

## Power losses in the screen of bifilar high current busduct

**Abstract.** Using the Poynting theorem and Joule-Lenz law the active and reactive power losses in the screen of bifilar high current busduct were determined. These power values were determined as analytical formulas expressed as Bessel's functions. Internal proximity effect was taken into account.

**Streszczenie.** W pracy wyznaczono straty mocy czynnej i biernej w ekranie bifilarnego toru wielkoprądowego. Obliczenia wykonano korzystając z twierdzenia Poyntinga oraz z prawa Joule-Lenza. Uwzględniono przy tym wewnętrzne zjawisko zbliżenia. Wyznaczone straty przedstawiono za pomocą zmodyfikowanych funkcji Bessela. (**Straty mocy czynnej i biernej w ekranie bifilarnego toru wielkoprądowego**)

**Keywords:** high current busduct, power losses, electromagnetic field.

**Słowa kluczowe:** tor wielkoprądowy, straty mocy, pole elektromagnetyczne.

### Introduction

A system of two conductors placed in common electrically conductive shell (fig. 1) is applied as a screened transmission line or as a screened high current busduct [1 - 5].

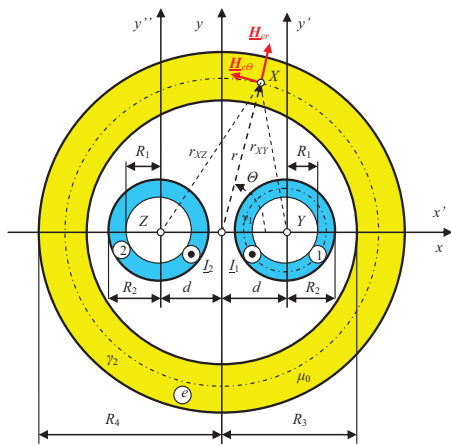


Fig. 1. Double-pole high current busduct with current  $\underline{I}_1$  and  $\underline{I}_2$

The design of the busducts used for high currents and voltages causes a necessity of precise describing of electromagnetic, dynamic and thermal effects. Knowledge of the relations between electrodynamics and constructional parameters is necessary in the optimization construction process of the high current busducts. Analysis of electromagnetic effects in the high current busducts is rather complicated. In devices these type, the power losses is emitted in the phase conductors and screens [1].

Information about distribution electromagnetic field and power losses is a base into analysis of electrodynamics and thermal effects in the high current busducts. Correct determination of the electrodynamics parameters has important practical meaning in the analysis of operation of every types of electrical devices. Determination of the power losses into high current busducts enables the calculation of temperature of these devices, which is a basic constructional parameter [1].

Power losses depend on value of currents, but for the large cross-sectional dimensions of the phase conductor, even for industrial frequency, skin and internal proximity effect (fig. 2) [2] should be taken into account.

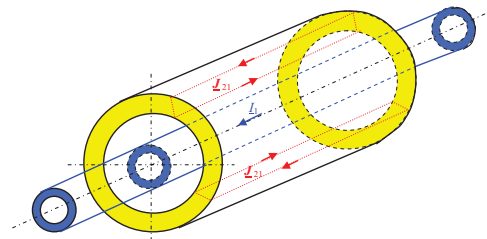


Fig. 2. Eddy currents induced on the screen by the magnetic field of the own current of the phase conductor – internal proximity effect.

### Electromagnetic field

Let us consider the electromagnetic field in tubular screens of the double-pole high current busduct presented in the fig 1.

Current density in the screen [2]

$$(1) \quad \underline{J}_e(r, \Theta) = \underline{J}_{e1}(r, \Theta) + \underline{J}_{e2}(r, \Theta)$$

