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Mathematical models and macromodels of electric power transformers

***Abstract.** In the paper different approaches used for creation of electric power transformers are discussed and main types of their mathematical models and macromodels are described. Mathematical model of transformer is created in the hybrid coordinate basis, and macromodel of the power transformer is developed using the "black box" approach in the form of state space equations, the features of such type models usage are analyzed.*

Streszczenie.** W artykule opisano alternatywne podejścia do konstrukcji transformatorów i podstawowe rodzaje ich matematycznych model i makromodeli. Omówiono tworzenie matematycznego model transformatora w hybrydowym układzie współrzędnych i budowę makromodelu w postaci równań stanu z wykorzystaniem podejścia typu ,,czarna skrzynka". Przedyskutowano własności opracowanych modeli. **(Matematyczne modele i makromodele transformatorów mocy)

Keywords: mathematical model, macromodel, power transformer, optimization
Słowa kluczowe: modele matematyczne, makromodele, transformatory mocy, optymalizacja

Introduction

Mathematical models of electric power system elements, including power transformers, intended for research of electromagnetic transient processes were considerably improved during last ten years and adapted for simulation using various computer programs. During simulation of electric power transformers it is necessary to consider such physical phenomena as deep saturation, hysteresis, losses due to eddy currents and thermal effects and etc. Development of transformer models with high level of adequacy based on their equivalent circuits causes their high complexity, and, consequently, considerable difficulties during their adaptation to modern computer programs and environments usually used for computer analysis of transient processes [1, 2].

If the researcher is interested just in reaction of the object to test signal just on its terminals then it is expedient to use principles of macromodelling, namely to create macromodels of real elements instead of their precise mathematical models. Therefore in this situation it is possible to use models of elements of the "black box" type widely used for creation of models of the electronic circuits' elements and electromechanical converters. Usage of macromodels can be explained by such considerable advantages as possibility to replace several elements of complex electric systems by single macromodel, simple adaptation to transient processes simulation programs and smaller time of their simulation.

The methods of macromodeling can be divided into two groups from the point of view of types of the models – they are the mathematical macromodels and circuit macromodels. Typical mathematical or logic relationships, graphs and etc are used for the first type models creation. In the second case dependences between voltages and currents are represented using connections of typical electric circuit elements.

During last years the considerable progress during power transformers mathematical models creation is observed and main attention is paid to the analysis of transient processes, fault regimes and periodic processes, which are rather adequate in describing the reaction to test signals. Such models can consist of several hundreds of nodes and branches that complicate the simulation from one side that is unpractical during investigation of the reaction of the "input-output" type. Macromodels of elements with different physical nature in the form of "black box" type using the state equations can solve this problem.

In the paper different approaches used for creation of electric power transformers are discussed and main types of their mathematical models and macromodels are described.

Mathematical models and macromodels of power transformers

Mathematical models intended for power transformer transient processes analysis typically are created based on division of magnetic flux into magnetic core main flux and leakage flux; using the equivalent circuits developed obtained due to the principle of duality or based on principle of uniform magnetic flux. These models are considered to be the most accurate ones, but their practical implementation using modern software is connected with substantial complications [2, 3].

The goal of the research is creation of adequate mathematical models and macromodels of power three-phase three-core transformer. The two types of models have been created:

1) mathematical model of three-phase three-core power transformer in the hybrid coordinate basis of the state coordinates, mesh coordinates and branch coordinates using the electromagnetic state equations of the transformer composed on the basis of its electric and magnetic equivalent circuits. The model was implemented into MATLAB/Simulink program using transformation of differential equations into integral based on mathematical operators from its libraries (integrators, multipliers, summators) [1]. If such type model is used it is necessary to know internal parameters of the researched object;

2) mathematical macromodel of three-phase three-core power transformer of the "black box" type in the form of discrete state equations based on transient characteristics of the transformer (winding currents and voltages obtained as a reaction of the object on step voltage applied to one of the windings). The macromodel was developed using experimental data obtained during short-circuit and no-load modes on high-voltage and low-voltage windings (computer experiment using detailed mathematical model of the object). The following discrete state equation is proposed as a tool for the macromodels mathematical description [4]:

$$\begin{cases} \vec{\mathbf{x}}^{(k+1)} = \mathbf{F} \cdot \vec{\mathbf{x}}^{(k)} + \mathbf{G} \cdot \vec{\mathbf{v}}^{(k)} + \vec{\Phi}(\vec{\mathbf{x}}^{(k)}, \vec{\mathbf{v}}^{(k)}) \\ \vec{\mathbf{y}}^{(k+1)} = \mathbf{C} \cdot \vec{\mathbf{x}}^{(k+1)} + \mathbf{D} \cdot \vec{\mathbf{v}}^{(k)} \end{cases} \quad (1)$$

where $\vec{\mathbf{x}}^{(k)}$ is a vector of discrete values of state variables, $\vec{\mathbf{v}}^{(k)}$ is a vector of input variables, $\vec{\mathbf{y}}^{(k)}$ is a vector of discrete values of output variables, $\mathbf{F}$, $\mathbf{G}$, $\mathbf{C}$, $\mathbf{D}$ are matrices of corresponding sizes, $\Phi(\vec{\mathbf{x}}^{(k)}, \vec{\mathbf{v}}^{(k)})$ is some vector-function of several variables, k is a discrete number.

The modeled object is described using “black box” approach. Vectors $\vec{\mathbf{v}}$ and $\vec{\mathbf{y}}$ are formed from the set of external variables in arbitrary way. To identify parameters of the model (1) or to find matrices $\mathbf{F}$, $\mathbf{G}$, $\mathbf{C}$, $\mathbf{D}$ and function $\Phi(\vec{\mathbf{x}}^{(k)}, \vec{\mathbf{v}}^{(k)})$ an a priori information in the form of transient characteristics $y_i(t)$ caused by input disturbance obtained experimentally or after computer simulation should be used.

The procedure of the macromodel creation consists of the following stages:

1) Formation of the task of macromodel development as a function of input variables important for their behavior. For macromodel creation initial input data can be estimated from the separate characteristics of electric circuits, and sets of input and output data and the macromodel type written down using unknown coefficient should be defined;

2) Planning an experiment for estimation of number of experiments and selection of the variables values;

3) Obtaining of the experimental data on experimental devices or using computer simulation. Sometimes creation of the macromodel requires to conduct measurements on several test installations, large number of experiments or causes complications during the measurements of the test variables, so this stage can be time consuming and has high cost.

4) The macromodel mathematical equations forming (here we have the discrete state equations) and estimation of the eventual range of the variables based on their physical sense;

5) During the macromodel creation the main problem consists in the complexity of the optimization procedure which not always can be solved even when good computers are used. It is caused by the complex character of the goal function that is typical for the macromodel creation and its high dimension because of large number of coefficients in developed macromodels. Therefore it is necessary to resample down the test date or to split macromodel on several parts (splitting relatively to typical range of the test data or relatively to output variables);

6) Creation of linear macromodel without vector-function $\Phi(\vec{\mathbf{x}}^{(k)}, \vec{\mathbf{v}}^{(k)})$ using optimization [5, 6];

7) Supplementing of linear macromodel using non-linear function $\vec{\Phi}$. Here it should be noted that the most complicated problem is how to develop the method of the macromodel construction for concrete object. Particularly, it is not always possible to separate the identification procedure of parameters of linear and nonlinear parts of the macromodel to make a choice of the mathematical form of their description. In such a case it is expedient to conduct the procedures of linear and nonlinear parts of the macromodel using different approaches of identification that complicates essentially the macromodel creation. Usually in order to conduct the optimization procedure of the parameters identification the diakoptic approach is used.

8) Conducting of the final optimization of the macromodel when all its coefficients are under optimization;

9) Verification of non-linear macromodel using independent set of test experimental data.

Obtained results

The power three-phase three-core transformer of ТДТН 110/35/10 type (in Ukraine) with rated power 10 MVA was used as the test example. An equivalent electric circuit used for creation of detailed mathematical model and obtaining test transient data [7] is shown in the Fig. 1.

During creation of the transformer macromodel in the form of discrete state equations (1) voltages of high-voltage and medium-voltage winding were used as input variables, and phase currents of primary and secondary winding were used as output variables. The macromodel was developed using experimental data obtained in short-circuit mode on high-voltage and medium-voltage windings (computer experiment using detailed mathematical model of the object) when direct voltage was applied from the side of high-voltage or medium-voltage winding.

