Peter CSURGAI, Miklos KUCZMANN

"Szécheny István" University, Lab. of Electromagnetic Field, Dept. of Telecommunications, Hungary

Comparison of various high-frequency models of RF chip inductors

***Abstract**. RF chip inductors are the most widely used inductors. Due to the miniaturization the continuous improvement of the electrical parameters are highly requested. This demand of the improvement makes the accurate electrical model of the inductors essential. The purpose of the paper is to present an evaluation and comparison of the currently available methods for calculating the inductor behaviour in frequency domain using analytic formulas, finite element simulations, and real measurements on market available RF chip inductors*

***Streszczenie.** Induktory z czipami o częstotliwościa radiowych RF są najszerzej używanymi induktorami. Dzięki miniaturyzacji ciągłe ulepszanie parametrów elektrycznych jest pożądane. Żądanie poprawy czyni, ze dokładne elektryczne modele induktorów zyskują na znaczeniu. Celem pracy jest pokazanie ewaluacji i porównania obecnie dostępnych metod dla obliczania działania induktora w dziedzinie częstotliwości przy użyciu wzorów analitycznych, symulacji metodą elementów skończonych, a także rzeczywistych pomiarów na dostępnych na rynku induktorach. (**Porównanie różnych modeli wysokoczęstotliwościowych induktorów RF**)*

Keywords: inductor modelling, inductor losses, stray capacitance.
Słowa kluczowe: modelowanie induktora, straty w induktorze, pojemność rozproszenia.

Introduction

RF chip inductors are the most widely used inductors in automotive, industrial, house hold and consumer electronic appliances. Some of the most common inductors can be seen in Fig. 1. Due to the miniaturization the continuous improvement of the electrical parameters are highly requested. This demand of the improvement makes the accurate electrical model of the inductors essential. On one hand the model shall be capable of identifying and separate the sources of the losses, on the other hand it shall be able to predict the exact behaviour of the inductor in frequency domain. The purpose of the paper is to present an evaluation and comparison of the currently available methods for calculating the inductor behaviour in frequency domain using analytic formulas, finite element simulations, and real measurements on market available RF chip inductors. For modelling electric components the analytic formulas are always preferred due to the lower computation time and lower hardware requirements. If the values of the electrical parameters have to be calculated not only in a few specific discrete frequencies, but on a frequency domain, analytic formulas can save a lot of time, and work. Of course the accuracy of the analytic models are not satisfactory in every case, therefore the best compromise between accuracy and computation time is a semi-analytic model, where some parameters are calculated by analytic formulas while others - mainly coefficients for frequency functions - are computed by finite element method, like the calculation of inductance due to the nonlinear characteristic of the permeability.

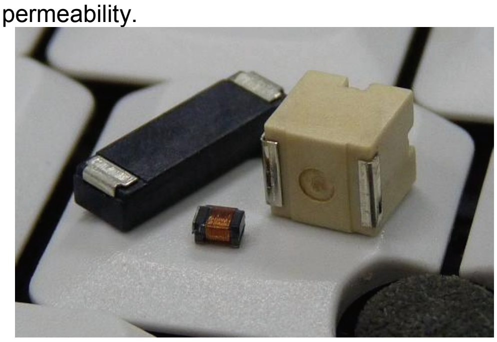

Fig. 1. RF chip inductors in various size

Model structure

The used model consists of the three basic elements of the electric circuits, like inductance, resistance and capacitance, as it can be seen in Fig. 2. By these elements the simulated impedance can be changed in either direction in the complex plane according to the real behaviour of an inductor. As the values of all the three parameters of the equivalent circuit depend on the frequency, they ought to be a function of frequency instead of a constant value. It should be noted, that these three elements will determine the impedance together [1].

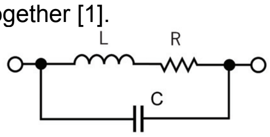

Fig. 2. Lossy inductor equivalent circuit

Numerous models are known for simulating the lossy inductor, but most of the models are applicable only to simulate the measured electrical curves based on measurement performed prior the simulation, and not applicable to determine the behaviour of the inductor prior the measurement. In those models the circuit components usually does not represents any specific physical phenomenon.

The impedance analysers simplify the 3 element model structure to a 2 element model – like can be seen in Fig. 3. – structure for the measurement. During the measurement only the complex impedance is measured (*R* and *X*), and the other parameters like impedance (*Z* and *φ*), inductance (L_s) and quality-factor (*Q*) are calculated based on the 2 element model [1].

