal first and total signal systems of polyphase recurrent signals for M=26

Generation of Frank code

The latest investigations in the field of theory of signals have shown that the next step in signals systems progress is use of more complicated polyphase signals with more phase levels as recurrent signals with, signals based on Frank codes and other codes

Frank code is a discrete complex signal which consists of the sequence of elementary signals, amplitude and phase of which we get according to the algorithm:

$$\theta_n = \theta_{pM+m} = pm\frac{2\pi}{N}, 0 \le N-1, 0 \le m \le K-1;$$

$$A_n = \begin{cases} 1, 0 \le m \le L-1 \\ 0, L \le m \le K-1 \end{cases}$$

where N is a number of phases levels in signal.

Frank code is formed as follows. A time domain equal to the duration of the signal T_S is divided in $M=NK$ temporal positions of duration $T_0=T_S/M$. On each of these temporal positions radio signals with frequency f_0 are formed, rounding and initial phase if which are chosen according to the algorithm

Forming the Frank code signal system

The systems of Frank code signals are also formed with the help of the shift registers, spanned by feedback, and a module N summation device. So, we can say that such signals system is also recurrent. Scheme of system formation device is shown on figure 8.

Frank code itself is predefined sequence. Formation of signals system goes through the following steps. First code element and the next one are summed on module N. The result is being memorized as the second code element of new signal (the second formative of signals system). The process continues till the last two elements being summed on N module.

To get the next formative of signals system, code values have to be used like the first formative.

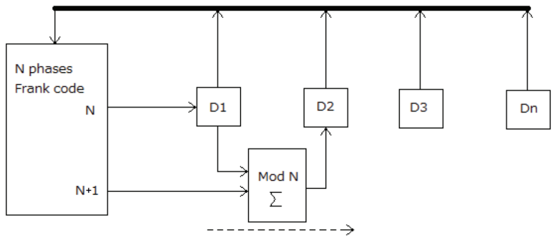

Fig.8. Block diagram of Frank code signals system generator

Frank code itself is predefined sequence. Formation of signals system goes through the following steps. First code element and the next one are summed on module N. The result is being memorized as the second code element of new signal (the second formative of signals system). The process continues till the last two elements being summed on N module.

To get the next formative of signals system, code values have to be used like the first formative.

Basic properties of the system of Frank code signals

The functions of autocorrelation and functions of crosscorrelation for the three-phase signals from such system of signals acquire the kind represented on figure 9, 10, 11, 12, 13.

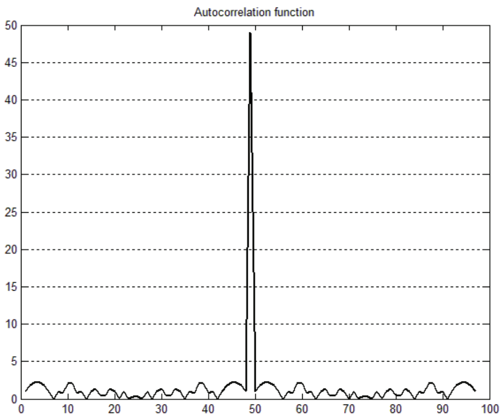

Fig.9. Function of autocorrelation of the first formative of Frank code (7 phase levels) signals system

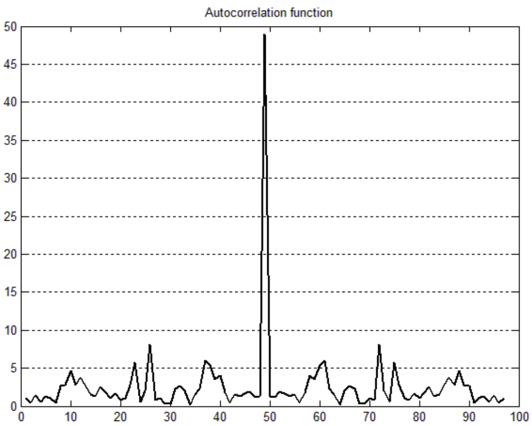

Fig.10. Function of autocorrelation of the third formative of Frank code (7 phase levels) signals system

