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Dynamic behavior of electromagnetic brake system consisting of permanent magnets

***Abstract**. Dynamic interaction between a pair of permanent magnets is analyzed. The numerical solution of the corresponding mathematical model is performed by a higher-order finite element method. Computations are realized by own hp-FEM based codes Hermes and Agros. The methodology is illustrated by an example whose results are discussed.*

Streszczenie.** Celem pracy jest analiza dynamicznyego oddziaływania pomiędzy dwoma magnesami trwałymi. Numeryczny model zjawiska rozwiązano przy pomocy metody elementów skończonych wykorzystującej approksymację wsokiego stopnia. Obliczenia wykonano przy pomocy specjalizowanych pakietów Hermes i Agros implementujących wspomnianą metodę. Omówiona metodyka jest wsparta przykładem, którego rezultaty przedyskutowano w zakończeniu pracy. **(Dynamika hamulca elektromagnetycznego z magnesami trwałymi)

Keywords: *hp*-FEM, dynamic behavior, permanent magnets, brake system, electromagnetic field.
Słowa kluczowe: MES wysokiego rzędu, dynamika, magnesy trwałe, systemy hamowania, pole elektromagnetyczne

Introduction

Magnetic brakes working on the principle of the force interaction between a pair of permanent magnets are nowadays commonly used in numerous industrial applications as a cheap alternative to the classical brake systems. Due to their simple design they are often used, for example, as dampers. The aim of this paper is to present the simplest kind of such a damper, derive its complete mathematical model, determine its characteristics and verify some theoretical results experimentally.

Description of the device

Consider an arrangement depicted in Fig. 1. It consists of a nonmagnetic basement **1**, wooden or plastic rod **2**, unmoving permanent magnet **3** and movable permanent magnet **4**. The ring-type axially magnetized permanent magnets **3** and **4** are oriented oppositely; magnet **4** can freely glide along the rod.

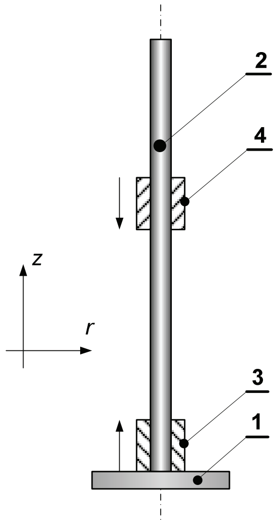

Fig. 1. Basic arrangement of the investigated system (orientation of magnets is marked by arrows): **1**–wooden or plastic rod, **2**–wooden basement, **3**–unmoving magnet, **4**–freely movable magnet

In the steady state the magnet **4** levitates in the position given by the balance of the repulsive force between both magnets and its weight. Suppose now that we lift the magnet **4** up and then we let it freely fall down. Its movement is then affected by its weight, strongly nonlinear repulsive force between the magnets, Lorentz force produced by currents induced in magnet **4** by movement, and also by the friction forces. The resultant movement of the magnet **4** is characterized by rapidly damped oscillations.

Mathematical model of the process

Magnetic field in the system is described in terms of magnetic vector potential $\boldsymbol{A}$ obeying the equation

$$\text{curl}\left(\frac{1}{\mu}\text{curl}\,\boldsymbol{A}-\boldsymbol{H}_\text{c}\right)-\gamma\boldsymbol{v}\times\text{curl}\,\boldsymbol{A}=\boldsymbol{0}\,, \tag{1}$$

where μ is the magnetic permeability (it must be considered in both permanent magnets, while everywhere else $\mu=\mu_0$), $\boldsymbol{H}_\text{c}$ is the remanence of the magnets, γ stands for their electric conductivity, and $\boldsymbol{v}$ denotes the velocity of the upper magnet.

Provided that the arrangement solved is axisymmetric, the vector $\boldsymbol{A}$ has only one nonzero component A_φ in the circumferential direction, vector $\boldsymbol{v}$ has the direction of the z-axis and vector $\boldsymbol{H}_\text{c}$ has two components in the r and z directions.

Magnetic field in the system produces two kinds of forces: repulsive magnetic force $\boldsymbol{F}_\text{m}$ acting between both magnets and Lorentz force $\boldsymbol{F}_\text{L}$ acting on the free magnet **4** that is produced by its movement in magnetic field of the unmoving magnet **3**.