Fig. 3. Equivalent circuit used by the impedance measuring instruments

The impedance which is represented by the three element model can be calculated by the following formulas for the 2 element model [2]:

(1) $$R_s = \frac{R_{ac}}{\left(1-\omega^2 L_{ac} C\right)^2 + \left(\omega C R_{ac}\right)^2}$$

(2) $$X_s = \frac{\omega L_{ac}\left(1-\omega^2 L_{ac} C - \dfrac{C R_{ac}{}^2}{L_{ac}}\right)}{\left(1-\omega^2 L_{ac} C\right)^2 + \left(\omega C R_{ac}\right)^2}$$

where R_{ac} is the resistive part of the model which represents the resistive losses, L_{ac} is the inductance part of the model, which represents the frequency dependent inductance, and C is the capacitor of the model which includes all the stray capacitances of the inductor.

Model components – Inductance (L)

The first part of the lossy inductor equivalent circuit is evidently the frequency dependent inductance. Based on the inductor construction two kinds of inductance calculation technique can be applicable. The fist part of the calculation – when the air-core inductance is calculated – is the same in both cases. The calculation of the air-core inductance is well known and explained either it is a single layer or a multilayer inductor, many closed formula is know which approximates the measured inductance value very well. The most common and precise formulas are the followings:

– Nagaoka's formula [3]:

$$L = K\mu \frac{r^2 \pi N^2}{l}, \tag{3}$$

where K is the Nagaoka's coefficient, μ is the permeability, r is the mean coil radius, N is the number of turns and l is the length of the coil.

– Wheeler's formula [4]:

$$L = \mu \frac{r^2 \pi N^2}{l(1+0{,}9r)} \tag{4}$$

– Improved solenoid formula [5]:

$$L = \mu \frac{N^2 r^2 \pi}{\sqrt{l^2 + (2r)^2}} \tag{5}$$

– Multilayer formula [6]:

$$L = \mu \frac{0{,}8 r^2 \pi N^2}{l + 0{,}9r + b}, \tag{6}$$

where b is the thickness of all layers.

– Improved multilayer formula [6]:

$$L = \mu \frac{r^2 \pi N^2}{l} \frac{1}{1 + 0{,}9\frac{r}{l} + 0{,}32\frac{b}{r} + 0{,}84\frac{b}{l}}. \tag{7}$$

Only the air core inductor can be assumed to have constant inductance in frequency range. If any ferromagnetic material is used as a ferrite core the inductance will be frequency dependent through the complex permeability.

The complex permeability, as the complex impedance, consists of two parts. The real part represents the inductance, and the imaginary part represents the resistive impedance or losses of the ferrite material [6].

$$\mu_{rs} = \mu_0 (\mu'_{rs} - \mu''_{rs}) \tag{8}$$

The real part of the complex series permeability can be approximated by the magnitude of a transfer function of a first order low-pass filter:

$$\mu'_{rs}(f) = \frac{\mu_{r0}}{\sqrt{1 + \left(\frac{f}{f_H}\right)^2}}, \tag{9}$$

where μ_{r0} is the low frequency permeability, f is the frequency, and f_H is the frequency threshold, where the permeability decreases by 3dB. The imaginary part of the permeability can be approximated by a magnitude of a transfer function of a second order band-pass filter:

$$\mu''_{rs}(f) = \frac{\mu_{r0}}{\sqrt{1 + Q^2\left(\frac{f}{f_H} - \frac{f_H}{f}\right)^2}}, \tag{10}$$

where the quality-factor (Q) is given by

$$Q = \frac{f_H}{\Delta f}, \tag{11}$$

where Δf is the -3 dB bandwidth of the μ'' characteristic.

Parameters like f_H and Δf is given in the datasheets of ferrite materials or can be determined by the given characteristic of the complex permeability.