In the above formula  $\underline{J}_{e1}(r, \Theta)$  is a density of current induced in the screen by the magnetic field of current  $\underline{I}_1$  and is defined by formula [2]

$$(2) \quad \underline{J}_{e1}(r, \Theta) = \underline{J}_0(r) + \sum_{n=1}^{\infty} \underline{J}_{e1n}(r, \Theta)$$

where  $\underline{J}_0(r)$  is given by formula [2]

$$(3) \quad \underline{J}_0(r) = \frac{\Gamma_e \underline{I}_1}{2\pi R_3} \frac{b_0 J_0(\Gamma_e r) + c_0 K_0(\Gamma_e r)}{d_0}$$

where

$$(3a) \quad d_0 = I_1(\Gamma_e R_4) K_1(\Gamma_e R_3) - I_1(\Gamma_e R_3) K_1(\Gamma_e R_4)$$

and

$$(3b) \quad b_0 = \beta_e K_1(\Gamma_e R_3) - K_1(\Gamma_e R_4)$$

$$(3c) \quad c_0 = \beta_e I_1(\Gamma_e R_3) - I_1(\Gamma_e R_4)$$

$$(3d) \quad \beta_e = \frac{R_3}{R_4} \quad (0 \leq \beta_e \leq 1)$$

And quantity  $\underline{J}_{e1n}(r, \Theta)$  has a form [2]

$$(4) \quad \underline{J}_{e1n}(r, \Theta) = \frac{\Gamma_e I_1}{\pi R_3} \underline{g}_n(r) \cos n\Theta$$

where

$$(4a) \quad \underline{g}_n(r) = -\left(\frac{d}{R_3}\right)^n \frac{K_{n-1}(\Gamma_e R_4) I_n(\Gamma_e r) + I_{n-1}(\Gamma_e R_4) K_n(\Gamma_e r)}{d_n}$$

and

$$(4b) \quad d_n = I_{n-1}(\Gamma_e R_4) K_{n+1}(\Gamma_e R_3) - I_{n+1}(\Gamma_e R_3) K_{n-1}(\Gamma_e R_4)$$

In the above formulas the complex propagation constant of electromagnetic wave in the screen

$$(5) \quad \underline{\Gamma}_e = \sqrt{j\omega\mu_0 \gamma_e} = \sqrt{\omega\mu_0 \gamma_e} \exp[j\frac{\pi}{4}] = k_e + jk_e$$

while  $I_0(\Gamma_e r)$ ,  $K_0(\Gamma_e r)$ ,  $I_1(\Gamma_e r)$ ,  $K_1(\Gamma_e r)$ ,  $I_n(\Gamma_e r)$ ,  $K_n(\Gamma_e r)$ ,  $I_{n-1}(\Gamma_e r)$ ,  $K_{n-1}(\Gamma_e r)$ ,  $I_{n+1}(\Gamma_e r)$  and  $K_{n+1}(\Gamma_e r)$  are modified Bessel's functions, 0, 1,  $n$ ,  $n-1$  and  $n+1$  order, calculated for  $r = R_3$  and  $r = R_4$ .

Current density  $\underline{J}_{e2}(r, \Theta)$  is generated by the magnetic field of current  $\underline{I}_2$  and has a form

$$(6) \quad \underline{J}_{e2}(r, \Theta) = \underline{J}_0(r) + \sum_{n=1}^{\infty} (-1)^n \underline{J}_{e1n}(r, \Theta)$$

The total current density  $\underline{J}_e(r, \Theta)$  depends on currents  $\underline{I}_1$  and  $\underline{I}_2$ . If  $\underline{I}_1 = -\underline{I}_2$  (currents of the same module and reverse turnings) then density current induced in the screen

$$(7) \quad \underline{J}_{e(2n-1)}(r, \Theta) = 2 \sum_{n=1}^{\infty} \underline{J}_{e(2n-1)}(r, \Theta)$$

where

$$(7a) \quad \underline{J}_{e(2n-1)}(r, \Theta) = \frac{\Gamma_e I_1}{\pi R_3} \underline{g}_{2n-1}(r) \cos(2n-1)\Theta$$

while

$$(7b) \quad \underline{g}_{2n-1}(r) = -\left(\frac{d}{R_3}\right)^{2n-1} \frac{K_{2n-2}(\Gamma_e R_4) I_{2n-1}(\Gamma_e r) + I_{2n-2}(\Gamma_e R_4) K_{2n-1}(\Gamma_e r)}{d_{2n-1}}$$

and

$$(7c) \quad d_{2n-1} = I_{2n-2}(\Gamma_e R_4) K_{2n}(\Gamma_e R_3) - I_{2n}(\Gamma_e R_3) K_{2n-2}(\Gamma_e R_4)$$

