

## The magnetic field of the tubular rectangular high current busduct of finite length

**Abstract.** The paper encloses the results of rectangle wires phenomena computation. The simulation was analyzed in the Femm 4.2. program. Numerical simulation was made for frequency 9835 Hz. The calculations of the electromagnetic field was made with the finite element method shows that FEMM software is useful tool for the initial analysis for electromagnetic field in the induction heaters. It can be successfully used for the calculation of approximate self-impedance values of rectangular cross section wires inclusive skin effect and proximity phenomenon.

**Streszczenie.** W pracy przedstawiono rozkład pola magnetycznego w przewodach szynowych o profilu prostokątnym wiodących prąd sinusoidalnie zmienny, służących do zasilania wzbudnika wewnętrznego układu nagrzewnicy indukcyjnej. Wykonano wielowariantową symulację komputerową w programie FEMM ver. 4.2. Wyznaczono rozkłady gęstości prądu i pola magnetycznego przewodach szynowych o profilu prostokątnym. Analizę przeprowadzono dla częstotliwości 9835 Hz. Uwzględniono zjawisko naskórkowości oraz efekt zbliżenia. (Pole magnetyczne rurowego prostokątnego toru wieloprądowego o skończonej długości)

**Słowa kluczowe:** Pole elektromagnetyczne, nagrzewanie indukcyjne, hartowanie indukcyjne, naskórkowość.

**Keywords:** Electromagnetic field, induction heating, induction hardening, skin effect.

### Introduction

In the process of induction heating of metals often inductor is powered by a single phase transmission line formed on the tubular rails with rectangular profile [1-7]. This allows simultaneous cooling of the track with water flowing inside the phase conductors. One example of the Induction heating systems of the internal coil in the induction hardened pipe loads [8-11] is the system shown in Figures 1 and 2. It uses busbars rectangular pipe.

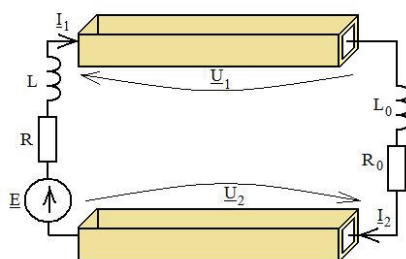


Fig.1. Induction heating diagram:  $E$ ,  $R$ ,  $L$  - power source inside parameters,  $R_0$ ,  $L_0$  workpiece and inductor parameters.

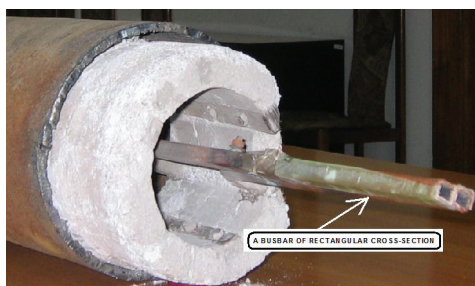


Fig. 2. A fragment of the real cross-section showing subway

In the study of the electromagnetic field of such a system should take into account the phenomenon of skin effect and the approximate due to the medium and high frequency current used in the process of heating. These phenomena give rise to uneven distribution of current density and magnetic field in busbars, which in turn significantly influence the size of losses in the track, the

temperature distribution, electrodynamic force between the wires and the replacement of the transmission line parameters [6, 12-20].

The distribution of the electromagnetic field in the vicinity of the high current busducts without regard and taking into account skin effect and proximity effects, and its electrical parameters was determined in a number of works by analytical methods, numerical and analytical-numerical characters used for the rectangular busbars in power [19, 21-27].

However, in the work of these and other busducts shall be extended, allowing you consider the electromagnetic field 2-D system. In the works [28-32] was determined distributions of the magnetic field of a single wire track of solid rectangular finite length, but, assuming even distribution of power.

This article attempts to develop algorithms and methods for determining the electromagnetic field in a single phase circuit of such busbars of finite length, taking into account skin effect and proximity effects. System implemented in the program FEMM ver. 4.2.

