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Image reconstruction with discontinuous coefficient in electrical impedance tomography

***Abstract**. The problem of the image reconstruction in Electrical Impedance Tomography (EIT) is a highly ill-posed inverse problem. There are mainly two categories of image reconstruction algorithms, the direct algorithm and the iterative algorithm which was used in this publication. The representation of the shape of the boundary and its evolution during an iterative reconstruction process is achieved by the level set function and the Chan-Vese model. The forward problem was solved by the finite element method.*

***Streszczenie.** Rekonstrukcja obrazu w tomografii impedancyjnej jest dokonywana poprzez rozwiązanie zagadnienia odwrotnego. Stosując algorytmy deterministyczne mamy do dyspozycji dwie kategorie rozwiązania: metodę bezpośrednią i model iteracyjny, który został wykorzystany w tej publikacji. W procesie rekonstrukcji została użyta funkcja zbiorów poziomicowych oraz model Chana-Vese (**Rekonstrukcja obrazu w zagadnieniu odwrotnym tomografii impedancyjnej**).*

Keywords: Electrical Impedance Tomography, Level Set Methods, Chan-Vese Model, Inverse Problem
Słowa kluczowe: tomografia impedancyjna, metoda zbiorów poziomicowych, model Chana-Vese, zagadnienie odwrotne

Introduction

In this paper was proposed a method based on the combination level set idea [5,7,8] and the finite element methods [2] to solve the inverse problem in the electrical impedance tomography. The presented method was based on a numerical scheme for the identification of piecewise constant conductivity.

Electrical Impedance Tomography

Electrical impedance tomography is a widely investigated problem with many applications in physical and biological sciences [3,4]. It is well known that the inverse problem is nonlinear and highly ill-posed. The forward problem in EIT is solved by Laplace's equation:

$$div(\gamma\, grad\, \mathbf{u}) = 0 \tag{1}$$

where: $\mathbf{u}$ - electric potential, γ - conductivity.

The following functional is minimized:

$$\mathrm{F} = 0.5\sum_{j=1}^{l}(\mathbf{U}-\mathbf{U_0})^{\mathrm{T}}(\mathbf{U}-\mathbf{U_0}) \tag{2}$$

where: l - the number of the projection angles.

The following steps are used in numerical algorithm:

- From the level set function $\phi(x,y)$ (initial) at a time level t, find the necessary interface information $\Gamma_o = \{\phi(x,y)=0\}$
- Calculate the electric potential (solving the Laplace equation by using the finite element method) $-\Delta\mathbf{u} = 0$
- Compute the difference of the computed solution with the observed data $\mathbf{u}(\Gamma_k) - \mathbf{u}_0$
- Solve the Poisson equation (adjoint equation) $-\Delta\mathbf{p} = \mathbf{u} - \mathbf{u}_0$
- Find the component of the normal velocity of the surface due to the electric potential
- Find the normal velocity of the level set function
- Calculate the velocity $\mathbf{v}_k = \nabla\mathbf{p}_k \cdot \nabla\mathbf{u}_k$
- Update the level set function
- Reinitialize the level set function.

Numerical Algorithms

A. Level set method

The level set method is known to be a powerful and versatile tool to model the evolution of interfaces. The idea is merely to define a smooth function ϕ, that represents the boundary. The interface information such as tangential and normal the derivatives and the curvature at projections are obtained from the values of the level set function at grid point plus a bilinear interpolations.

Thus, the interface is captured for all later time, by localization of the set Γ(t) for which ϕ vanishes. This deceptively trivial statement is of great significance for numerical computation primarily, because topological changes such as breaking and merging are well defined and performed. The motion is analyzed by the convection the ϕ values (levels) with the velocity field ν. The Hamilton-Jacobi equation of the form [7]:

$$\frac{\partial\phi}{\partial t} + \nu\cdot\nabla\phi = 0 \tag{3}$$

is describing this process.

