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Failure Intensity determination for system with standby doubling

***Abstract**. The paper is devoted to problem of failure intensity calculation for doubled repairable system with standby reduced redundancy. Failure intensity determination is suggested by using special method for extended Markov reliability model. The correctness for such approach is verified by Monte-Carlo method.*

Streszczenie.** W artykule opisano problem obliczania intensywności awarii zduplikowanych systemów o zredukowanej nadmiarowości. Zaproponowano nową metodę wyznaczania częstotliwości awarii przy pomocy rozszerzonego modelu niezawodności Markova. Prawidłowość zaproponowanej metody potwierdzono za pomocą metody Monte-Carlo. **(Wyznaczanie częstotliwości awarii systemów ze podwójną gotowością)

Keywords: Reliability, Markov Model, Failure Intensity, Standby Reserve
Słowa kluczowe: niezawodność, modele Markowa, częstotliwość awarii, rezerwa gotowości

Introduction

The failure intensity *z*(*t*) is a ratio of mean number repairable system failures for infinitesimal life to this life quantity. This index shows frequency of system operational state leaving. Both availability and failure intensity are characterized by repairable system reliability and availability property. This paper is devoted to failure intensity determination for repairable system with standby doubling. Doubling system consists from two identical subsystems: main and reserve. During normal mode system functioning is provided by main subsystem that is fully loaded. Reserve subsystem at this time is turn off, it's haven't any load and so can't to fail. If main subsystem breaks down, then reserve one changes unload mode to load mode. So in such state system functioning is provided by reserve subsystem. Using of standby doubled redundancy is the most effective approach for technical system reliability improving.

The practical aspect of such problem decision is agreed with accuracy improvement for failure intensity and other concerned indexes determination for repairable systems. And the theoretical aspect is provided further improvement Markov analysis approach for systems that have components with general failure and renewal distribution models.

Literature reviews

The problem of failure intensity determination is especially important for reliability analysis of electromechanical and power electrical systems. For such systems this index permanently is monitored and predicted [1, 2]. Analytical approach for failure intensity function derivation is concerned with linear Volterra integral equations. The method of such equations is known for the simplest case [3]. However, based on analytical equations in [4, 5] approximate approaches are formed for particular case and useful result can be achieved. If the system is structured, component failure models or external factors effect aren't known than failure intensity is predicted by means of statistical analysis of history process behaviour [6] or by neuron nets and fuzzy logic [7, 8]. This approach needs essential time expenses for data front-end processing or neuron net teaching. And result based on such approach has low authenticity for long-term forecasting. In [9] failure intensity function bounds are proposed to find by means Bayesian approach. But if curve function has essential amplitude on considered time interval than curve range bounds are very wide and so useless. Paper [10] is based on non-homogeneous Poisson process (NHPP) used for repairable system failure intensity. The main disadvantage of this approach is as follows: there is no method that NHPP model parameters and system component failure model parameters can univocal correspondence is supported. The Monte-Carlo method also is used for failure intensity simulation [3, 11, 12]. However, curves obtained by this method have stochastic error that complicating they quantity analysis. If Monte-Carlo number iteration is increased, than stochastic error have recessionary tendencies. But time needed for simulation is increased too.

It's well-known that reliability indexes for repairable systems are calculated by means of state space method. This method is based on ordinary homogeneous Markov reliability models [13, 14] and extended ordinary Markov reliability models [15 — 17]. In [14, 17] it's shown how failure intensity for repairable system can be determined using ordinary Markov model. Yet, this approach is limited only by exponential distribution for failure and renewal models of system components. Perspective study consists in how extended Markov reliability model must be applied for failure intensity determination. Such approach can provide calculation failure intensity for general failure and renewal models of system components.

For problem solving such statements should be assumed:

1. It's to suggest that the method for failure intensity determination for repairable system with standby doubling that based on extended Markov model.
2. Its necessary to verify the result correctness by means model for treated system using Monte-Carlo method.

Failure intensity determination

The system of differential equations written by means of state space equations (using vectors and matrixes) is understood as the Markov model:

$$\frac{d}{dt}\mathbf{p}(t) = \mathbf{\Lambda}\,\mathbf{p}(t)\,,$$

where *d/dt* — symbol, that means time derivation for each vector component; *t* — time (or life); $\mathbf{p}(t)$ — state or phase probability function vector; $\mathbf{\Lambda}$ — state or phase intensity transition matrix.