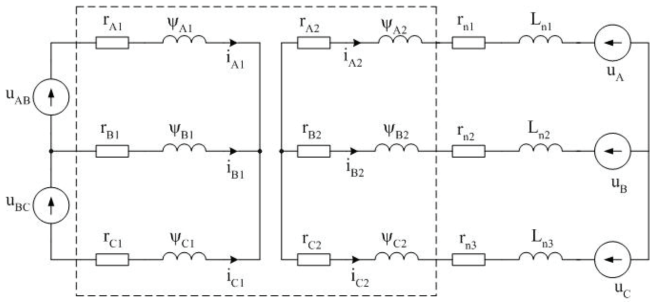

Fig.1. An equivalent electric circuit of the power transformer.

For simplification of the optimization task the resampling of experimental data was conducted in the following way:
$\vec{\mathbf{x}}^{(km)} = \tilde{\vec{\mathbf{x}}}^{(k)}$, $\vec{\mathbf{v}}^{(km)} = \tilde{\vec{\mathbf{v}}}^{(k)}$, $\vec{\mathbf{y}}^{(km)} = \tilde{\vec{\mathbf{y}}}^{(k)}$

where $\vec{\mathbf{x}}$, $\vec{\mathbf{v}}$, $\vec{\mathbf{y}}$ are vectors of original data, $\tilde{\vec{\mathbf{x}}}$, $\tilde{\vec{\mathbf{v}}}$, $\tilde{\vec{\mathbf{y}}}$ are vectors of resample data, *m* is a resample coefficient. To accelerate the calculation the linear macromodel for resample data using optimization was carried out in such form:

$$\begin{cases} \tilde{\vec{\mathbf{x}}}^{(k+1)} = \tilde{\mathbf{F}} \cdot \tilde{\vec{\mathbf{x}}}^{(k)} + \tilde{\mathbf{G}} \cdot \tilde{\vec{\mathbf{v}}}^{(k)} \\ \tilde{\vec{\mathbf{y}}}^{(k+1)} = \tilde{\mathbf{C}} \cdot \tilde{\vec{\mathbf{x}}}^{(k+1)} \end{cases} \tag{2}$$

where $\tilde{\mathbf{F}}, \tilde{\mathbf{G}}, \tilde{\mathbf{C}}$ are matrices of corresponding sizes (we consider that $\tilde{\mathbf{D}} = 0$). Matrix $\tilde{\mathbf{F}}$ was diagonal. For common usage of developed macromodel with mathematical models of other elements and further simulation it is necessary to conduct transformation to the initial data. Therefore matrices $\tilde{\mathbf{F}}, \tilde{\mathbf{G}}, \tilde{\mathbf{C}}$ developed from the resample data were converted to matrices **F**, **G**, **C** correct for the initial data range using the following transformation:

$$x_i^{(k+1)} = \sqrt[m]{\tilde{f}_{ii}}\, x_i^k + \frac{1 - \sqrt[m]{\tilde{f}_{ii}}}{1 - f_{ii}} \sum_j \tilde{g}_{ij} v_j^{(k)} \tag{3}$$

$$y_i^{(k+1)} = \sum_j \tilde{c}_{ij} x_j^{(k+1)} \tag{4}$$

$$g_{ij} = \tilde{g}_{ij} \frac{1 - \sqrt[m]{\tilde{f}_{ii}}}{1 - \tilde{f}_{ii}}, \; f_{ii} = \sqrt[m]{\tilde{f}_{ii}}, \;, \; c_{ij} = \tilde{c}_{ij} \tag{5}$$

After this liner macromodel was enlarged with nonlinear function $\vec{\Phi}$ of special type and final optimization of the macromodel when all coefficients were under optimization was conducted.

To create the macromodel the resample data sets were used (m=25). Linear macromodel was obtained using the Rastrigin's direct cone method with adaptation of parameters [5]. After transformation from matrices $\widetilde{\mathbf{F}},\widetilde{\mathbf{G}},\widetilde{\mathbf{C}}$ developed for resample data to matrices **F**, **G**, **C** correct for the macromodel in the field of original experimental data using equations (3) and (4). Obtained linear macromodel looks as:

$$\vec{\mathbf{x}}^{(k+1)} = \begin{pmatrix} 0.995 & 0 & 0 & 0 \\ 0 & 0.995 & 0 & 0 \\ 0 & 0 & 0.989 & 0 \\ 0 & 0 & 0 & 0.989 \end{pmatrix} \vec{\mathbf{x}}^{(k)} + $$