Here ν is the desired velocity on the interface, and is arbitrary elsewhere. Actually, only the normal component of ν is needed $\nu_N = \nu\cdot\frac{\nabla\phi}{|\nabla\phi|}$, so (3) becomes:

$$\frac{\partial\phi}{\partial t} + \nu_N\cdot|\nabla\phi| = 0 \tag{4}$$

In the level set representation, the interface which is the set of points (*x,y*) satisfying $\phi(x,t)=0$ is not explicitly given. There is only information $\phi(x_i,y_i)$ at each grid point.

The expression for the curvature of the zero level set assigned to the interface itself is given by:

$$\kappa = \nabla\cdot\frac{\nabla\phi}{|\nabla\phi|} = \frac{\phi_{xx}\phi_y^2 - 2\phi_y\phi_x\phi_{xy} + \phi_{yy}\phi_x^2}{\left(\phi_x^2+\phi_y^2\right)^{3/2}} \tag{5}$$

Update the level set function ϕ (*x,t*) by solving the Hamilton-Jacobi equation:

(6) $$\frac{\phi^{k+1}-\phi^{k}}{\Delta t}+\mathbf{v}_k\left|\nabla\phi^{k}\right|=0$$

B. Chan-Vese model

The Chan-Vese algorithm set formulation and minimization problem in image processing, to compute piecewise-smooth optimal approximations of a given image. The proposed model follows and fully generalizes work [1], where there was proposed an active contour model without edges based on a 2-phase segmentation and level sets γ_{I} and γ_{II}. Conductivity γ is represented as:

(7) $$\gamma=\gamma_{\mathrm{I}}\mathbf{H}(\phi)+\gamma_{\mathrm{II}}(1-\mathbf{H}(\phi))$$

where **H** is the Heaviside function.
The functional in the Chan-Vese model is following:

(8) $$\mathrm{F}(\phi,\gamma_1,\gamma_2)=\omega\int_{\Omega}\left|\nabla\mathbf{H}(\phi)\right|d\Omega+\eta_1\int_{\Omega}(\mathrm{s}_o-\gamma_1)^2\mathbf{H}(\phi)d\Omega+\eta_2\int_{\Omega}(\mathbf{s}_o-\gamma_2)^2(1-\mathbf{H}(\phi))d\Omega$$

where: ω ,η_1, η_2 – coefficients, $\mathbf{s}_o$ – the original image.
The functional minimization provides to:

(9) $$\frac{\partial\phi}{\partial t}=\delta_{\varepsilon}(\phi)\left[\omega\nabla\cdot\left(\frac{\nabla\phi}{|\nabla\phi|}\right)-\eta_1(\mathbf{s}_o-\gamma_1)^2+\eta_2(\mathbf{s}_o-\gamma_2)^2\right]$$

In the electrical impedance tomography the derivative of F with respect to γ is given by:

(10) $$\left[\frac{\partial \mathrm{F}}{\partial\gamma}\right]=-\sum_{j=1}^{p}\nabla\mathbf{u}_j\nabla\mathbf{p}_j$$

where: **u** – the electric potential, **p** – the adjoint variable.

The final equation of the level set function was solved following:

(11) $$\phi^{k+1}=\phi^{k}-\mu(\nabla\mathbf{u}_k\cdot\nabla\mathbf{p}_k)(\gamma_{\mathrm{I}}-\gamma_{\mathrm{II}})\delta(\phi)$$

where μ - the coefficient

The iterative algorithm with the Chan-Vese model and the reinitialization was shown below (fig.1).

Results

The figure 2 presents an image reconstruction in EIT. The numerical model was inserted in the inside of the examined object. The grid was used by 16×16 elements solution, the number of these variables are equal only to 289. The representation of the shape of the boundary and its evolution during an iterative reconstruction process is achieved by the level set method. Numerical algorithm is a combination of the level set method for following the evolving step edges and the numerical method for computing the velocity. The pictures show different objects and the process reconstruction. The original object is noted by the blue line, zero level set function is red, the following iterations are green and the final figure is pink. The images show the original object and reconstruction after the process iterations. In the example the zero level set function as an initial condition representing by the circle was used. The final contour represents the zero value of the level set function. The process reconstruction is good, because the region borders are located nearly the object edges. Using the different the zero level set function, the object function achieves the minimum after the various number of iterations. The object in the figure (a) achieves the minimum after 416 iterations, whereas the same object in the figure (b) achieves the minimum after 631 iterations. Next figures (c), (d) show the process reconstruction with the different zero level set function. The shape and size of the zero level set function does not influence on the final form of the finding object.