The repulsive force $\boldsymbol{F}_\text{m}$ acting on magnet **3** is given by the formula [1]

$$\boldsymbol{F}_\text{m}=\frac{1}{2}\int_{S_4}\left[\boldsymbol{H}\left(\boldsymbol{n}\cdot\boldsymbol{B}\right)+\boldsymbol{B}\left(\boldsymbol{n}\cdot\boldsymbol{H}\right)-\boldsymbol{n}\left(\boldsymbol{B}\cdot\boldsymbol{H}\right)\right]\text{d}S \tag{2}$$

where $\boldsymbol{B}$ and $\boldsymbol{H}$ are the field vectors and $\boldsymbol{n}$ is the unit vector of the outward normal to the surface S_4 of magnet **4**. The integration is performed just over that surface. The force acts exclusively in the direction of the z-axis.

The Lorentz force $\boldsymbol{F}_\text{L}$ is of volumetric nature and follows from the interaction of currents induced in the free magnet and with magnetic field:

$$\boldsymbol{F}_\text{L}=\int_{V_4}\boldsymbol{J}_\text{ind}\times\boldsymbol{B}\,\text{d}V\,, \tag{3}$$

where

$$\boldsymbol{J}_\text{ind}=\gamma\boldsymbol{v}\times\text{curl}\,\boldsymbol{A} \tag{4}$$

and V_4 is the volume of magnet **4**. This force acts against the movement.

Important for the movement is also the drag force acting on magnet **4** and its weight. The drag force consists of two components: aerodynamic resistance $\boldsymbol{F}_\mathrm{a}$ and friction resistance. The first of them is described by the formula

$$\boldsymbol{F}_\mathrm{a} = -\boldsymbol{v}\frac{1}{2}\rho c S v\,, \tag{5}$$

where c is the friction coefficient (dependent on geometry of the body), ρ denotes the density ambient air, S is the characteristic surface of the moving magnet and v stands for the module of its velocity. This force also acts against the movement similarly as the following friction force $\boldsymbol{F}_\mathrm{f}$ caused by gliding of the magnet along the rod that is supposed to be a linear function of velocity $\boldsymbol{v}$

$$\boldsymbol{F}_\mathrm{f} = -k\boldsymbol{v}\,, \tag{6}$$

k being a constant. The weight $\boldsymbol{F}_\mathrm{g}$ of this magnet is

$$\boldsymbol{F}_\mathrm{g} = m\boldsymbol{g}\,, \tag{7}$$

where m is its mass and $\boldsymbol{g}$ is the gravitational acceleration. This force acts only in the $-z$ direction.

The equation describing motion of the magnet **4** then reads

$$m\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = \boldsymbol{F}_\mathrm{m} + \boldsymbol{F}_\mathrm{L} + \boldsymbol{F}_\mathrm{a} + \boldsymbol{F}_\mathrm{f} + \boldsymbol{F}_\mathrm{g}\,, \quad \boldsymbol{v} = \frac{\mathrm{d}\boldsymbol{s}}{\mathrm{d}t}\,, \tag{8}$$

where $\boldsymbol{s}$ stands for the position. These equations are supplemented with initial conditions

$$\boldsymbol{v}(0) = 0, \quad \boldsymbol{s}(0) = \boldsymbol{s}_0\,. \tag{9}$$

Numerical solution and principal features of the code

The numerical solution of the model is realized by a higher-order finite element method *hp*-FEM [2]. The *hp*-FEM is a modern version of the finite element method combining finite elements of variable size (*h*) and polynomial degree (p) in order to obtain fast exponential convergence. This approach leads to a significant reduction of the size of the discrete problem and significantly accelerates the whole computation. Our own numerical software Agros2D [3] (freely available under the GPLv2 license as a part of the modular higher-order finite element C++ library Hermes2D [4] is used for the computation. The main features of the SW follow:

- Hermes puts a strong emphasis on credibility of results, i.e., on evolution of the error in the process of automatic adaptivity and its control. Everybody who deals with computer modeling knows well how complicated it is to use automatic adaptivity together with standard lower-order approximations such as linear or quadratic elements–the error may decrease during a few initial adaptivity steps, but then it slows down and investing more unknowns or CPU time is practically of no use. This is typical for low-order methods. In contrast to this, the exponentially-convergent adaptive *hp*-FEM and *hp*-DG do not have this problem–the error drops steadily and fast during adaptivity to the prescribed accuracy.
- Hermes is completely PDE-independent. Many FEM codes are designed to solve some narrow class of tasks (such as elliptic, parabolic or hyperbolic problems, etc.). In contrast to that, Hermes does not employ any technique or algorithm that would only work for some particular class of the above PDE problems. Automatic adaptivity is guided by a universal computational a-posteriori error estimate that works in the same way for any PDE. Of course, this does not mean that it performs equally well on all PDE–some equations simply are more difficult to solve than others. However, Hermes allows tackling an arbitrary PDE or multiphysics PDE system.
- Hermes has a unique original methodology for handling arbitrary-level hanging nodes. This means that extremely small elements can be adjacent to very large ones. When an element is refined, its neighbors are never split forcefully as in conventional adaptivity algorithms. It is well known that approximations with one-level hanging nodes are more efficient compared to regular meshes. However, the technique of arbitrary-level hanging nodes brings this to perfection.
- Various physical fields or solved quantities in multiphysics problems can be approximated on individual meshes, combining quality H^1, H(curl), H(div), and L^2 conforming higher-order elements. Due to a unique original methodology, no error is caused by operator splitting or transferring data between different meshes.
- In time-dependent problems, different physical fields or solution components can be approximated on individual meshes that dynamically change in time independently of each other. Due to a unique original technology of mapping, no error is caused by transferring solution data between different meshes and time levels. No such transfer takes place in the multimesh *hp*-FEM–the discretization of the time-dependent PDE system is monolithic.

Illustrative example

An experimental arrangement of the problem is depicted in Fig. 2, together with the principal dimensions in meters. The permanent magnets are of NdFeB type whose remanent induction is $B_\mathrm{r} = 1.111$ T and relative permeability is $\mu_\mathrm{r} = 1.0628$. Mass of each magnet $m = 0.0298$ kg.

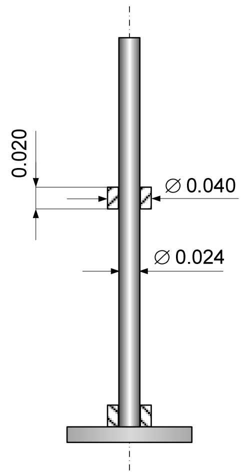

Fig. 2. Principal dimensions of the arrangement

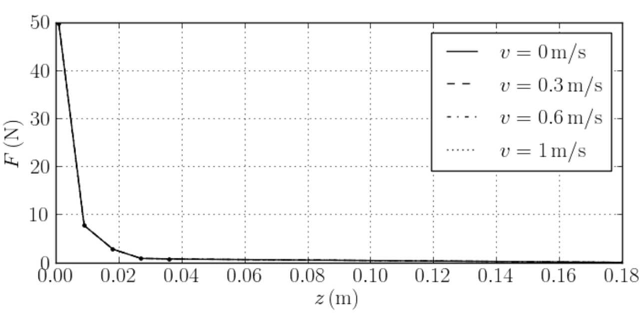

Fig. 3. Static characteristics of the system for several different velocities of the free magnet

For illustration, Fig. 3 shows the static characteristics ($F = F_{m,z} + F_{L,z}$) of the system for different values of velocity of the free magnet 4. It is evident that velocities smaller than 1 m/s play no role and can be neglected (the principal reason being a poor electric conductivity of the magnets not exceeding about 6.25×10^5 S/m).

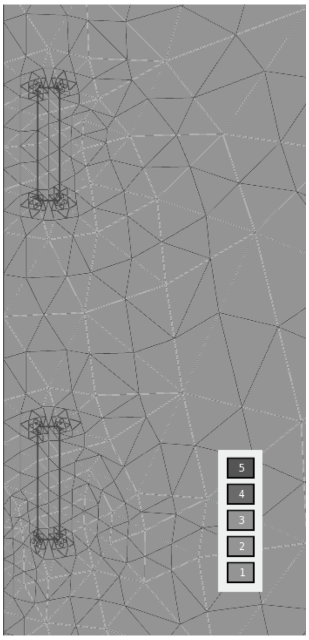

Fig. 4. Finite element adaptive mesh at the beginning of the process (black lines) and at the end of the adaptation process (white lines), distance between magnets being 0.04 m (different colours of elements show the corresponding degree of the polynomial)