Magnetic field in the screen is defined by formula [3]

$$(8) \quad \underline{H}_e(r, \Theta) = \underline{H}_{e1}(r, \Theta) + \underline{H}_{e2}(r, \Theta)$$

Radial component of the magnetic field  $\underline{H}_{e1}(r, \Theta)$  is expressed by formula [3]

$$(9) \quad \underline{H}_{e1r}(r, \Theta) = \frac{I_1}{\pi R_3} \frac{1}{\Gamma_e r} \sum_{n=1}^{\infty} n \underline{g}_n(r) \sin n\Theta$$

but tangent component is a sum [3]

$$(10) \quad \underline{H}_{e1\theta}(r, \Theta) = \underline{H}_{e1\theta0}(r) + \sum_{n=1}^{\infty} \underline{H}_{e1\theta n}(r, \Theta)$$

where

$$(10a) \quad \underline{H}_{e1\theta0}(r) = \frac{I_1}{2\pi R_3} \frac{b_0 I_1(\Gamma_e r) - c_0 K_1(\Gamma_e r)}{d_0}$$

and

$$(10b) \quad \underline{H}_{e1\theta n}(r, \Theta) = \frac{I_1}{\pi R_3} \frac{1}{\Gamma_e r} f_{-n}(r) \cos n\Theta$$

while

$$(10c) \quad f_{-n}(r) = \left(\frac{d}{R_3}\right)^n \frac{1}{d_n} \{K_{n-1}(\Gamma_e R_4) [n I_n(\Gamma_e r) - \Gamma_e r I_{n-1}(\Gamma_e r)] + I_{n-1}(\Gamma_e R_4) [n K_n(\Gamma_e r) + \Gamma_e r K_{n-1}(\Gamma_e r)]\}$$

Radial component of the magnetic field  $\underline{H}_{e2}(r, \Theta)$  has a form [3]

$$(11) \quad \underline{H}_{e2r}(r, \Theta) = \frac{I_2}{\pi R_3} \frac{1}{\Gamma_e r} \sum_{n=1}^{\infty} (-1)^n n \underline{g}_n(r) \sin n\Theta$$

And tangent components [3]

$$(11a) \quad \underline{H}_{e2\theta}(r, \Theta) = \underline{H}_{e2\theta0}(r) + \sum_{n=1}^{\infty} \underline{H}_{e2\theta n}(r, \Theta)$$

where

$$(11b) \quad \underline{H}_{e2\theta0}(r) = \frac{I_2}{2\pi R_3} \frac{b_0 I_1(\Gamma_e r) - c_0 K_1(\Gamma_e r)}{d_0}$$

and

$$(11c) \quad \underline{H}_{e2\theta n}(r, \Theta) = \frac{I_2}{\pi R_3} \frac{1}{\Gamma_e r} (-1)^n f_n(r) \cos n\Theta$$

If  $\underline{I}_1 = -\underline{I}_2$  the components of magnetic field in the screen

$$(12) \quad \underline{H}_{er(2n-1)}(r, \Theta) = 2 \frac{I_1}{\pi R_3} \frac{1}{\Gamma_e r} \sum_{n=1}^{\infty} (2n-1) \underline{g}_{(2n-1)}(r) \sin(2n-1)\Theta$$

$$(13) \quad \underline{H}_{e\theta(2n-1)}(r, \Theta) = 2 \frac{I_1}{\pi R_3} \frac{1}{\Gamma_e r} \sum_{n=1}^{\infty} f_{-(2n-1)}(r) \cos(2n-1)\Theta$$

(13a)

$$\underline{f}_{2n-1}(r) = \left(\frac{d}{R_3}\right)^{2n-1} \frac{1}{\underline{d}_{2n-1}} \times \\ \times \left\{ K_{2n-2}(\underline{\Gamma}_e R_4) \left[ (2n-1) I_{2n-1}(\underline{\Gamma}_e r) - \underline{\Gamma}_2 r I_{2n-2}(\underline{\Gamma}_e r) \right] + \right. \\ \left. + I_{2n-2}(\underline{\Gamma}_e R_4) \left[ (2n-1) K_{2n-1}(\underline{\Gamma}_e r) + \underline{\Gamma}_e r K_{2n-2}(\underline{\Gamma}_e r) \right] \right\}$$