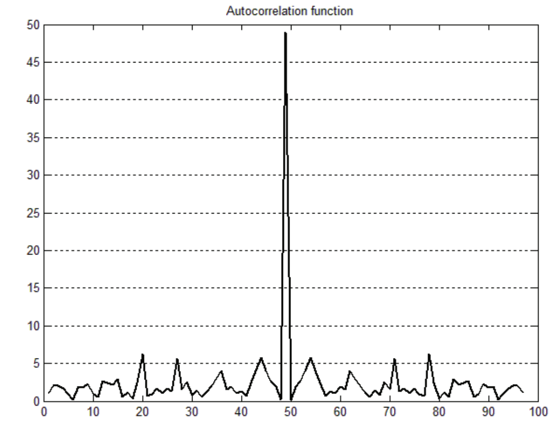

Fig.11. Function of autocorrelation of the tenth formative of Frank code (7 phase levels) signals system

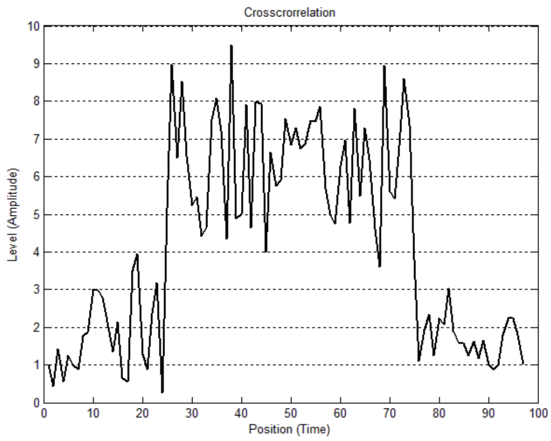

Fig.12. Function of crosscorrelation of the first and the third formative of Frank code (7 phase levels) signals system

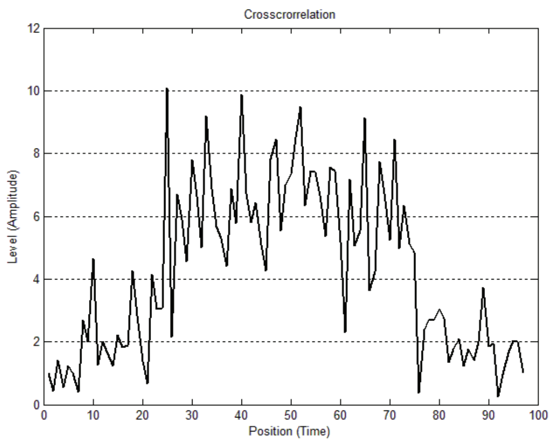

Fig.13. Function of crosscorrelation of the third and the tenth formative of Frank code (7 phase levels) signals system

As can be seen from these figures, the signals based on Frank code have less levels of side lobes. The first formative has level of approximately 5%. Investigations have shown that the more phase levels signal has, the less level side lobes exist. The next formatives have greater values of side lobes levels. However, they don't exceed the level of 10%

Conclusions

The use of signals systems is possible in many devices and systems. This investigation has shown that the more phase levels signal has, the more accurate is its autocorrelation function and bigger system of signals can be built. As the main "users" are cell phone communication systems, radar systems and some others specific applications. Introduction of polyphase signals systems in these systems will make them more flexible and accurate.

REFERENCES

[1] Church R., Tables of irreducible Polynominals for the First Four Prime Moduli, *Ann. Of Math.* Vol.36,1935

[2] Elspas B., The Theory of Autonomous Linear Sequential Networks, *Trans. IRE*, vol.CT-6, 1959

[3] Nadav Levanon, Eli Mozeson, Radar Signals, *John Wiley & Sons, Inc,* 2004

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