### Construction of mathematical model

The magnetic field is time-varying, eddy currents can be induced in materials with a non-zero conductivity. Several other Maxwell's equations related to the electric field distribution must also be accommodated. Denoting the electric field intensity as  $E$  and the current density as  $J$  obey the constitutive relationship:

$$(1) \quad J = \sigma E$$

We use also the Maxwell's equations:

$$(2) \quad \nabla \times H = J + \frac{\partial D}{\partial t}$$

$$(3) \quad \nabla \times E = -\frac{\partial B}{\partial t}$$

$$(4) \quad \nabla \cdot B = 0$$

$$(5) \quad \nabla \cdot D = \rho$$

Then we take a system of two straight wires of length  $L$  and rectangular cross section with sides  $b$ ,  $h$ ,  $l$  the

thickness, electrical conductivity  $\sigma$ , made of material with a permeability  $\mu_0$  non ferromagnetic (Fig. 3).

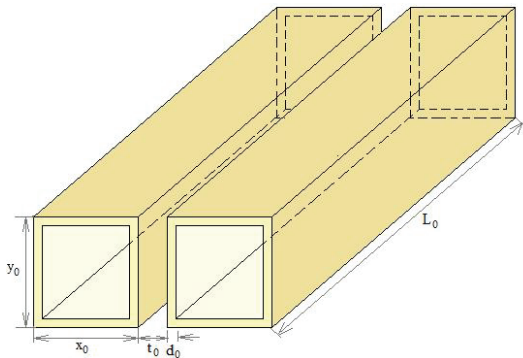


Fig.3. A busbar of pipes rectangular cross-section

Let between the ends of these wires are forced sinusoidally time-varying potential differences in the pulsation  $\omega$  whose values are complex,  $\underline{U}_1$  and  $\underline{U}_2$  (Fig. 1). Forced in this way the electric field inside the pipe –  $\text{grad } \underline{V}(X)$  causes the rate of flow of a current density of the complex  $\underline{J}(X)$ , which in turn induces the wires inside and outside the complex electric field induction equal  $-j\omega \underline{A}(X)$ . Accident complex electric field then takes the form [34]:

$$(6) \quad \underline{E}(X) = -\text{grad } \underline{V}(X) - j\omega \underline{A}(X)$$

where the composite magnetic vector potential

$$(7) \quad \underline{A}(x_1, x_2, x_3) = \frac{\mu_0}{2\pi} \cdot \sum_{p=1}^2 \int_{V^{(p)}} \frac{\underline{J}(y_1, y_2, y_3)}{\sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2 + (x_3 - y_3)^2}} dy_1 dy_2 dy_3$$

$V^{(p)}$  - is an area in which the p-th conductor. Taking into account Ohm's law

$$(8) \quad \underline{J}(x_1, x_2, x_3) = \sigma \underline{E}(x_1, x_2, x_3)$$

and equation (7) and (8), obtained by the equation

$$(9) \quad -\text{grad } \underline{V}(x_1, x_2, x_3) = \frac{1}{\sigma} \underline{J}(x_1, x_2, x_3) + j \frac{\omega \mu_0}{2\pi} \cdot \sum_{p=1}^2 \int_{V^{(p)}} \frac{\underline{J}(y_1, y_2, y_3)}{\sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2 + (x_3 - y_3)^2}} dy_1 dy_2 dy_3$$

The point  $X(x_1, x_2, x_3)$  we determine the magnetic field strength formula

$$(10) \quad \underline{H} = \frac{1}{\mu_0} \text{rot } \underline{A} = \frac{1}{\mu_0} \text{rot } \underline{A} \mathbf{1}_z = \underline{H}_x \mathbf{1}_x + \underline{H}_y \mathbf{1}_y$$

where  $\mathbf{1}_x$ ,  $\mathbf{1}_y$  and  $\mathbf{1}_z$  are unit vectors of Cartesian coordinates.

Module of the total magnetic field has the form

$$(11) \quad H(x_1, x_2, x_3) = \sqrt{H_x^2(x_1, x_2, x_3) + H_y^2(x_1, x_2, x_3)}$$

This module can be expressed in relative units, ie in the form

$$(12) \quad h(x_1, x_2, x_3) = \frac{H(x_1, x_2, x_3)}{H_0}$$

where volume  $H_0 = \frac{I}{2(h+b)}$ .

### Example number

As marked on Figure 3, the following dimensions of rectangular wires:  $x_0=20$  mm,  $y_0=25$  mm, wall thickness of the rectangular profile  $t_0=2$ mm, profile length  $L = 2$ m. The distance between conductors is  $d_0=3$  mm. Supply voltage frequency is 9835 Hz. Assumed that the cable is made of copper with a relative permeability  $\mu_r=1$ , and conductivity of  $\sigma=5,6 \cdot 10^7$  S/m. Inductor current is supplied with a value of effective 2kA. Numerical model of the high current busducts analyzed with a grid Figure 4 illustrates. For the discretization using triangular elements with linear approximation. The grid was generated independently for the busway and the surrounding airspace and consisted of 48.361 nodes and 96.686 elements.