**Power losses in the screen of the bifilar high current busduct**

Apparent power of the screen is equal

$$(14) \quad \underline{S}_e = -\iint_S \left[ \underline{E}_e(r) \times \underline{H}_e^*(r) \right] \cdot \underline{dS} = P_e + jQ_e$$

If  $\underline{I}_1 = -\underline{I}_2$  then from (13) we have

$$(15) \quad \underline{S}_{e(2n-1)} = 16 \frac{j I_1^2 l}{\pi \gamma_e R_3^2} \sum_{n=1}^{\infty} V_{(2n-1)}$$

where

(15a)

$$\underline{V}_{(2n-1)} = \underline{g}_{(2n-1)}(R_4) \underline{f}_{(2n-1)}^*(R_4) - \underline{g}_{(2n-1)}(R_3) \underline{f}_{(2n-1)}^*(R_3)$$

In the above formula we can not isolate the real part (as an active power) and the imaginary part (as a reactive power). It is impossible on account of the complex propagation constant and complex modified Bessel's functions. Therefore the active power will be calculated from formula

$$(16) \quad P_e = \iiint_V \frac{1}{\gamma} \underline{J}_e(r) \underline{J}_e^*(r) dV$$

From the above formula we get

$$(17) \quad P_e = 4 \frac{l \underline{\Gamma}_e^* I^2 R_4}{2\pi \gamma_e R_3^2} \sum_{n=1}^{\infty} \left(\frac{d}{R_3}\right)^{4n-2} \frac{\underline{a}_{nn(2n-1)}}{\underline{d}_{2n-1} \cdot \underline{d}_{2n-1}^*}$$

where

(17a)

$$\underline{a}_{nn(2n-1)} = K_{2n-2}(\underline{\Gamma}_e R_4) K_{2n-2}^*(\underline{\Gamma}_e R_4) \times \\ \left\{ \left[ I_{2n-1}^*(\underline{\Gamma}_e R_4) I_{2n}(\underline{\Gamma}_e R_4) + j I_{2n-1}(\underline{\Gamma}_e R_4) I_{2n}^*(\underline{\Gamma}_e R_4) \right] - \right. \\ \left. \left[ \beta_e \left[ I_{2n-1}^*(\underline{\Gamma}_e R_3) I_{2n}(\underline{\Gamma}_e R_3) + j I_{2n-1}(\underline{\Gamma}_e R_3) I_{2n}^*(\underline{\Gamma}_e R_3) \right] \right] \right\} \\ + I_{2n-2}(\underline{\Gamma}_e R_4) I_{2n-2}^*(\underline{\Gamma}_e R_4) \times \\ \left\{ \left[ -K_{2n-1}^*(\underline{\Gamma}_e R_4) K_{2n}(\underline{\Gamma}_e R_4) - j K_{2n-1}(\underline{\Gamma}_e R_4) K_{2n}^*(\underline{\Gamma}_e R_4) \right] + \right. \\ \left. \left[ \beta_e \left[ K_{2n-1}^*(\underline{\Gamma}_e R_3) K_{2n}(\underline{\Gamma}_e R_3) + j K_{2n-1}(\underline{\Gamma}_e R_3) K_{2n}^*(\underline{\Gamma}_e R_3) \right] \right] \right\} \\ + K_{2n-2}(\underline{\Gamma}_e R_4) I_{2n-2}^*(\underline{\Gamma}_e R_4) \times \\ \left\{ \left[ I_{2n}(\underline{\Gamma}_e R_4) K_{2n-1}^*(\underline{\Gamma}_e R_4) - j I_{2n-1}(\underline{\Gamma}_e R_4) K_{2n}^*(\underline{\Gamma}_e R_4) \right] - \right. \\ \left. \left[ \beta_e \left[ I_{2n}(\underline{\Gamma}_e R_3) K_{2n-1}^*(\underline{\Gamma}_e R_3) - j I_{2n-1}(\underline{\Gamma}_e R_3) K_{2n}^*(\underline{\Gamma}_e R_3) \right] \right] \right\} \\ + I_{2n-2}(\underline{\Gamma}_e R_4) K_{2n-2}^*(\underline{\Gamma}_e R_4) \times \\ \left\{ - \left[ I_{2n-1}^*(\underline{\Gamma}_e R_4) K_{2n}(\underline{\Gamma}_e R_4) - j K_{2n-1}(\underline{\Gamma}_e R_4) I_{2n}^*(\underline{\Gamma}_e R_4) \right] + \right. \\ \left. \left[ \beta_e \left[ I_{2n-1}^*(\underline{\Gamma}_e R_3) K_{2n}(\underline{\Gamma}_e R_3) - j K_{2n-1}(\underline{\Gamma}_e R_3) I_{2n}^*(\underline{\Gamma}_e R_3) \right] \right] \right\}$$

