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Simulation of spatial coordinate of movable ferromagnetic solid object in induced magnetic field by diakoptic methods

***Abstract**. In the paper transient processes of the electromagnetic-mechanical system are researched. Numerical experiments for transient analysis were carried out by software intended for simulation of electrical and electronic circuits by diakoptic methods where the large circuit is split on several parts – sub-circuits. Every part of the circuit is simulated independently on some time step. Numerical experiments have shown wide potential possibilities of proposed simulation method and its effectiveness.*

Streszczenie. *W pracy omówiono badanie stanów nieustalonych w systemach elektromagnetyczno-mechanicznych. Do numerycznego modelowania stanów nieustalonych wykorzystano oprogramowanie wykorzystujące metody diakoptyczne, w których złożony system jest rozbijany na pod-systemy analizowane niezależnie dla każdego kroku czasowego. Eksperymenty numeryczne pokazały wiele potencjalnych zastosowań tej metody i jej efektywność.* ***(Symulacja ruchu obiektu ferromagnetycznego w zewnętrznym polu elektromagnetycznym z wykorzystaniem diak optyki)***

Keywords: large complex system, diakoptic methods, subcircuit.
Słowa kluczowe: złożone systemy, metody diakoptyczne, pod-obwody

Introduction

The problem of calculation of trajectory of moving body in magnetic field of composite configuration has theoretical and practical importance (Fig.1). Many researchers paid a lot of attention to this problem [1, 2, 3 etc]. The main difficulty of this task is available complex nonlinear dependencies in the system equations, the simplification which leads to significant distortions of the solution. Therefore, effective way is to calculate the transient processes and finding the optimum conditions of complex system represented by an equivalent electrical circuit, affordable modelling software of electrical circuits by diakoptic methods.

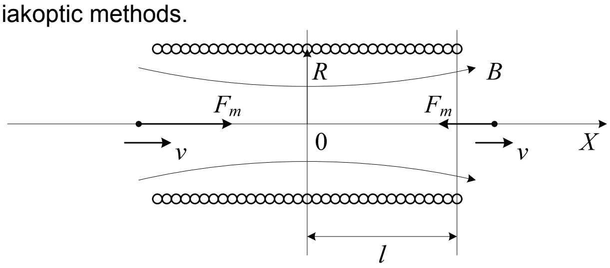

Fig.1. Trajectory of moving body in magnetic field

Analysis of large complex systems with lumped and distributed components usually is carried out by simulation of the same type system by essential simplification of alternative model circuit part. However, in order to solve such problem diakoptic approach allows taking into consideration special features of electrical, magnetic or mechanical part of the researched system from all points of view. Therefore it is reasonable to use program software for simulation of electrical circuits which allows implementation of diakoptic algorithms.

The mathematical formulation of the problem

Complex system (Fig.2) separated on three parts – subsystems: 1) electrical part – sequential RLC-loop. Loop current created magnetic field of solenoid, in which body is moving; 2) magnetic part – solenoid of specify structure, its magnetic flux is forming feedback with current of RLC-loop; 3) mechanical part – moving body is exposed into magnetic field of solenoid, which can be changing structure of magnetic field on certain conditions.

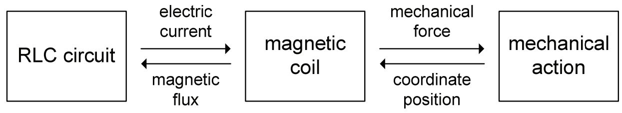

Fig.2. Complex system separated on three subsystems

Each subsystem described such equations. Therefore the *1st subsystem*:

$$\begin{cases} C\dfrac{du_C}{dt} = i_L; \quad L\dfrac{di_L}{dt} = -u_C - Ri_L \end{cases} , \tag{1}$$

where $L = \dfrac{d\Psi}{di_L}$ – inductance, which is adjusted during the calculation of transient process. If we assume that the magnetic flux is directly proportional to the current of the solenoid, then the inductance of the solenoid is constant and independent of the current. Otherwise necessary it's correcting of the inductance at simulation process.

In general the *2nd subsystem* is a component with distributed parameters, which described by differential equations in partial derivatives. However, there is sufficient for the first time to consider a simplified model of a solenoid, in our case, where to be given consideration on solution of the system as a whole and test diakoptic approach, providing capacity to replace it on the full system model with distributed parameters a further.