Initial condition probability function vector $\mathbf{p}(0)$ must be added to this differential equation system. Markov model construction come to only intensity transition matrix $\mathbf{\Lambda}$ and initial condition probability function vector $\mathbf{p}(0)$ determination. Such model can be graphically represented by state transition diagram that have a single meaning with $\mathbf{\Lambda}$ and $\mathbf{p}(0)$.

Failure intensity function is determined using the rule described in [14, 17, and 18]. System failure intensity is calculated as sum of products transition intensity, that cross system from operational phases to system failed phases, and according probability functions of such operational phases.

The system functions according to the maintainability algorithm shown in the fig 1.

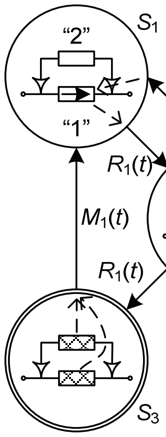

Fig. 1. State transition diagram for repairable system with standby doubling

In the initial time there is an operational state S_1. In this state both components are operational, main — "1" and reserve — "2". First component lifetime is distributed by failure model $R_1(t)$. Second component in this state is not loaded and therefore can't fault. The system crosses to state S_2 after fist component failure. It's assumed that technical diagnostics facilitates are ideal. This means that component failure is tested instantaneously. After repair renewed component is "as good as new". In the operational state S_2 fist component is operational and second component — is not operational. First component repair time is distributed by renewal model M_1(t), and second component lifetime is distributed by failure model $R_1(t)$. If the first component is renewed, than system returns to state S_1. If second component is fault, than system crosses to state S_3. In nonoperational state S_3 both components are nonoperational. It's assumed that both components repair time is distributed by renewal model M_1(t) too and after renewal the system comes back to state S_1. So system is continuously moved between discrete state set $\{S_1, \ldots S_3\}$.

Life for main and reserve system components under full load are distributed according phase-type failure model $R_1(t)$ (fig. 2.a) and c_0, c_1, c_2 and λ_1 are its parameters.

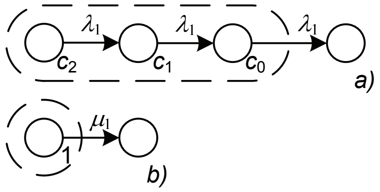

Fig. 2. State transition diagram for failure and renewal models of system components

Phase-type distributions are special set of distributions that are used for general distribution series expansion, similarly to Taylor series expansion. Phase-type distributions are important components for extended Markov model [16 — 19]. Analytical expression for failure model has such type form:

$$R_1(t) = \left(1 + (c_1 + c_2)\lambda_1 + \frac{c_2 \lambda_1^2}{2}\right) e^{-\lambda_1 t},$$

$$c_0 + c_1 + c_2 = 1.$$

Renewal model is given by exponential distribution with μ_1 parameter. Exponential distribution is the simplest phase-type distribution (fig. 2.b):

$$M_1(t) = 1 - e^{-\mu_1 t}.$$

As repair time is much less than life time therefore an error related with exponential distribution treatment is neglected.

System failure intensity function $z_1(t)$ is determined using extended Markov reliability model. State transition diagram (fig. 3) for this model is formed using phase-type distributions conversion and state space extension. Transition intensity matrix and initial condition vector for this model are so bulky and than are not shown. In details such models are described in [15].

Using mentioned designations a failure intensity function for repairable system with standby doubling is obtained as a product of Ph_{10} phase probability function $p_{10}(t)$ and transition intensity λ_1 from such phase:

$$z_1(t) = \lambda_1 \, p_{10}(t) .$$

For result correctness verifying system failure intensity $z_2(t)$ is computed by Monte-Carlo method (fig. 4). More information about this approach can be found in [3].