If the examined inductor contains ferrite core and the magnetic circuit is not closed (flux lines pass through air, or non ferromagnetic material) the complex permeability curve will deform. The permeability – both real and imaginary parts – will drop and the threshold frequency will increase. In these cases the following formula can give a good approximation to determine the threshold frequency, when the effective permeability of the entire magnetic loop is known [6]:

$$f_H = \frac{6{,}25 \cdot 10^9}{\mu_r}, \tag{12}$$

where μ_r is the effective permeability of the open magnetic loop. It is very hard to find the effective permeability, and in most cases the finite element method is the only way to find it. But after once the effective permeability of the open magnetic loop is known, it can be calculated to any frequency without further complicated time and hardware consumptive manipulations. The frequency dependent inductance of the inductor can be calculated by the above mentioned air cored and solenoid formulas using the frequency dependent complex permeability.

Model components – Resistance (R)

The AC resistance of conductors is well known and well described results of the skin and proximity effects. The frequency dependence of the resistance can be calculated based on Dowell's approximation [7]. The applied formula (15) takes into account independently the skin and the proximity effects as well. The skin effect part (13) determines the resistance increase based on the geometry of the conductor, while the proximity effect part (14) calculates the resistance raise based on the conductor geometry and the layer number of the multilayer inductor:

$$F_S = A \cdot \left[\frac{\sinh(2A) + \sin(2A)}{\cosh(2A) - \cos(2A)}\right], \tag{13}$$

$$F_P = A \cdot \left[\frac{2(N_l^2 - 1)}{3} \frac{\sinh(A) - \sin(A)}{\cosh(A) + \cos(A)}\right], \tag{14}$$

$$F = \frac{R_W}{R_{WDC}} = F_S + F_P, \tag{15}$$

where F is the AC resistance factor which gives the resistance increase, N_l is the number of layers of multilayer inductor, and A is the conductor thickness normalized to the skin depth:

$$A = \left(\frac{\pi}{4}\right)^{\frac{3}{4}} \frac{d}{\delta_W} \sqrt{\frac{d}{p}}, \tag{16}$$

where d is the thickness of the round conductor (winding wire), p is the pitch (distance between two adjacent turns), and δ_W is the skin depth:

$$\delta_W = \sqrt{\frac{2\rho}{\omega\mu}}, \tag{17}$$

where ρ is the resistivity of the conductor material, μ is the permeability, and ω is the angular frequency.

All losses increase the resistive part of the impedance, therefore during the calculation of the resistance other factors like eddy current losses and ferrite hysteresis losses shall be considered.

The ferrite hysteresis loss can be calculated based on the complex permeability and the inductance of the coil:

$$R_C = \omega \mu'' L, \tag{18}$$

where R_C is the core equivalent resistance, ω is the angular frequency, μ'' is the imaginary part of the complex permeability in given frequency, and L is the inductance of the coil [6]. The overall resistance of the inductor can be calculated by simply summing the winding resistance and the core equivalent resistance.

Model components – Capacitance (C)

The overall stray capacitance of an inductor consists of the capacitance between turns (turn-to-turn), capacitance between turns and the shielding and capacitance between turns and conducting core. As the examined signal use surface mounted inductors do not have shielding, and the ferrite core is made of NiZn ferrite, whose resistivity is in the 10^6 Ωm range the capacitance between shielding and between conducting core can be neglected. However the terminals due to the large surface (compared to the external dimensions of the inductor) might have significant stray capacitance if they take place sufficient close to the winding turns.

Most single layer turn-to-turn capacitance models assume the capacitance constant over the whole frequency range and so thus the analytic formulas are created assuming electrostatic problem and environment [8-9]. The calculation of these capacitances can be done based on the capacitance of two infinite long straight and parallel wires with round cross section in homogenous medium. In this case the curvatures of the winding turns and the capacitances between non-adjacent turns are neglected. The capacitance between two adjacent turns can be calculated as follows [9]:

(19)
$$C_{tt} = \frac{\pi^2 D \varepsilon_0}{\ln\left(\frac{p}{2r} + \sqrt{\left(\frac{p}{2r}\right)^2 - 1}\right)},$$

where D is the diameter of the winding turns, p is the pitch, and r is the radius of the winding wire. As the turns are connected series in the coil, so thus the capacitance between them are also connected series. Therefore the overall capacitance can be calculated as follows:

(20)
$$C_S = \frac{C_{tt}}{N-1},$$

where C_{tt} is the turn-to-turn capacitance and N is the number of turns. The above described capacitance calculation can be applicable only for single layer coil, where the capacitance between layers and the wire coating are negligible [9].

There can be found analytic formulas also for multilayer inductors, but those manipulations can be applicable only for very limited cases [6].

