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Peculiarities of frequency symbolic method applied to parametric circuits analysis

Abstract. *The article deals with the assessment of asymptotic stability of linear parametric circuits by frequency symbolic method and approximation of transfer circuit function by the truncated Fourier series. Assessment of asymptotic stability is carried out according to the transfer function of one frequency. The examples of assessment of asymptomatic stability of single-circuit and double-circuit parametric amplifiers are given.*

Streszczenie. *W artykule oceniono możliwość oszacowania asymptotycznej stabilności liniowych obwodów parametrycznych przy pomocy częstotliwościowej metody symbolicznej i aproksymacji transmitancji przez szereg Fouriera. Do oszacowanie asymptotycznej stabilności wykorzystywana jest zwyczajna transmitancja. Zaprezentowano przykład oszacowania stabilności jednooczkowego i dwooczkowego obwodu parametrycznego.* ***(Osobliwości zastosowania metody symbolicznej do analizy obwodów parametrycznych)***

Keywords: symbolic analysis, linear parametric circuits, assessment of asymptotic stability.
Słowa kluczowe: analiza symboliczna, parametryczne obwody liniowe, oszacowanie stabilności asymtotycznej.

Introduction

Linear parametric circuit of time t is characterized by impulse transfer function $w(t,\xi)$ as a circuit response to delta-impulse given at the time period ξ. The function of two variables $w(t,\xi)$ is often used. However, one variable is often considered as a parameter. At the same time the researcher should deal with so-called conjugate impulse reaction when the parameter is t or normal impulse reaction when the parameter is ξ [1]. This condition divides the sums of research of linear parametric circuits into two classes. So, the sums connected with the stability analysis in the supervision period [1] are done with the help of normal impulse reaction and their representations according to Laplace that are normal parametric transfer functions $W(s,\xi)$ but the sums connected with the transfer of signals are done with the help of conjugate impulse reactions and their representations according to Laplace domain that are conjugate parametric transfer functions $W(s,t)$. The assessment of asymptotic stability is carried out according to so-called bifrequency transfer function $W(s,r)$ [1,2], that represents the result of double application of Laplace transformation to function $w(t,\xi)$.

The article shows the possibility of assessment of asymptotic stability of linear parametric circuit described by parametric transfer function $W(s,\xi)$ that is formed with much less computational burden than $W(s,r)$.

The purpose of the article

The paper [1] shows that spectral research methods of stability of linear circuits with constant parameters which are usually used by researchers can be transferred to linear circuits with variable parameters but with some peculiarities. We will consider these peculiarities and their application. So, this article deals with the problem of assessment of stability of linear parametric circuits performed by analysis of frequency symbolic method [3].

This problem can be solved with the help of transfer function of poles that quite frequently and usually is used for the stability assessment carried out by symbolic analysis of linear circiuts with constant parameters.

To reach this aim the following criteria of stability of linear parametric circuit given in [1] are used.

Criterion 1. Linear parametric circuit on the given observation interval T is stable if:

(1) $$\int_{\xi}^{\infty}\left|w(t,\xi)\right|dt<\infty\,,\quad 0<\xi<T$$

This leads to the following definition of stability: *linear system with variable parameters is stable on the interval only when its normal parametric transfer function $W(s,\xi)$ does not have poles in the right half-plane and on the imaginary axis of complex plane s by all ξ, which are in the interval under consideration.*

Criterion 2. Linear parametric circuit is stable asymptotically $0<\xi<\infty$ if the integral

(2) $$\int_{0}^{\infty}\int_{0}^{\infty}\left|w(t,\xi)\right|dtd\xi$$

is convergent absolutely. It demands the determination of all analiticity infringements of bifrequency transfer function $W(s,r)$ on the plane $\rho\sigma$ and making so-called coincident characteristic $\rho=\chi(\sigma)$ and area D_1 on this basis [1,2]. If area D_1 includes points with $\sigma<0$, the circuit with such characteristics $\rho=\chi(\sigma)$ is asymptotically stable.

The possibilities of application of critiria (1) and (2) for assessing the stability of linear parametric circuits when analysed by frequency symbolic method are considered and the peculiarities of such application are determined.