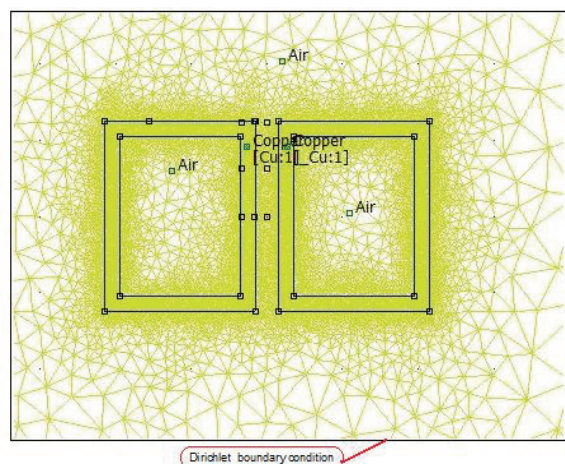


Fig. 4. Two-dimensional model of wire track with a visible grid

In such an approach is preserved continuity of the network. On the boundary of the defined boundary conditions Mixed type. This denotes a boundary condition of the form:

$$(13) \quad \epsilon_r \epsilon_0 \frac{\partial A}{\partial n} + c_0 A + c_1 = 0, \text{ where } c_0 = 0, c_1 = 0.$$

By the choice of coefficients, this boundary condition can either be a Robin or a Neumann boundary condition.

The Robin boundary condition is sort of a mix between Dirichlet and Neumann, prescribing a relationship between the value of  $A$  and its normal derivative at the boundary. An example of this boundary condition is:

$\frac{\partial A}{\partial n} + cA = 0$ . The Dirichlet boundary condition - the value

of potential  $A$  or  $V$  is explicitly defined on the boundary, e.g.  $A = 0$ . The most common use of Dirichlet-type boundary conditions in magnetic problems is to define  $A = 0$  along a boundary to keep magnetic flux from crossing the boundary.

The Neumann boundary condition specifies the normal derivative of potential along the boundary. In magnetic problems, the homogeneous Neumann boundary condition,

$\frac{\partial A}{\partial n} = 0$  is defined along a boundary to force flux to pass the boundary at exactly a 90 angle to the boundary [35].

### The results of calculations

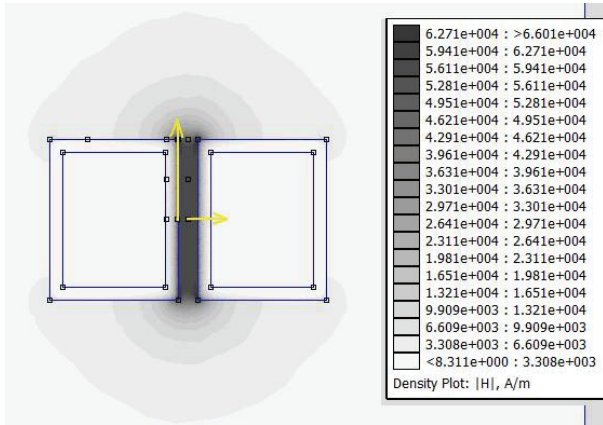


Fig. 5. Magnetic field intensity with market two straight lines. Along this lines is appointed distributions of field intensity.