Model validation by measurement

The accuracy of the above explained 3 element model was evaluated on three different basic constructions of surface mounted inductors like solenoid type inductors, air core multilayer inductors, and ferrite core multilayer inductors.

Solenoid type inductors:

As this kind of inductor construction is not very common only one type was evaluated with the model. This type was a 0805 ceramic core type with the inductance value of 150nH. This construction consists of only a ceramic bobbin and the winding, which results the lowest losses and the simplest geometry. The winding is single layer winding, and as there is not ferrite core, the core losses was eliminated from the model.

The "Simplified model" named curve assumes all parameter (L, C, R) constant, and shows a big difference compared to the measured curve. The "Complete model" named curve in this specific case assumes only the resistance to be frequency dependent, the inductance and capacitance is assumed to be constant in frequency range. The results of the different models can be seen on the Fig. 4.

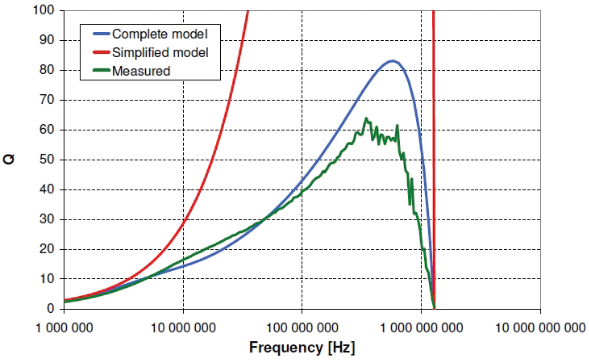

Fig. 4. 0805 size 150nH chip inductor Q curves

The results of the inductance calculation of 0805 size 150nH by different formulas can be seen in Table 1.

Table 1. Inductance calculation for 0805 – 150nH

Wheeler formula (4)	168,12 nH
Improved solenoid formula (5)	159,91 nH
Multilayer formula (6)	134,50 nH
Improved Multilayer formula (7)	168,12 nH

Air core multilayer type inductors:

The second, air core multilayer type inductor in the evaluation was a special 11mm long transponder coil without ferrite core. This type differs from the previous type in the number of turns and in the number of layers in the winding. The transponder coil due to its main function has much higher number of turns and layers. The "Simplified model 2" curve in Fig. 5. assumes all parameter (L, C, R) constant, the "Simplified model 1" includes only the proximity effect calculation, and the "Complete model" contains both skin and proximity effect calculation as well.

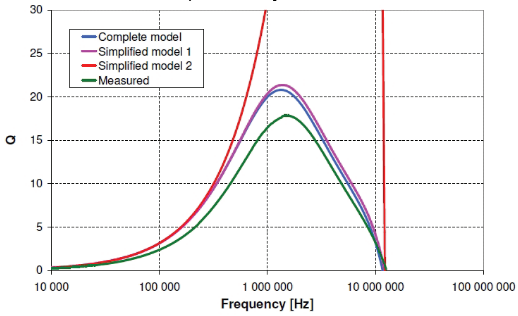

Fig. 5. Transponder 11mm (air core) chip inductor Q curves

The results of the inductance calculation of 11mm air core transponder coil can be seen in Table 2.

Table 2. Inductance calculation for 11mm Transponder – 84 μH

Wheeler formula (4)	99,88 μH
Improved solenoid formula (5)	107,4 μH
Multilayer formula (6)	76,63 μH
Improved Multilayer formula (7)	85,79 μH

Ferrite core multilayer inductors:

These kinds of inductors are the most common constructions in the SMD inductor spectrum. For these constructions the model was completed with the losses of the ferrite core and the drift of the inductance due to the drop of the permeability in frequency.

The tested inductor in the evaluation was the 2220 size 4,7 μH. The resulting curves are shown in Fig. 6. The

“Simplified model 3” assumes all parameter (L, C, R) constant, the “Simplified model 2” includes only the proximity effect calculation, the “Simplified model 1” includes the losses of the core and proximity effect, and the “Complete model” includes the complete Dowell formula and the losses of he core as well.