Main body

To calculate function $W(s,t)$ in [3] its approximation $\hat{W}(s,t)$ in the form of truncated Fourier series is successfully used:

(3) $$\hat{W}(s,t)=W_0(s)+\sum_{i=1}^{k}\begin{bmatrix}W_{ci}(s)\cos(i\Omega t)+\\+W_{si}(s)\sin(i\Omega t)\end{bmatrix}$$

or in complex form

(3a) $$\hat{W}(s,t)=W_0(s)+\sum_{i=1}^{k}\begin{bmatrix}W_{-i}(s)\cdot\exp(-j\cdot i\cdot\Omega\cdot t)+\\+W_{+i}(s)\cdot\exp(+j\cdot i\cdot\Omega\cdot t)\end{bmatrix},$$

where $W_0(s)$, $W_{ci}(s)$, $W_{si}(s)$ та $W_0(s)$, $W_{-i}(s)$, $W_{+i}(s)$ - independent on time t fractionally rational functions of complex variable s, k-number of harmonics in the series, $\Omega = 2\pi/T$, T - period of parameter change of parametric element of the circuit under the influence of pumping signal.

Such approximation has the following positive calculation properties:

1. it provides unique simplicity of determination of functions $W_0(s)$, $W_{-i}(s)$, $W_{+i}(s)$ or $W_0(s)$, $W_{ci}(s)$, $W_{si}(s)$ in the symbolic form due to the fact that it leads to doing the system of linear algebraic equations [3];
2. it allows to get exact expression for $\hat{W}(s,t)$ (from the point of view of methodic mistake) in any way by increasing the number of k included in series (3) terms [5];
3. it forms the expression $\hat{W}(s,t)$ which is much more compact than the expressions received by other, for example, approximate methods from [6];
4. it proved its high efficiency and convenience during the further research of linear parametric circuits [3,5] by researching the function $W(s,t)$ through the research of its approximation $\hat{W}(s,t)$.

The task of stability determination as mentioned above demands the previous determination of functions $W(s,\xi)$ or $W(s,r)$. Because of the mentioned above positive peculiarities of approximation (3) and (3a) the formation of function $W(s,\xi)$ should be carried out by analogy, for example in the form of Fourier trigonometic series:

$$\hat{W}(s,\xi) = W_0(s) + \sum_{i=1}^{k} \begin{bmatrix} W_{ci}(s)\cos(i\Omega\xi) + \\ + W_{si}(s)\sin(i\Omega\xi) \end{bmatrix}, \quad (4)$$

where $W_0(s)$, $W_{ci}(s)$, $W_{si}(s)$ - fractionally rational functions of s with the same denominator $\Delta(s)$ according to frequency symbolic method.

But according to criterion 1 the approximation (4) has a serious drawback for the assessing stability. It turned out that finding the poles of the function $W(s,\xi)$ defined in the form of Fourier trigonometric series does not lead to the desired aim and it is supported by the following arguments.

Argument 1. At different meanings ξ in the general case the circuit can be stable or unstable as its impulse transfer characteristic $w(t,\xi)$ can be falling at some values of ξ and not falling time function t at the others.

Argument 2. As the position of poles in function $W(s,\xi)$ (4) or denominator root of function $W(s,\xi)$ on the plane s determines stability or unstability of the circuit, the argument shows that the value of these poles (denominator roots) must depend on ξ.

Argument 3. The approximations (4) and (3,a) have denominators only in expressions $W_0(s)$, $W_{ci}(s)$, $W_{si}(s)$, $W_{-i}(s)$, $W_{+i}(s)$ which do not depend on ξ. This means that these denominator roots do not depend on ξ either.

The content of arguments 1 and 3 leads to the conclusion that according to criterion 1 the assessment of circuit stability with the help of denominator roots of function $W(s,\xi)$ approximated by Fourier series does not lead to desired result.

To solve this problem it is necessary to offer the other approximation of function $W(s,\xi)$ that by-turn makes its definition not always possible.

But the situation is not hopeless. As approximation (4) has the mentioned above positive calculating parts of approximation (3), it can be used to determine the function $W(s,r)$ to which the criterion 2 can be applied. So, we will use the dependence given in [1]:

$$W(s,r) = \int_0^{\infty} W(s,\xi)e^{-r\xi}d\xi. \quad (5)$$

In this case without denominators dependent on ξ (argument 3) approximation (4) provides quite ordinary calculation of integral (5) and as it is shown in [4] when using frequency symbolic method it changes the problem of assessment of asymptotic stability of the circuit from the analysis of characteristic of convergence $\rho = \chi(\sigma)$ of function $W(s,r)$ to ordinary finding of denominator root of $\Delta(s)$ of normal transfer function of cirzuit $W(s,\xi)$ in (4) with the biggest real part. At that for the assessment of asymptotic stability of linear parametric circuit according to criterion 2 it is enough [4]:

1. to find normal transfer function $\hat{W}(s,\xi)$ of this circuit by frequency symbolic method in the form of truncated Fourier series(4);
2. to find the roots of denominator $\Delta(s)$ of function $\hat{W}(s,\xi)$;
3. to define the availability of roots with zero or positive real parts among the roots of polynomial $\Delta(s)$. If such roots exist, the circuit is unstable, but if they do not exist, the circuit is stable asymptotically.