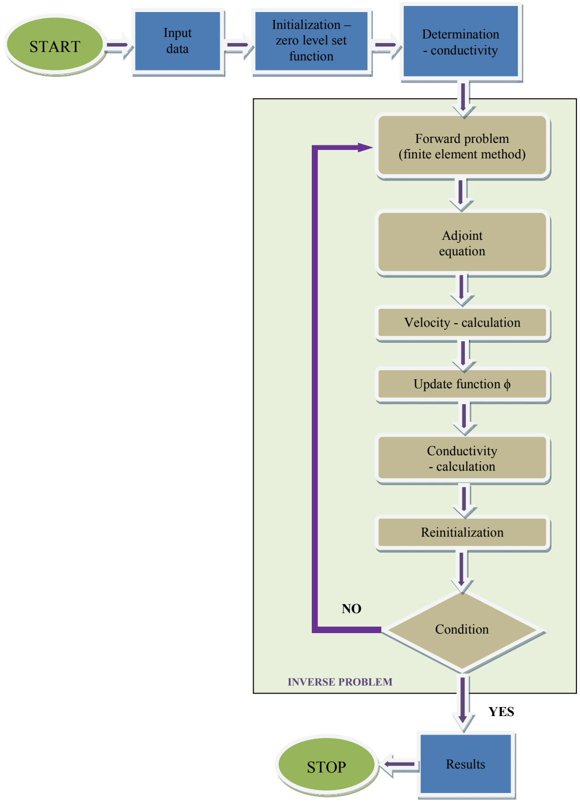


Fig. 1. The iterative algorithm with the Chan-Vese model

The figure 3 presents an image reconstruction in the electrical impedance tomography, too. It can be observed that a better quality image is obtained for the higher spatial image resolution 32×32. The level set function for identifying the unknown shape of an interface was used. The pictures show different objects and the process reconstruction with the reinitialization. The original object is noted by the blue line, the zero level set function is red, the following iterations are green and the final figure is pink. In the example the zero level set function as initial condition representing different shapes was used. The number of iterations were increased. For example: the object in the figure (a) achieves the minimum after 4636 iterations, whereas the other objects in the figures (b), (c), (d) achieve the minimum after different iterations.

Figure 4 shows the image reconstruction with different objects. The grid was used by 16×16 elements solution. The representation of the boundary shape and its evolution during an iterative reconstruction process is achieved by the level set method. The Laplace equation is calculated by the numerical method. The original object is noted by the black line, and the final figure is pink. Figure (a) shows two objects. The pictures: (b) has two objects in the corners, (c) three objects, (d) four objects.

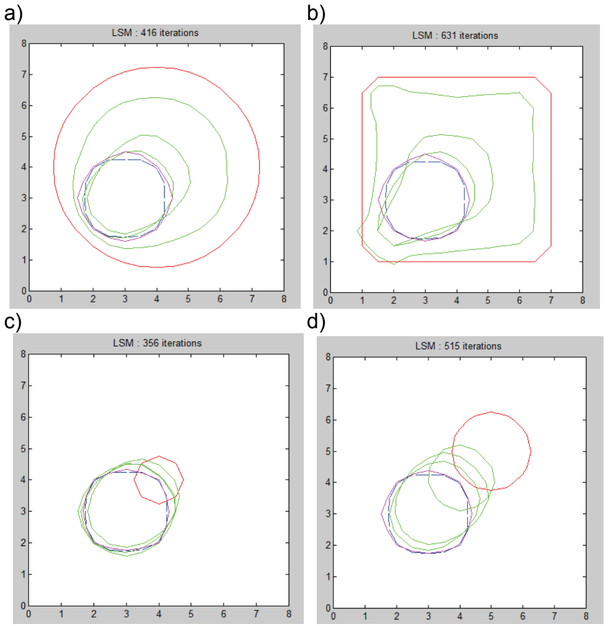


Fig. 2. The image reconstruction 16×16 (mesh size): a, b, c, d) different zero level set function