If we introduce the reference active power

$$(18) \quad P_{0ew} = \frac{l I^2}{\pi \gamma_e (R_4^2 - R_3^2)}$$

then we the relative active power losses in the screen can be expressed by the formula

$$(19) \quad k_e^{(P)} = \frac{P_e}{P_{0ew}}$$

Dependence of the coefficient (19) on parameter  $\alpha_e$  for different values of the relative walls thickness  $\beta_e$  of the tubular screen and different values of parameter  $\lambda$  (where  $\alpha_e = k_e R_4$  and  $\lambda = \frac{d}{R_3}$  ( $0 \leq \lambda < 1$ )) is presented in the fig 3.

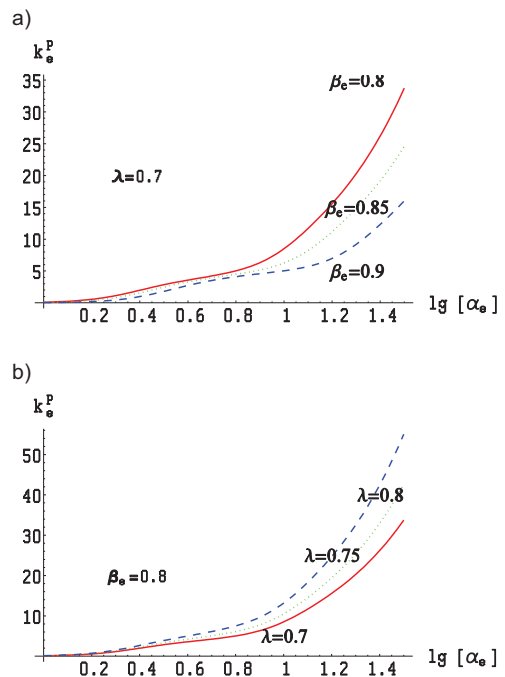


Fig. 3. Dependence of the relative active power of the tubular screen on parameter  $\alpha_e$ : a) for different values of parameter  $\beta_e$ , b) for different values of parameter  $\lambda$

The reactive power emitted on internal reactance of tubular screen, we calculate from (14), thus

$$(20) \quad Q_e = 4 \frac{l I^2}{\pi \gamma_e R_3^2} \left\{ \sum_{n=1}^{\infty} V_{(2n-1)} + \right. \\ \left. + j \frac{\underline{\Gamma}_e^* R_4}{2} \sum_{n=1}^{\infty} \left(\frac{d}{R_3}\right)^{4n-2} \frac{\underline{a}_{nn(2n-1)}}{\underline{d}_{2n-1} \underline{d}_{2n-1}^*} \right\}$$

The reference reactive power has a form

$$(21) \quad Q_{0ew} = \omega \frac{\mu_0 l}{2\pi} \left[ \frac{R_4^4}{(R_4^2 - R_3^2)^2} \ln \frac{R_4}{R_3} - \frac{1}{4} \frac{3R_3^2 - R_4^2}{R_4^2 - R_3^2} \right] I_1^2$$

Then the relative reactive power

$$(22) \quad k_c^{(Q)} = \frac{Q_e}{Q_{0ew}}$$

Dependence of the coefficient (22) on parameter  $\alpha_e$  is presented in the fig 4.

