Antonín PŘEDOTA, Zdeňka BENEŠOVÁ

University of West Bohemia

Fast transient overvoltage in transformer winding

***Abstract:** This paper deals with very fast transient phenomena in the transformer winding. The model of the transformer winding is created as a circuit with distributed parameters. The mathematical model is given by a system of partial differential equations and it is solved numerically. The transformer model respects Joule´s and dielectric losses; it enables to consider space varying parameters. Illustrative examples show the time-space voltage and current distribution in the transformer winding.*

Streszczenie:** Artykuł dotyczy szybkich stanów nieustalonych w uzwojeniach transformatora. Model uzwojenia to obwód o parametrach rozłożonych obejmujący straty w postaci ciepła Joule'a, straty w dielektryku oraz umożliwiający uwzględnienie przestrzennego rozkładu parametrów elektrycznych. Matematyczny opis za pomocą cząstkowych równań różniczkowych został rozwiązany numerycznie. Przykłady wyników obliczeń pokazują zmiany napięcia w czasie i przestrzeni oraz rozkład prądów w uzwojeniach. **(Szybkie stany przejściowe w uzwojeniach transformatora)

Keywords: surge wave, transformer winding, voltage distribution
Słowa kluczowe: impulsy przepięciowe, uzwojenie transformatora, rozkład napięcia.

Introduction

The fast and very fast transient phenomena cause a high overvoltage in a transformer winding, this situation is very dangerous for an isolation system. The transients are produced due to switch-on and switch-off processes, in systems with invertors with very quick transistors or can be transmitted from connected transmission lines after a lightning stroke. To study this problem we need an adequate model of a transformer winding. Analyzing such a system we can simulate many dangerous situations and state recommendations for design of winding namely for determination the properties of the isolation system. Many authors dealt with this problem and they used various approaches. In [1] authors solved a lossless transformer winding that was modeled with a circuit with distributed parameters. Results were obtained analytically under very strong simplified considerations. Modern approach is usually based on the RLC ladder network model or the multiconductor transmission line model [2 - 5]. The analysis of these models is obviously carried-out in the frequency domain and it is applied only on one phase transformer. Very important and difficult question is the determination of model parameters they should be evaluated for very high frequency and their values are dependent on a manner how the winding structure is carried-out [6, 7].

Our method is based on the winding model created as a circuit with distributed parameters. The parameters are considered as space varying along the winding. In this way it is possible to respect a graduated isolation or different inductive and capacitive linkage given by the layout of turns in coils. The mathematical model is a system of partial differential equations with space varying coefficients which is solved numerically in the time domain. The result of this analysis is the time-space distribution voltage and current along the winding, it allows us to find the place of maximal peak value of voltage which is important for the isolation design.

Basic element of model

The model of transformer winding is created as a network with distributed parameters and respects resistance, inductive and capacitive coupling. Both, constant or varying parameters (for non-homogenous line) can be considered. The basic element is depicted on fig. 1 it can be described by three partial differential equations

(1) $$-\frac{\partial u(t,x)}{\partial x} = L(x)\frac{\partial i_{\mathrm{L}}(t,x)}{\partial t} + R(x)\, i_{\mathrm{L}}(t,x)$$

(2) $$-\frac{\partial u(t,x)}{\partial t} = \frac{1}{C(x)}\frac{\partial i_{\mathrm{L}}(t,x)}{\partial x} + \frac{1}{C(x)}\frac{\partial i_{\mathrm{K}}(t,x)}{\partial x}$$

(3) $$-\frac{\partial^2 u(t,x)}{\partial x \partial t} = \frac{1}{K(x)}\, i_{\mathrm{K}}(t,x)$$

This system of equations is supplemented by initial condition and boundary conditions – eq. (4). In general, boundary conditions express relations among the voltage $u(t,x)$ and the current $i(t,x)$ at the input and the output of the winding, in our case they were formulated in the form

(4) $$\begin{aligned} &t = 0; \quad u(0,x) = 0 \\ &t > 0; \quad x = 0; \quad u(t,0) = u_0(t) - R_0\, i_0(t,0) \\ &t > 0; \quad x = l; \quad u(t,l) = R_Z\, i(t,l) \end{aligned}$$

A voltage source with an inner resistance R_o connected at the input and a resistor R_z linked at the output has been assumed. The shape of the input voltage can be arbitrary: a step voltage function, a pulse with various rate of rise or a surge voltage wave can be considered. The value of the output resistor has simulated various ways of the winding ending: a short-circuited line, an unloaded line or a matched line. The similar situations can be simulated by the input resistance value.