Thus, the 2nd subsystem is described by

$$F_m = m\frac{dB_X}{dX} , \tag{2}$$

where F_m – magnetic force, m – magnetic dipole moment, B_X – magnetic flux density, X – distance from centre of solenoid to body point. For example diamagnetic and paramagnetic materials overall value of dipole moment is proportional to of field B, then force will be proportional to multiplication of B on dB_X/dX, i.e. the square of the solenoid current. In ferromagnetic materials dipole moment approaches its extreme value. On basis of information from [4] for 1g of iron $m = 0{,}2353$ J/(T·g), for 1g of aluminium

$$m = 0.01\cdot 10^{-3}\frac{B_X}{1.8}, \quad [m] = \frac{\mathrm{J}}{\mathrm{T}\times\mathrm{g}}, \quad [B_X] = \mathrm{T} .$$

From the physics base course (electricity and magnetism) [4] it is known, that axial magnetic field of the solenoid can be defined using the formula:

$$B_X = \frac{\mu_0 wi}{2}\left(\cos\theta_1 - \cos\theta_2\right) =$$

(3) $$= \frac{\mu_0 wi}{2}\left(\frac{l-X}{\sqrt{(l-X)^2+R^2}} + \frac{l+X}{\sqrt{(l+X)^2+R^2}}\right) =$$

$$= \frac{\mu_0 wi}{2}\left(\frac{1-x}{\sqrt{(1-x)^2+r^2}} + \frac{1+x}{\sqrt{(1+x)^2+r^2}}\right),$$

where i – solenoid current, w – number of coils per length unit, $2l$ – length of solenoid, X and R – axial distance from centre of solenoid to point and radius of solenoid, respectively, $x = X/l$ and $r = R/l$ – relative distance of point from the centre of solenoid and its relative radius.

Define the derivative of magnetic flux density because it affects the force (2), acting on the body

$$\frac{dB_X}{dX} = -\frac{\mu_0 wi}{2}R^2\left(\frac{1}{\left((l-X)^2+R^2\right)^{3/2}} - \frac{1}{\left((l+X)^2+R^2\right)^{3/2}}\right) =$$

(4) $$= -\frac{\mu_0 wi}{2}\frac{r^2}{l}\left(\frac{1}{\left((1-x)^2+r^2\right)^{3/2}} - \frac{1}{\left((1+x)^2+r^2\right)^{3/2}}\right).$$

This derivative, respectively, and the force (2) is nonlinear, as evidenced by its regular tabs for different r (Fig.3).

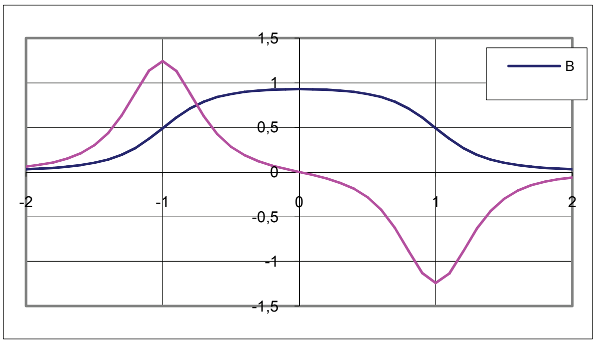

Fig.3. The relative values of flux density and its derivative for r=0.4

The 3rd *subsystem* (Fig.1) is described by

(5) $$m\frac{dv}{dt} = F_m - F_j(v),$$

where $v = \frac{dX}{dt}$ – body velocity, $F_j(v) = k_j v$ – friction force.

Electrical equivalent circuit is based on the replacement of mechanical variables and parameters on the electrical as show in Table 1.

Table 1. The correspondence between the mechanical and electrical variables and parameters

Mechanical	Electrical
$[x]$	$[u_{C3}]$ = V
$[v]$ = m/s	$[i_{L3}]$ = mA
$[m_1]$ = g	$[L_3]$ = H
$F_j(v) = k_j v$	$R_3 i_{L3}$
$[k_j]$=N·s/m	$[R_3]$ = kΩ
$[F_m]$ = N	$[FE_{32}] = [E_{n2}]$ = V

Electrical equivalent scheme of mechanical subsystem is $x \equiv u_{C3}$, $v \equiv l\frac{i_{C3}}{C_3} = \frac{k_5 l}{C_3}i_{L3}$, $i_{L3} \equiv \frac{C_3}{k_5 l}v$, $L_3\frac{di_{L3}}{dt} = FE_{32} - R_3 i_{L3}$, $L_3\frac{C_3}{k_5 l}\frac{dv}{dt} = FE_{32} - R_3\frac{C_3}{k_5 l}v$, $F_m \equiv FE_{32}$.