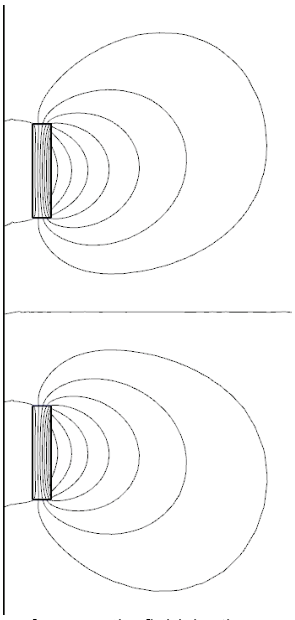

Fig. 5. Distribution of magnetic field in the system for the same case

Next three figures illustrate computation of magnetic field in the system when the distance between both magnets is 0.04 mm. The number of DOFs in the rough mesh is 3058, in fine mesh after adaptive process 3670 DOFs, maximum order of polynomials is 5, time of computation about 13.5 s and the number of adaptive steps was 6.

Figure 7 shows the experimental stand with both magnets in the initial (rest) position and Figures 8 and 9 show the time evolution of the position (this quantity was also validated experimentally) and velocity of the moving magnet for s_0 = 0.18m, (c = 0.4, k = 0.05, S = 0.000197m^2) m. After 8 seconds the oscillating transient can be declared for finished.

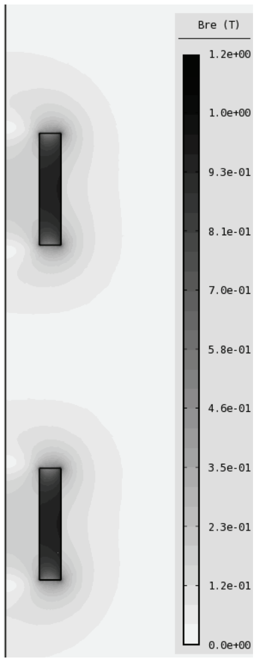

Fig. 6. Distribution of the module of magnetic flux density in the system for the same case

Fig. 7. Experimental stand

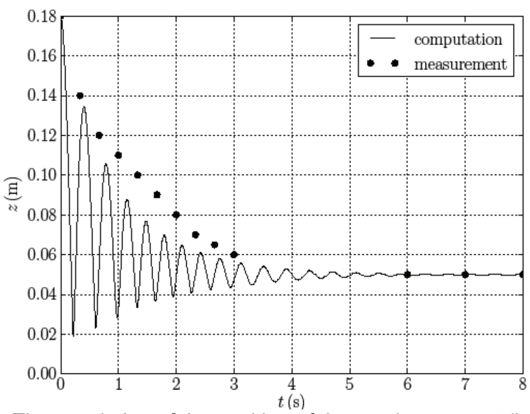

Fig. 8. Time evolution of the position of the moving magnet (its balanced position is in the distance of 0.05 m above the unmoving magnet)

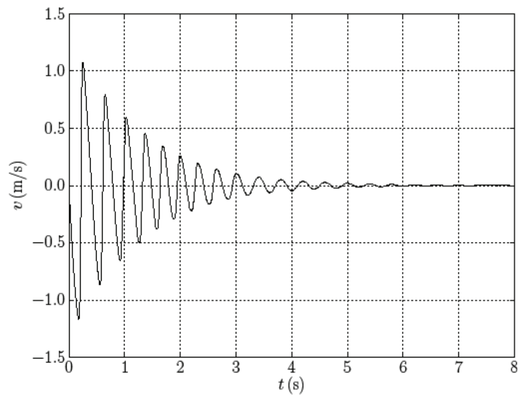

Fig. 9. Time evolution of the velocity of the moving magnet

The graph of time evolution of the position well corresponds with the experiment (the differences do not exceed about 15%),

Acknowledgment

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REFERENCES

[1] Stratton, J. A. (2007), *Electromagnetic Theory*, John Wiley & Sons, Inc., Hoboken, NJ.
[2] Solin, P. (2005), *Partial Differential Equations and the Finite Element Method*, Wiley & Sons, NY.
[3] Karban, P. et al: Agros2D - An application for the solution of physical fields (http://hpfem.org/agros2d).
[4] Solin, P. et al: Hermes2D - Higher-Order Modular Finite Element System (http://hpfem.org/hermes2d).

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