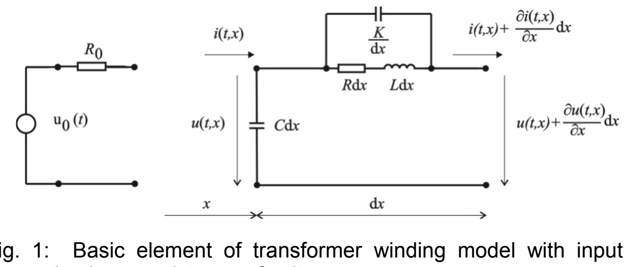

Fig. 1: Basic element of transformer winding model with input respecting inner resistance of voltage source

The system of eq. (1) – (3) together with boundary conditions (4) has been solved numerically using the implicit Wendroff's formula. More detailed is this algorithm for the numerical solution described in [8] and [9]. Using the first accuracy order of approximation for eq. (3) we obtain the following recurrent formula

(5) $$\mathbf{A} \cdot \mathbf{v}^{(l)} = \mathbf{B} \cdot \mathbf{v}^{(l-1)} + \mathbf{D}$$

Vector **v** consists of nodal values of voltage and current elements of matrices **A** and **B** are space-varying and they are dependent on parameters $R(x)$, $L(x)$, $C(x)$ *and* $K(x)$ of the transformer windings. Vector **D** respects the type of voltage source and the boundary conditions form.

Illustrative examples

In the following examples two important phenomena were investigated: firstly, the influence of an inner resistance of voltage source on the voltage and current distribution was studied. The reflected waves arising at the input and the output can strongly disturb the shape of the origin input signal travelling along the winding. In the next example the winding with the varying turn-to-turn capacitance $K(x)$ was considered and some interesting results were obtained. Both examples were solved under these conditions: the winding is supplied from the voltage source of a step voltage U_0 = 300 V, the output is short-circuited, parameters of transformer winding are:
R = 20.7 mΩ/m, L = 660 μH/m, C = 20.7 nF/m, K = K_0 = 2.07 pF/m.

Example 1

On the picture fig. 2, 3, 4 and 5 are given results when the ideal voltage source is connected to the winding input.

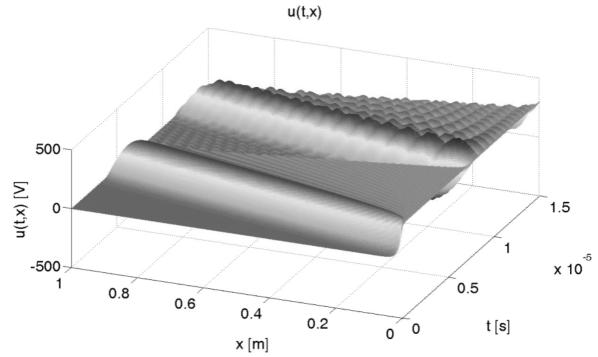

Fig. 2. Voltage time-space distribution, U_{max} = 436.9 V

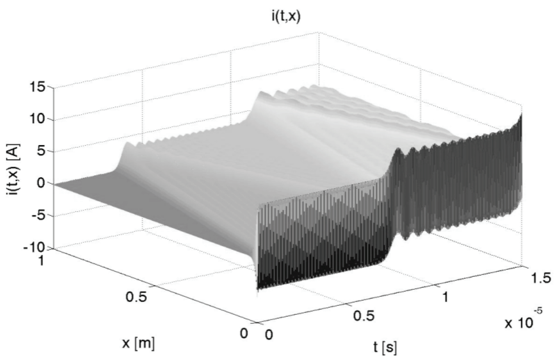

Fig. 3. Current time-space distribution

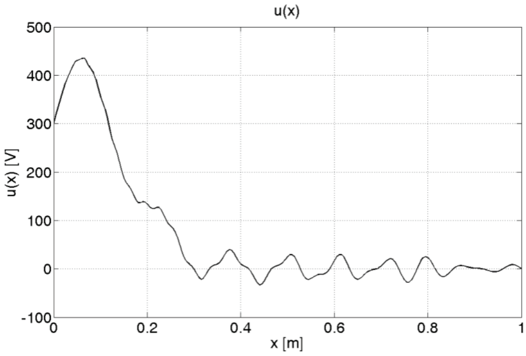

Fig. 4. Space-voltage distribution along the line, cut at t = 8.2 μs

The results prove the known fact that the maximal value of voltage does not occur at the input but at the point near to winding beginning. The detailed graphs on fig. 4 and 5 show that the point with peak value of the voltage is placed at the distance x = 0.08 ℓ from the input and after two wave reflections