The resulting electrical equivalent circuit is shown in Fig. 4

It should be noted that the sources of FE12, FJ21, FJ23, FE32 necessary to establish the coupling between input and output values of subcircuits. Controlled sources Ji1, Ju3, Eu4, Ji5 used for the same purpose, but inside subcircuits. This structure allows the introduction of electric circuit model of a solenoid with distributed parameters. At this initial stage of research done simplification, namely, the solenoid is represented by a model with lumped parameters.

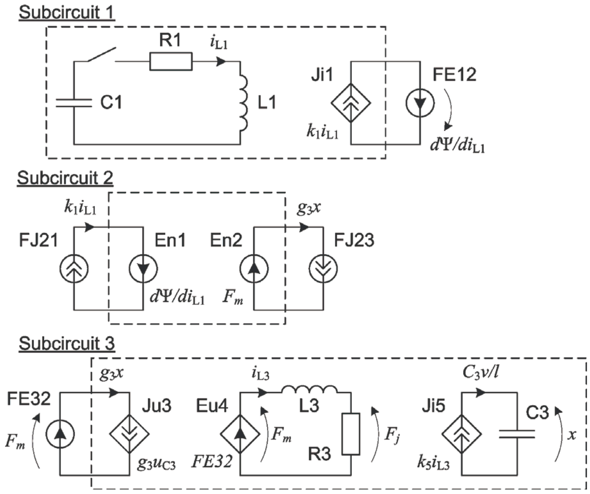

Fig.4. Electrical equivalent circuit of complex system

Obtained results of calculation of transient processes

Calculations of the trajectories of the point body mass of 1g were carried out on the example of the solenoid is contained in the book [4]. Several parameters of the solenoid are listed there, some parameters can be determined indirectly. Thus, the final list of required parameters is as follows:

– magnetic flux density in the centre of the solenoid B_0 = 30000 G = 3,0 T;

– magnetic flux density at the edge of solenoid on its axis B_1 = 18000 G = 1,8 T;

– derivative of the magnetic flux density at the edge of solenoid on its axis dB_1/dX = 1700 G/sm = 17 T/m;

– active power P = 400 kW, which correspond, for example, such quantities: the number of winds w=50000, winding resistance 111 Ω, the solenoid current 60 A;

– geometrical parameters: the radius R = 5-20 sm; length of solenoid $2l$ = 40 sm (r = 0.25÷1).

From expressions (3) and (4) is composed a system of equations:

$$B_0 = \frac{\mu_0 w I_1}{2}\frac{2}{\sqrt{1+r^2}};\ B_1 = \frac{\mu_0 w I_1}{2}\frac{2}{\sqrt{4+r^2}};$$

$$\frac{dB_1}{dX} = -\frac{\mu_0 w I_1}{2}\frac{r^2}{l}\left(\frac{1}{r^3} - \frac{1}{\left(4+r^2\right)^{3/2}}\right),$$

from which we find the average relative radius $r \approx 0.54$ and a multiplier $\frac{\mu_0 w I_1}{2} \approx 1.86\ \mathrm{T}$. According to the formulas (2) and (4) the force acting on 1g of iron is equal to

$$F_m \approx -k_0|i|\frac{r^2}{l}\left(\frac{1}{\left((1-x)^2+r^2\right)^{3/2}} - \frac{1}{\left((1+x)^2+r^2\right)^{3/2}}\right),$$

where $k_0 = m\frac{\mu_0 w}{2} \approx 7.38$.

Simulated calculation results of such regime of researched system are presented in mentioned coordinates in the Fig. 5, where u(C3) [V] ≡ x, u(FE32) [V] ≡ Fm [N], i(L3) ≡ v [m/s].

Obviously, the practical interest is the case when the body gets maximum momentum or velocity v. The simulation results in this direction are summarized in Table 2, which shows

– initial coordinate of the body x_0;

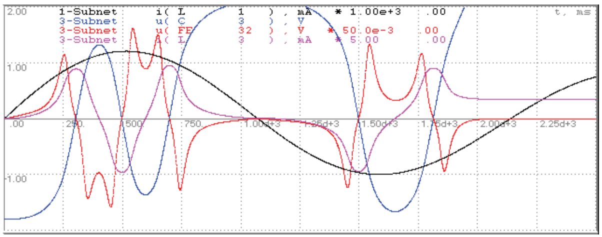

Fig.5. Simulated results of researched system

– voltage capacitor before switching U_{C1};

– capacitance C_1 (it is chosen so that the point of half-period of current oscillation of RLC-loop i_{L1} approximately coincides with the second maximum of magnetic force F_m, which is opposite to the velocity direction (Fig. 6)

– and velocity of the body v_{max} outside the solenoid (approximately at a distance $x = 5$).