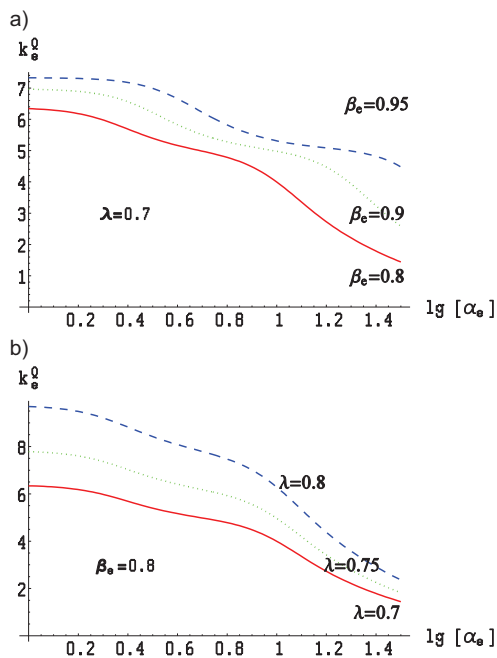


Fig. 4. Dependence of the internal reactive power of the tubular screen on parameter  $\alpha_e$ : a) for different values of parameter  $\beta_e$ , b) for different values of parameter  $\lambda$

We should add that the total reactive power of tubular screen is a sum of the determined above reactive power connected with internal inductance of the screen, of the reactive power connected with external inductance of the screen and of the reactive power connected with mutual inductance. Because we consider the bifilar busduct so the external reactive power depends only on external inductance defined by formula

$$(23) \quad L_z = \frac{\mu_0 l}{2\pi} \left( \ln \frac{2l}{R_4} - 1 \right)$$

Thus the external reactive power

$$(24) \quad Q_z = X_z I^2 = \omega L_z I^2 = \omega \frac{\mu_0 l}{2\pi} \left( \ln \frac{2l}{R_4} - 1 \right) I^2$$

The above power can be compare with the internal reactive power by the coefficient

$$(25) \quad k_{cz}^{(Q)} = \frac{Q_z}{Q_e}$$

Dependence of the coefficient (25) on parameter  $\alpha_e$  is presented in the fig 5 (where  $\eta_e = \frac{l}{R_4}$ ).

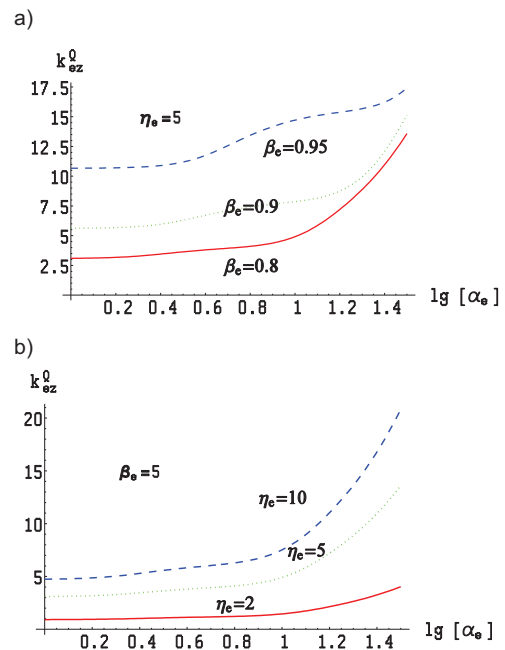


Fig. 5. Dependence of the external reactive power of the tubular screen on parameter  $\alpha_e$ : a) for different values of parameter  $\beta_e$ , b) for different values of parameter  $\eta$

## Conclusions

Presented research work shows that the active and reactive powers in the screen of bifilar high current busduct determine internal proximity effect.

Figure number 3 shows that the active power in the screen is from 5 to 10 times bigger than the active power calculated without taking into account proximity effect.

The internal reactive power in the screen of bifilar high current busduct is from 5 to 8 times smaller than the reactive power without taking into account proximity effect.

Figure number 5 shows that the reactive power connected with mutual and external inductances is ten times bigger than the reactive power connected with internal inductance.

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