Control of calculation accuracy

The approximation of transfer functions $\hat{W}(s,t)$ and $\hat{W}(s,\xi)$ of the circuit performed by Fourier cut series imposes some peculiarities on the control of result accuracy. So, in general case it does not matter how many harmonics are accounted in approximation as there is no certainty that all significant harmonics are included into approximation expression.For the practical application of frequency symbolic method we can present the following method of accuracy control: *the accuracy of received transfer function with accounted harmonics in approximation k is sufficient if increase of number of harmonics in approximation n does not lead to considerable change of results received on its basis but it is insufficient –if vice versa.*

In the last case it is necessary to increase k and repeat the calculation. For example, if we have two versions of output signal of the circuit determined by product of input signal on approximation of transfer function $\hat{W}(s,t)$ when the number of harmonics is k and $k+n$ respectively and at that both output signals are within given deviation δ we consider transfer function $\hat{W}(s,t)$ to be determined quite accurately. If not, we do calculation of transfer function again with higher value of k as it is mentioned above.

We can speak about determination of roots $\Delta(s)$ of denominator of function $\hat{W}(s,\xi)$ by analogy. The increase in number of harmonics in approximation $\hat{W}(s,\xi)$ from k to $k+n$ leads to the increase in degree of polynomial $\Delta(s)$ that is to the increase in number of roots in it. According to frequency symbolic analysis [3] the degree of polynomial $\Delta(s)$ increases when number of harmonics included in approximation increases. But the number of different roots or at least the number of different real parts of roots is limited, that is why they begin to repeat after some k. At that we have two versions of roots determined from polynomial $\Delta(s)$ when the number of harmonics in $\hat{W}(s,\xi)$ is k and $k+n$ respectively. Since the most number of harmonics must lead to the increase in calculation accuracy, we consider that polynomial $\Delta(s)$ of higher degree must include "specified" roots of polynomial $\Delta(s)$ of lower degree and additional roots. When comparing the same roots between these versions in turn we determine if they (real and real parts) are situated within the given deviation δ. If so, we consider the given root to be determined with enough accuracy and we go to the following root. If not, we repeat the calculation of $\Delta(s)$ with higher value of k as it is mentioned above. This process finishes when the roots with new real parts stop to appear among accurate roots. Then the assessment of asymptotic stability of circuit is carried out according to real parts of the found "accurate" roots.

Obviously, the mentioned approach does not provide any guarantee of result accuracy but the higher n is, there are more chances that the suggested way of accuracy control gives exact result with the given number of harmonics k. We think that the final determination of number of necessary harmonics as the answer is up to researcher of the given linear parametric circuit .

Example 1.

To carry out the assessment of stability of single-circuit parametric amplifier shown in Figure 1.

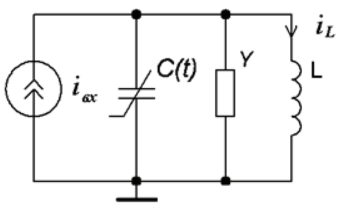

Fig.1. Single-circuit parametric amplifier, $i_{bx}=I_m\cdot\cos(\omega\cdot t+\varphi)$; $I_m=1A$; $\omega=1\,\text{rad/s}$; $Y=0{,}25\,\text{Sm}$; $L=1H$; $C(t)=C_0\cdot(1+m\cdot\cos(\Omega\cdot t))$; $C_0=1F$; $\Omega=2\cdot\omega$.