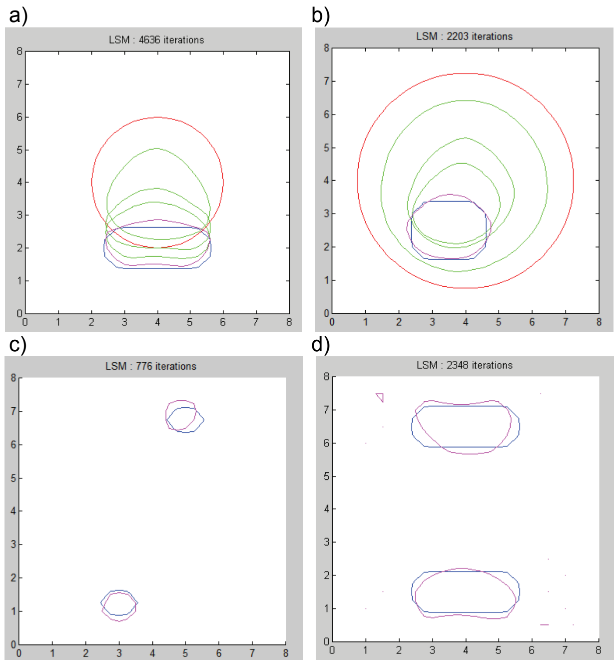
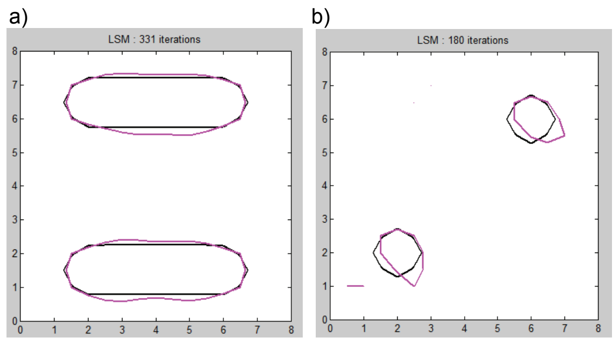


Fig. 3 The image reconstruction 32×32 (mesh size): a, b) different zero level set function – one object, c, d) two objects



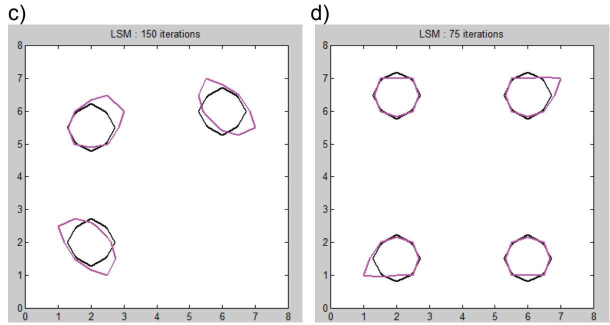


Fig. 4. The image reconstruction in EIT (16×16) ε=0, α =0.5: a) two objects, b) two objects in the corners, c) three objects, d) four objects

Conclusion

This paper has introduced the new method of approximation of the material coefficient (conductivity). There was discussed the application of the level set function for identifying the unknown shape of an interface in the inverse problem. Level set methods and the Chan-Vese model were chosen for the electrical impedance tomography. It is sometimes more important to recover the shape of the domains containing different materials than to recover the values for the materials. There is required to identify unknown conductivities from near-boundary measurements of the potential in the model problem of the electrical impedance tomography. It is assumed that the value of the conductivity is known in subregions whose boundaries are unknown. The Chan-Vese model for identifying the unknown conductivities in the object was used. The level set function techniques was shown to be successful to identify the unknown boundary shapes. The presented techniques were shown to be successful in the electrical impedance tomography.

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