$$+ \begin{pmatrix} 0.0008 & -0.0004 & -0.0016 & 0.0008 \\ -0.0004 & 0.0008 & 0.0008 & -0.0016 \\ -0.0032 & 0.0016 & -0.0154 & 0.0077 \\ 0.0016 & -0.0032 & 0.0077 & -0.0154 \end{pmatrix} \vec{\mathbf{v}}^{(k)}$$

$$\vec{\mathbf{y}}^{(k+1)} = \begin{pmatrix} 0.7165 & 0.0079 & -0.1816 & -0.0031 \\ 0.0084 & 0.7151 & -0.0024 & -0.1822 \\ 1.1479 & 0.0056 & 0.7209 & 0.0007 \\ 0.0064 & 1.1490 & 0.0010 & 0.7209 \end{pmatrix} \vec{\mathbf{x}}^{(k+1)}$$

Obtained linear macromodel was enlarged with nonlinear function

$$\vec{\Phi}(\vec{x},\vec{y}) = \sum_k \vec{\alpha}_k x_{ik}^{\gamma} v_{jk}^{3-\gamma} \quad (6)$$

where $\vec{\alpha}_k$ are coefficients of the model, $\gamma \in \{2,3\}$.

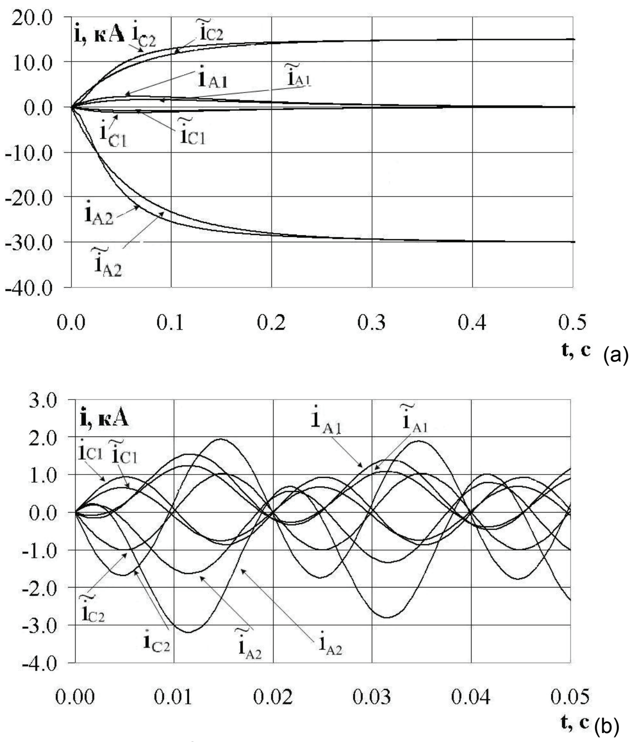

Fig.2. Reaction of obtained model to the test data (time characteristics of current in high-voltage and medium-voltage winding) when direct voltage was applied to low-voltage winding (a) or sinusoidal voltage to high-voltage winding (b)

Reaction of the model to test signals obtained during the short-circuit mode when direct voltage with value equal to the amplitude of the sinusoidal voltage when the voltage was applied from the side of low-voltage winding is shown in the Fig 2 (a). The model was tested when the sinusoidal nominal voltage was applied to the high-voltage winding of the transformer. Reaction of obtained model to the test data in this mode is depicted in the Fig. 2 (b), where $i_{A1}, i_{A2}, i_{C1}, i_{C2}$ are currents obtained using original model, currents $\widetilde{i}_{A1}, \widetilde{i}_{A2}, \widetilde{i}_{C1}, \widetilde{i}_{C2}$ – using developed macromodel.

Obtained discrete macromodel can be used for electric and power system elements analysis with the purpose of their dynamic modes calculation along with detailed mathematical model and can be introduced into modern software and applied for complex system research with the purpose of improving of quality and reliability of the modeling process[8].

Simulation results obtained due to mathematical macromodel testing were compared with transient characteristics obtained using detailed mathematical model of transformer of the same type introduced into MATLAB/Simulink software; the features and advantages concerning usage of each model were analyzed.

Conclusions

So, the methods of mathematic macromodels creation developed in the dynamic systems theory can be spreaded for analysis of electromechanical and electric power ssystems. Macromodels created using the "black box approach" in the form of discrete state equations can be used for analysis of electric system transients and introduced into the modern computer programs for this purpose.

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