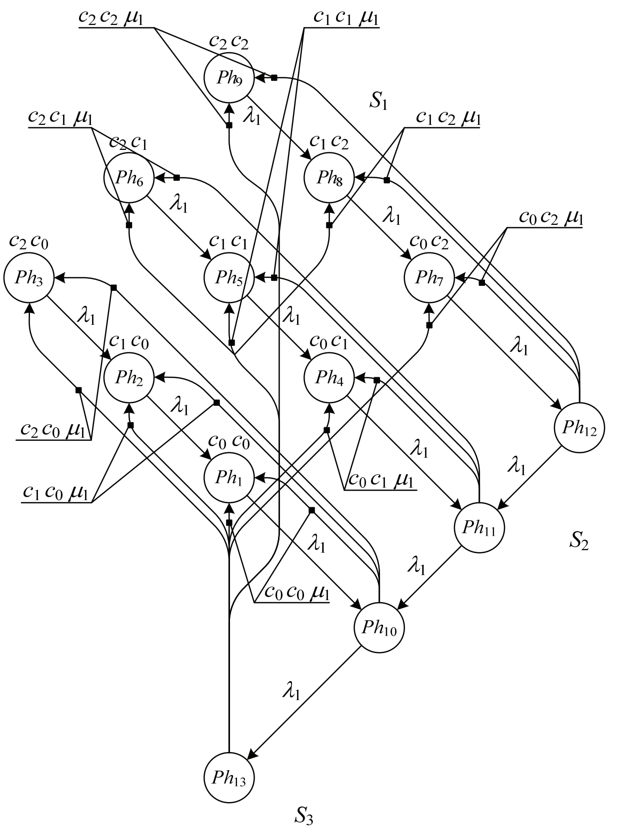

Fig. 3. Extended state transition diagram for repairable system with standby doubling

The Monte-Carlo model convergence is proved. As it can be seen from fig. 4 a — c, if number Monte-Carlo iteration *Nr* is increased, than mean square error *ERR* between failure intensity bold solid curve 1 computed using extended Markov model $z_1(t)$ and failure intensity solid curve 2 computed based on Monte-Carlo model $z_2(t)$ is tend to zero.

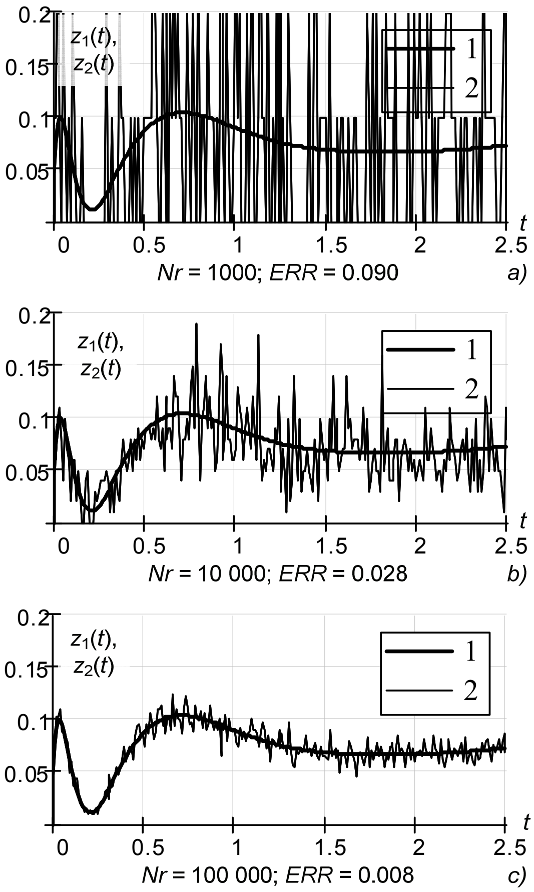

Fig. 4. Failure intensity curves for repairable system with standby doubling

The mean square error ERR between system failure intensity $z_1(t)$ and $z_2(t)$ is determined by formula:

$$ERR = \sqrt{\frac{1}{Nt}\sum_{i=0}^{Nt}\left(z_1(t_i) - z_2(t_i)\right)^2} \ ,$$

where Nt — time axis point number.

Conclusions

The method of failure intensity determination for repairable system with standby doubling based on phase-type distribution and extended Markov reliability model was improved. Failure intensity determination is provided for such systems with arbitrary failure and renewal models for components. Both failure intensity curves calculated by suggested approach and by Monte-Carlo method are agreed with permissible error that confirms correctness for proposed approach.

Further investigation is directed to spare parts and useful lifetime determination for repairable system with standby doubling using both acyclic Markov model and electrical transition processes taking into account [20].

The paper is written in the framework of "PNRL" research project framework and it's sponsored from Ukrainian state budget.

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