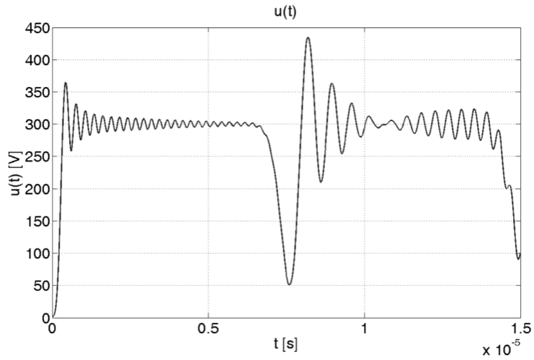

Fig. 5. Time-voltage distribution at the output, cut at x = 0.08 ℓ

It is seen that because of reflections the current wave at the input strongly oscillates. The value of the source inner resistance strongly affects the current wave distribution. It is depicted for the inner resistance R_0 = 2 Ω on the fig. 6. The oscillations are more attenuated but repeated reflections can cause dangerously increasing value of the current wave – fig. 7.

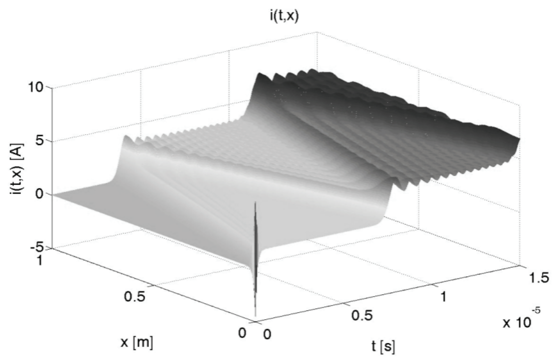

Fig. 6. Current time-space distribution, R_0 = 2 Ω, t_{max} = 15 μs

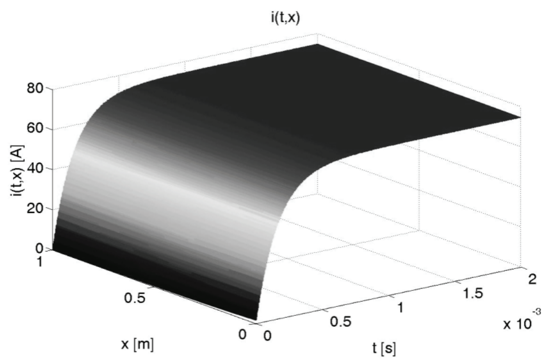

Fig.7. Current time-space distribution, R_0 = 2 Ω, t_{max} = 2 ms

Example 2

To reduce a dangerous stress of an isolation system caused due to surge overvoltage a graduated system of the isolation has been often used. If this procedure is projected without detailed analysis it can even enhance the isolation stress caused by overvoltage. For this reason the various expressions for the space-varying isolation were investigated. The inner source resistance R_0 = 2 Ω was considered and the end of the winding is short-circuited. Firstly, the continuously varying isolation properties were considered then only two isolations with different values of capacitances were assumed.

a) The continuously varying isolation is assumed the graph of $K(x)$ is shown on fig. 8. The turn-to-turn capacitance is given in the form

$$K(x) = 0.2 * K_0 \left(1 - x/l\right)^2 + K_0 \quad (6)$$

where K_0 = 2.07e^{-12} F. The voltage distribution is given on fig.9.

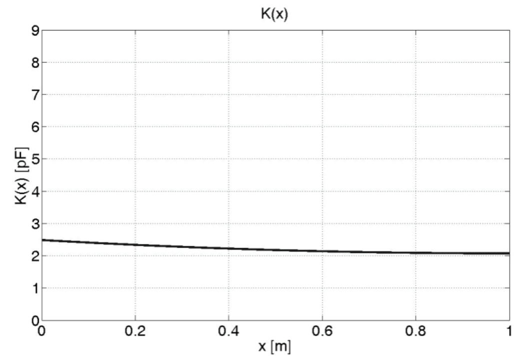

Fig. 8. Graph of turn-to-turn capacitance, $K(x)$ according to eq. (6) - K_{max} = 2.48 pF/m

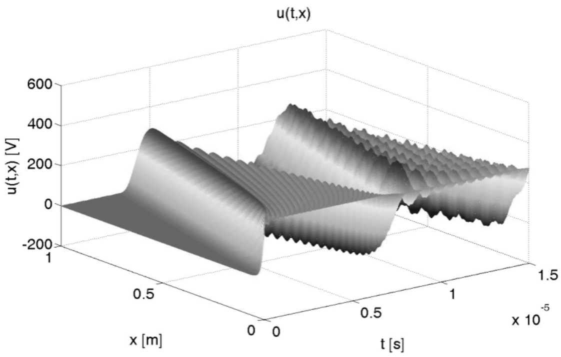

Fig. 9 Voltage time-space distribution, U_{max} = 412.2 V , $K(x)$ according to fig. 8

b) The continuously varying isolation is assumed the graph of $K(x)$ is shown on fig. 10. The turn-to-turn capacitance is given in the form

$$K(x) = 3 * K_0 \left(1 - x/l\right)^2 + K_0 \quad (7)$$

where K_0 = 2.07e^{-12} F. The voltage distribution is given on fig.11.