From current of signal $i_{вх}(t)$ to current of inductor $i_L(t)$ for the circuit from Figure 1 the normal parametric transfer function $W(s,\xi)$ is determined by frequency symbolic method [3] in the form of truncated Fourier series, for example, when $k=1$ it is:

(6)
$$\hat{W}(s,\xi)=\frac{A_0(s)}{\Delta(s)}+\frac{A_{c1}(s)}{\Delta(s)}\cdot\cos(i\Omega\xi)+ +\frac{A_{s1}(s)}{\Delta(s)}\cdot\sin(i\Omega\xi),$$

where

$A_0(s)=A_0(s,m)=-0.0625\cdot(4\cdot s^3+ +s^2+20\cdot s+4)\cdot s\cdot m;$

$A_{c1}(s)=A_{c1}(s,m)=2.3125+0.625\cdot s+ +2.515625\cdot s^2+0.125\cdot s^3+0.25\cdot s^4;$

$A_{s1}(s)=A_{s1}(s,m)=0.5\cdot s\cdot m\cdot(s^2+3);$

$\Delta(s)=\Delta(s,m)=2.3125+(0.25-0.125\cdot m^2)\cdot s^6+ +(0.1875-0.03125\cdot m^2)\cdot s^5+ +(2.796875-1.125\cdot m^2)\cdot s^4+ +(1.378906-0.125\cdot m^2)\cdot s^3+ +(4.984375-1.5\cdot m^2)\cdot s^2+1.203125\cdot s,$

m - intensity of modulation of capacity $C(t)$.

Expression (6) is solution to the following differential equation [1]:

(7)
$$\left[1+\left(-LC'(t)+LY\right)s+LC(t)s^2\right]\cdot W(s,\xi)+ +\left(LC'(t)-LY-2LC(t)s\right)\cdot W'(s,\xi)+ +LC(t)\cdot W''(s,\xi)=1,$$

which by-turn [1] issues from differential equation that describes circuit from Figure 1:

(8)
$$c(t)L\cdot i_L''(t)+[c'(t)L+yL]\cdot i_L'(t)+ +i_L(t)=i_{вх}(t).$$

Figures 2,a-2,f show trajectories of roots in the plane $\sigma j\omega$ that are received during the change of m from 0,15 to 0,7 and during the change of number of harmonics in approximation $\hat{W}(s,\xi)$ from one to six respectively. The beginning of every trajectory is signed by symbol «×» and at the end of trajectory there is root number which makes it. Complex conjugate root corresponds to every root from Figure 2. Trajectories of these conjugate roots are symmetric relative to the axis σ and they are not shown in the Figure 2. Analysing Figure 2 we can see the following:

- trajectories of roots 1 and 2 when changing in Figure 2,a and Figure 2,b, in Figure 2,c – Figure 2,f they are practically the same that is "specified";
- trajectories of roots 3 and 4 when changing in Figure 2,b and Figure 2,c, in Figure 2,d – Figure 2,f are practically the same ("specified") and equal to trajectories of roots 1 and 2 respectively;
- trajectories of roots 5 and 6 when changing in Figure 2,c and Figure 2,d, in Figure 2,e – Figure 2,f are practically the same ("specified")and equal to trajectories of roots 1,3 and 2,4 respectively;
- when increasing the number of harmonics in Δ(s) trajectories of roots 7,9 and 8,10 are considered to become equal to trajectories 1, 3, 5 and 2, 4, 6 respectively and so on;
- there are only two trajectories of "specified" roots with different real parts and they are within real values σ from – 0,17 to – 0,36 and from – 0,09 to + 0,02, respectively;
- trajectories of "specified" roots that cross the axis $j\omega$, have the same real part, that is why we consider them on the example of root 2 in more details.