Figure 6-7 present the magnetic field along the line

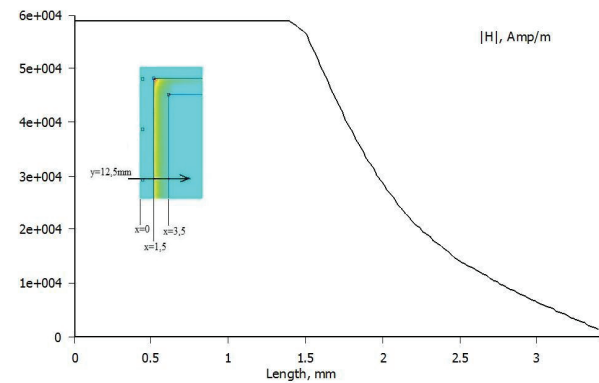


Fig. 6. Magnitude of magnetic field intensity

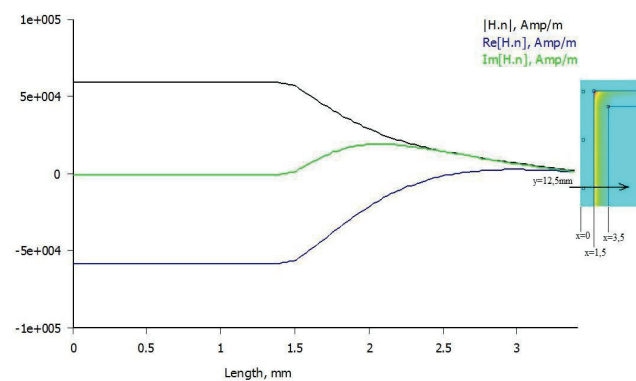


Fig. 7. Normal magnetic field intensity

Fig. 8-10 shows distributions of magnetic field intensity along the line vertical.

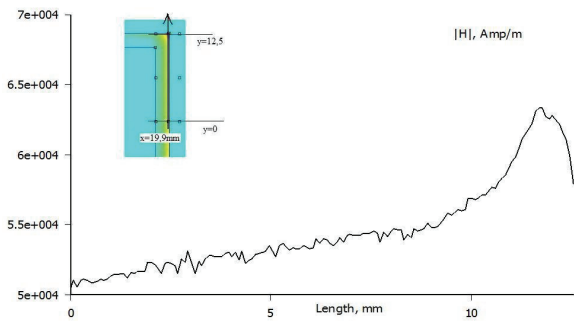


Fig. 8. Magnitude of magnetic field intensity

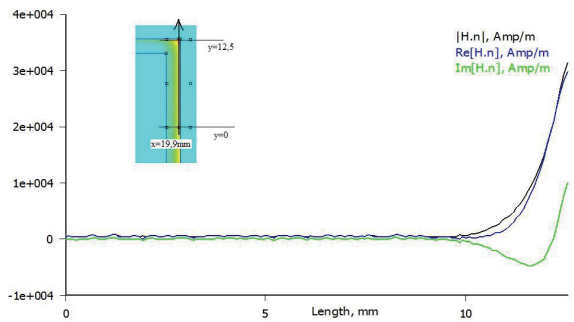


Fig. 9. Normal magnetic field intensity

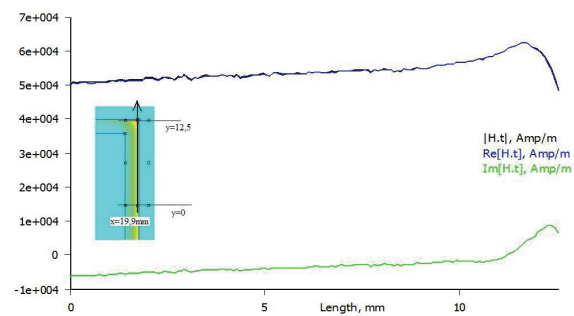


Fig. 10. Tangential magnetic field intensity

Fig. 11 shows distributions of magnetic field intensity.

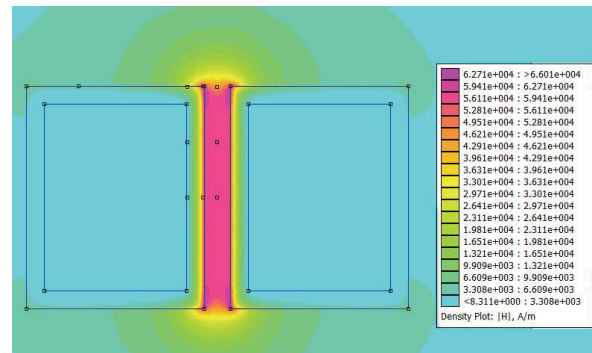


Fig.11. Magnetic field intensity

### Conclusions

Using FEMM computer simulation was carried out multi-variant. In this paper we present only the results obtained for the frequency of the power source of 9835 Hz. The analysis shows that for high frequencies above 5kHz inductor current, the conductor made of copper pipe cross-



section of magnetic field strength is greatest in thin wires, and then rapidly disappears. Large values of the field strength are also present in the inner corners of the wires. The highest values of magnetic field strength equal to 62.71 kA/m was obtained for the space between the two rails leading current in the opposite direction to the frequency of 9835 Hz.

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