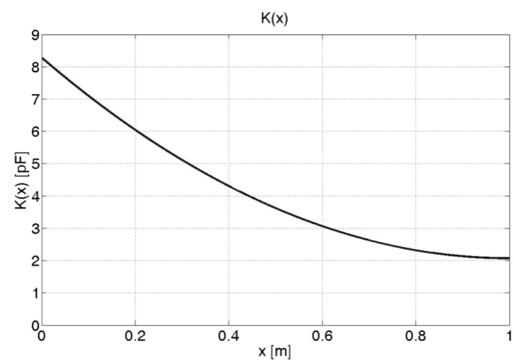

Fig. 10. Graf of turn-to-turn capacitance, $K(x)$ according to eq. (7) – K_{max} = 8.28 pF/m

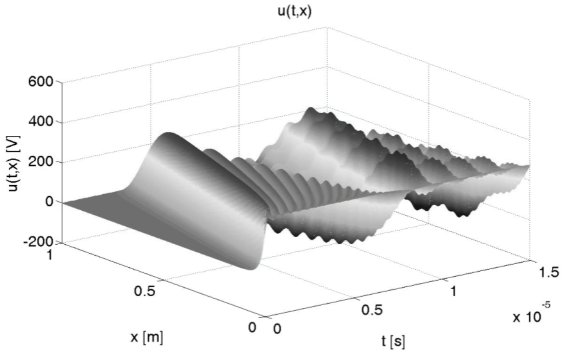

Fig. 11 Voltage time-space distribution, U_{max} = 409 V, $K(x)$ according to fig. 9

c) Two levels of the isolation were considered. The graph on fig. 8 is replaced by two levels of isolation with capacitances K_1 = 2.4 pF/m, K_2 = 2.2 pF/m – fig. 12. The voltage distribution is shown on fig. 13.

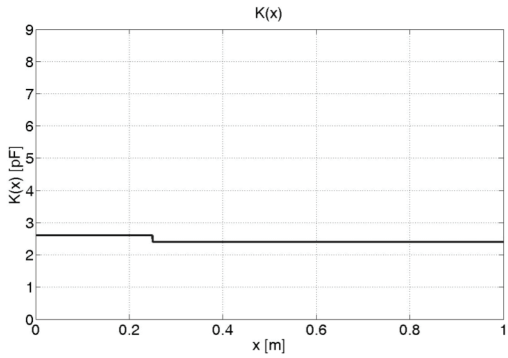

Fig. 12. Graf of turn-to-turn capacitance, K_1 = 2.4 pF/m, K_2 = 2.2 pF/m

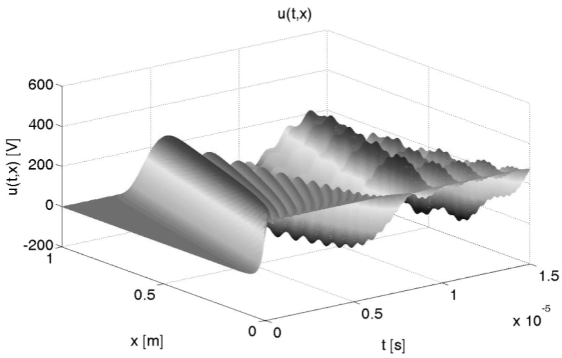

Fig. 13. Voltage time-space distribution, U_{max} = 412.3 V, $K(x)$ according to fig. 12

d) The graph on fig. 10 is replaced by two levels of isolation with capacitances, K_1= 8.28 pF/m, K_2 = 2.07 pF/m - fig. 14. The voltage distribution is on fig. 15.