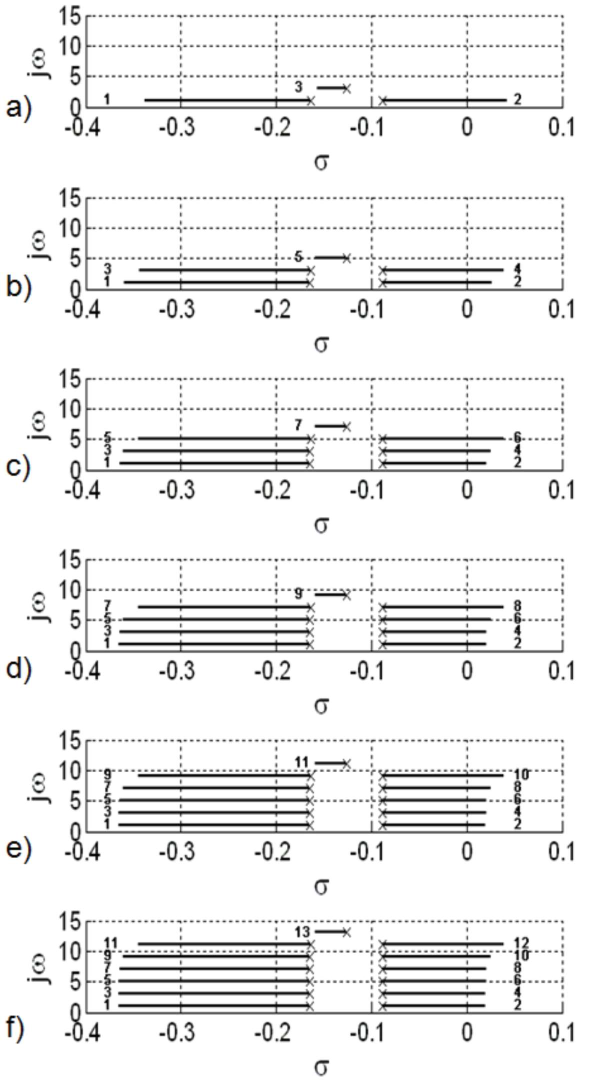

Fig.2. Trajectories of roots of denominator $\Delta(s,m)$ of function $\hat{W}(s,\xi)$ of the circuit in Figure 1, during the change of m from 0,15 to 0,7 and for the number of harmonics in its approximation from one to six: a – one harmonic, b – two harmonics, c – three harmonics, d – four harmonics, e – five harmonics, f – six harmonics.

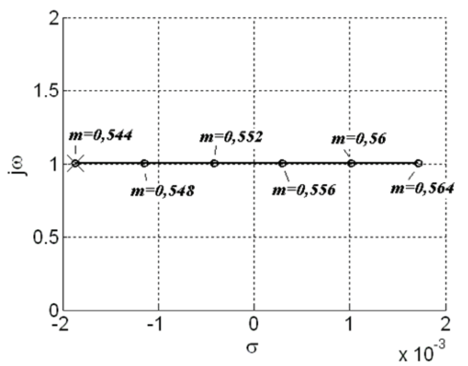

Fig.3. Trajectory of root 2 during the change of m from 0,544 to 0,564 with lead 0,004. The beginning of trajectory signed by symbol «×».

The fragment of trajectory of the root 2 for value m from 0,544 to 0,564 across 0,004 is shown in Figure 3 on an enlarged scale and as it can be seen from the figure it crosses axis $j\omega$ when $m = 0{,}554 \pm 0{,}002$. This means that at $m < (0{,}554 \pm 0{,}002)$ the circuit from Figure 1 is stable asymptotically, but at $m < (0{,}554 \pm 0{,}002)$ it is unstable. This result coincides absolutely with the result from the Figure 4 that is received for the circuit from Figure 1 by numerical method with the help of MicroCAP programme which analyses electric circuits.

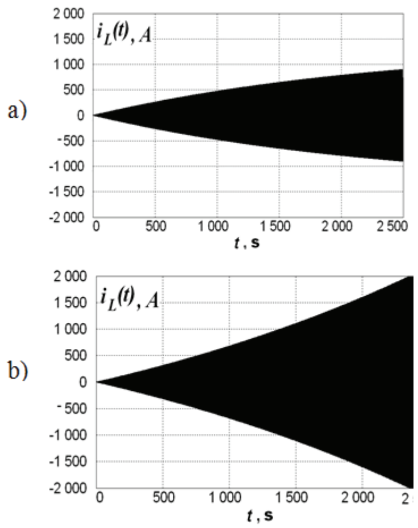

Fig.4. Time dependences of current of inductor $i_L(t)$ for the circuit from Figure 1 received with the help of MicroCap: a - $m = 0.552$, stable circuit; b - $m = 0.556$, unstable circuit.

Example 2.

To carry out the assessment of stability of double-circuit parametric amplifier shown in Figure 5. Signal circuit L1C1 icluding its shunting elements C0, L2, C2 is prepared for the frequency $\omega_0 = 2\pi \cdot 10^8$ rad/s. The frequency of input signal is $\omega_{c0} = \omega_0$ and the frequency of pumping is $\Omega_0 = 2 \cdot \pi \cdot 298{,}573$ rad/s .