The simulation results show the dependences of the maximum speed from the initial voltage of the capacitor before switching and from the initial coordinates of the body. The maximum speed v_{max} = 8.23 m/s is achieved when x_0 = –1.8.

Table 2. The maximum speed of the body $\boldsymbol{v}_{\mathbf{max}}$ [m/s] outside the solenoid

U_{C1} [V]	C_1 [µF]	x_0						
		-1.5	-1.6	-1.7	-1.8	-1.9	-2.0	-2.1
7071	10	3.73						
6742	11	4.12						
6455	12	**4.28**	3,73					
6202	13	**4.28**	4.13					
5976	14	4.19	4.36					
5774	15	4.04	**4.44**	3.73				
5590	16		4.41	4.14				
5423	17		4.32	4.39				
5270	18		4.19	**4.50**				
5130	19			**4.50**	3.71			
5000	20			4.45	4.09			
4880	21			4.37	4.33			
4767	22			4.27	4.47			
4663	23				**4.52**			
4564	24				**4.52**			
4472	25				4.47	3.95		
4385	26				4.40	4.20		
4303	27				4.30	4.37		
4226	28				4.21	4.46		
4152	29				4.12	**4.49**		
4082	30				4.02	**4.48**		
4016	31					4.45		
3953	32					4.39	3.97	
3727	36					4.11	4.42	
3676	37					4.04	**4.43**	
3627	38						4.41	
3581	39						4.37	
3536	40						4.33	3.82
3371	44						4.11	4.31
3333	45						4.05	4.34
3297	46							**4.35**
3262	47							4.34

Simulated calculation results of regime C_1 = 24 µF, U_{C1} = 4564 V, x_0 = –1.8 are presented in mentioned coordinates in the Fig. 6.

Options for the suppression of the magnetic force in the opposite direction through a period or one and a half period of current oscillation of RLC-loop i_{L1} leads to a degradation of results, i.e. to reduce the maximum velocity of the body (Fig. 7)

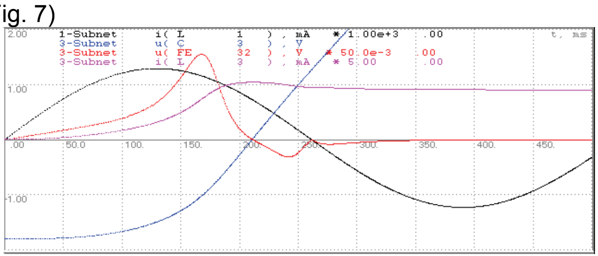

Fig.6. Results of regime C_1 = 24 µF, U_{C1} = 4564 V, x_0 = –1.8

(a)

(b)

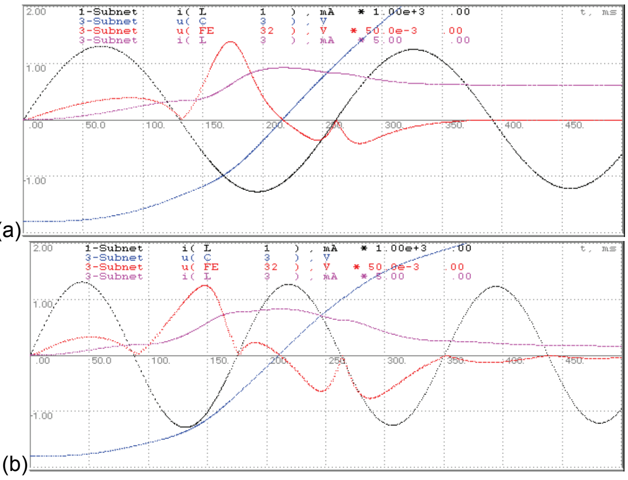

Fig.7. The suppression of the magnetic force in the opposite direction through a period (a) and one and a half period (b) of current oscillation of RLC-loop

Conclusions

Numerical experiments for transient analysis were carried out by program software intended for simulation of electrical and electronic circuits by diakoptic methods where the large circuit is split on several parts – subcircuits. Every part of the circuit is simulated independently on some time step. Numerical experiments have shown wide potential possibilities of proposed simulation method and its effectiveness.

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