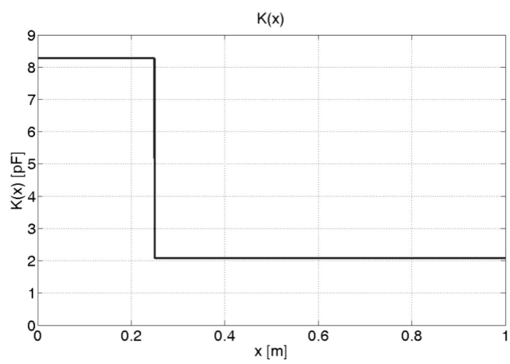

Fig.14.Graf of turn-to-turn capacitance, K_1= 8.28 pF/m, K_2 = 2.07 pF/m

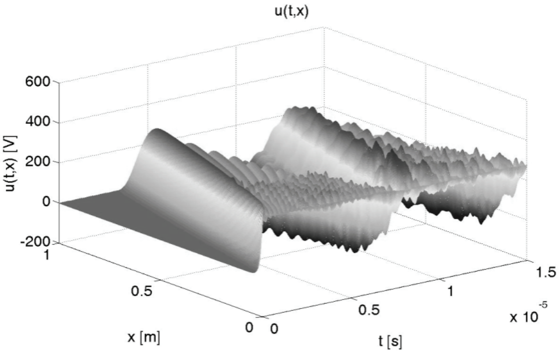

Fig. 15 Voltage time-space distribution, U_{max} = 439.4V, $K(x)$ according to fig. 14

Fig. 15 shows substantial voltage adjustment at the isolation interface. The detail of the time-space voltage distribution is depicted by fig. 16.

A pulse voltage signal applied at the transformer winding input is propagating through the first isolation level to the isolation interface (x = 0.25 m) where is reflected. Because of the reflected wave the result voltage distribution has higher or lower peak values according to isolation properties (it depends on the rate of permittivity and thickness of isolation).

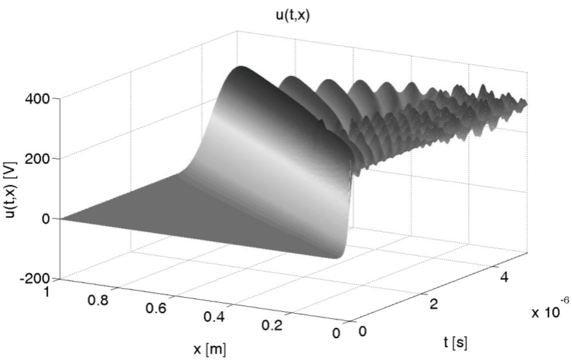

Fig. 16 Voltage time-space distribution at isolation interface. Detail of Fig. 15

Comparing the graphs of the voltage distribution which are shown on fig. 9, 11, 13 and 15 it is seen that the continuously varying isolation provides the best results. The only two step isolation induced reflections at the point of the transition change.

The influence of these phenomena is the stronger as the bigger is the difference between the capacitance values i.e. between a quality of the isolation. In the case that this difference is to big the new situation can be even worse then by an usage the isolation with constant properties. The comparison of the voltage peak value is given in Table 1.

Tab. 1. Peak values summary

Example	$K(x)$ [pF/m]	U_{max} [V]
1	K = 2.07	436.9
2 a)	$K(x)$ - eq.(6) : K_{max}= 2.48, K_{min}= 2.07	412.2
2 b)	$K(x)$ - eq.(7) : K_{max}= 8.28, K_{min}= 2.07	409.0
2 c)	K_1= 2.40, K_2= 2.20	412.3
2 d)	K_1= 8.28, K_2= 2.07	439.4

Conclusion

The paper connects to the previous work of authors. The algorithm for the numerical solution of very fast transients in the transformer winding was extended. The presented algorithm enables to take into account the space varying parameters of the transformer winding. The influence of the inner resistance of the voltage source on the voltage and current distribution was investigated. Results unequivocally indicate that the value of inner resistance of the voltage source (the value of an input resistance of a winding) has a great influence on the voltage and current distribution. It can be used for a correct setting up of voltage and current conditions in the transformer winding. The second reason for the introducing of the inner resistance to the mathematical models is that it has a fecund effect for the numerical algorithm. Very interesting results were obtained for the case of non-homogenous isolation systems. It was found that only two-steps isolation which has been often used to reduce isolation stress can have a negative influence. Owing to the wave reflection on isolation interface the electric stress of the isolation can be higher than in the case that only the one-step isolation system is used. The usage of isolation system with continuously varying properties is more efficient and gives assurance of non-reflection isolation system designing.

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***Authors**:*
Ing. Antonín Předota, Prof. Ing. Zdeňka Benešová,CSc, Department of Theory of Electrical Engineering, University of West Bohemia in Pilsen, Univerzitní 8, 306 14 Plzeň, The Czech Republic, E-mail: antonin@predota.eu,bene@kte.zcu.cz.