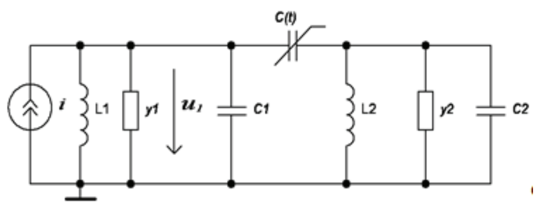

Fig.5. Double-circuit parametric amplifier :
$i(t) = I_m \cdot \cos(\omega \cdot t + \varphi)$; $\varphi = 45^\circ$; $\mathrm{Im} = 0{,}1mA$;
$y1 = y2 = 10^{-4} Sm$; $C1 = C2 = 68pF$; $L1 = 36{,}70795$ nH ;
$L2 = 9{,}312609$ nH ; $C(t) = C0 \cdot (1 + m \cdot \cos(\Omega \cdot t))$;
$C0 = 1pF$; $\Omega = 2 \cdot \pi \cdot 298573000$ rad/s.

Figure 6 shows the given trajectories of roots in plane $\sigma j\omega$ that are received during the change of m from 0,16 to 0,28 at approximation $\hat{W}(s,\xi)$ by two harmonics. The beginning of every trajectory in Figure 6 is signed by symbol "×".

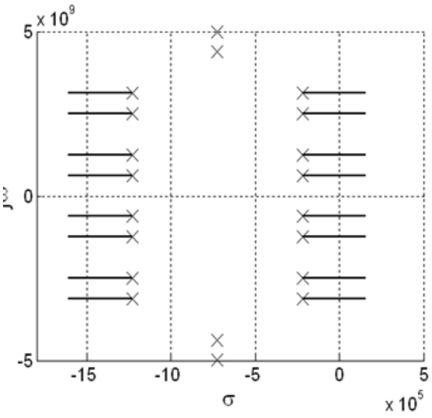

Fig.6. Trajectories of roots of denominator $\Delta(s,m)$ of function $\hat{W}(s,\xi)$ of the circuit from Figure 5 during the change of m from 0,16 to 0,28 for two harmonics of its approximation.

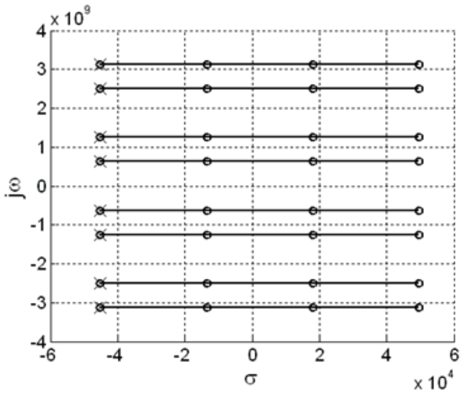

Fig.7. Trajectories of roots during the change of m from 0,215 to 0,245 with lead 0,01. The beginning of trajectory is signed by symbol"×".

The trajectories of the roots that cross axis $j\omega$ for the values m from 0,215 to 0,245 with lead 0,01 (the position of the root on every lead is signed by the symbol "o") are presented in Figure 7. As it can be seen from the figure the trajectories cross the axis $j\omega$ when $m = 0{,}23 \pm 0{,}005$. This means that at $m < (0{,}23 \pm 0{,}005)$ the circuit from Figure 5 is asymptotically stable but at $m \geq (0{,}23 \pm 0{,}005)$ it is unstable. This result coincides with the result received for the circuit from Figure 5 with the help of MicroCAP programme.

Conclusion

From the material presented in the article the following conclusion can be drawn.

1. Opposite to the assessment of stability of the interval, the assessment of asymptotic stability can be performed according to frequency symbolic method of analysis of linear parametric circuits during approximation of normal function of circuit transfer performed by truncated Fourier series.
2. When using frequency symbolic method of analysis the approximation $\hat{W}(s,\xi)$ in the form of (4) leads the problem of assessment of asymptotic stability of circiut from the analysis of convergence characteristic $\rho = \chi(\sigma)$ of function $W(s,r)$ to ordinary finding of denominator root $\Delta(s)$ of normal function of circuit transfer $W(s,\xi)$ with the biggest real part.
3. According to accuracy of stability assesment a number of harmonics k in approximation $\hat{W}(s,\xi)$ is considered to be sufficient if it provides the absence of roots with "new" real parts at increasing k.
4. The results of experiments in assessment of asymptotic stability carried out on the circuit in Figure 1 and the circuit in Figure 5 on the basis of the method offered in the article and MicroCAP programme coincide and it proves the correctness of the stated material.

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