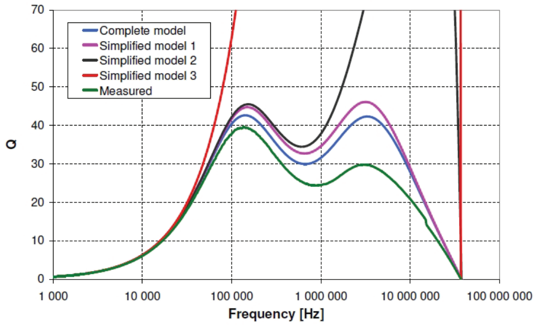

Fig. 6. 2220 size 4,7μH chip inductor Q curves

The inductance calculation (using the effective permeability computed by finite element method) of the 2220 size 4,7 μH can be seen in Table 3.

Table 3. Inductance calculation for 2220 – 4,7 μH

Wheeler formula (4)	6,868 μH
Improved solenoid formula (5)	6,470 μH
Multilayer formula (6)	4,260 μH
Improved Multilayer formula (7)	5,001 μH

Conclusions

Inductance calculation:

The inductance calculation of chip inductors may be sufficient accurate, only the appropriate formula has to be used. In case of single layer coil the improved solenoid formula (5) should be used, in case of multilayer inductor the improved multilayer formula (7) gives the most accurate approximation. In constructions where ferrite core is used the inductance calculation becomes difficult due to the open magnetic loop, and the unknown overall permeability. Although some experimental approximation and charts [10] helps to determine the effective permeability, the most accurate results can be computed only by finite element method. Once the effective permeability is known the inductance of the inductor can be further calculated by using the formulas (5) or (7). Through the characteristic of the permeability in frequency range the inductance drift can be calculated. If exact chart about the permeability curve is not available the permeability of the ferrite can be approximated by the transfer function of a first order low-pass filter.

Resistance calculation:

From the performance point of view the resistance of the inductor has the highest importance, because the losses (winding losses, core losses, eddy current losses) increase the resistive part of the impedance. The resistance has the highest influence on the quality factor as well. As can be seen in the charts above (Fig. 4.-6.) slight difference in the resistance curve results significant difference in the quality factor curve. Therefore all kind of losses and the exact behaviour of the losses in frequency domain should be approximated to get realistic Q curve. It can be seen that in case of air core inductors, where the losses are mainly the winding AC resistance, the Q curve can be approximated easily by a quite simple model. Using a ferrite core makes the model more intricate and so the resistive losses will be more difficult to determine. Although the above simulated Q curves stand close to the realistic curves, it can be seen that some sources of losses are still neglected or disregarded. One of these losses is the eddy current losses. In chip inductors the terminals are quite large compared to the external dimensions of the inductor, and in some cases it takes place close to the winding. As the geometry of the terminals, like thickness and shape, might be very different among the sizes and constructions of chip inductors, stand alone closed analytic formulas can not be assigned to this loss. However a semi-analytic formula where some coefficients are calculated by finite element method, while the complete curve of the eddy current loss are approximated by an analytic formula might be made.

Capacitance calculation:

The capacitance calculation is the most difficult task in the modelling of inductors. As the voltage distribution between turns are different in each constructions and sizes of chip inductors, the stray capacitance might be different in each cases despite of the winding turns fill the winding camber with the same pattern. For example in case of a hexagonal pattern the stray capacitance of the winding will be significantly different if the turns are placed in heaps next to each other instead of smooth layers on the top of each others. As the pattern and the geometry are the same in both cases and the self resonant frequency is different it can be stated that the stray capacitance is different. Therefore the stray capacitance is defined not only by the geometry, but by the voltage distribution among the winding turns as well. This will result not only that the winding technique will define the stray capacitance, but inductors with sufficient small external dimensions (sufficient low stray capacitances and low inductance values, so thus with very high self resonance frequencies) have lower self resonance frequency than it is expected using the analytic formulas. Due to the small external dimensions of the inductor the wavelength of the exciting signal will be in the same range as the length of the winding turns, which results non-linear voltage distribution among the turns.

REFERENCES

[1] Agilent Technologies, “Impedance Measurement Handbook”, 4th edition, June 2009
[2] Marian K. Kazimierczuk, Giuseppe Sancineto, Gabriele Grandi, Ugo Reggiani, Antonio Massarini, „High-Frequency Small-Signal Model of Ferrite Core Inductors”, *IEEE Transactions on Magnetics*, Vol. 35, No. 5, September 1999
[3] H. Nagaoka, "The Inductance Coefficients of Solenoids," *J. Coll. Sci.*, vol. 27, pp. 18-33, 1909
[4] H.A. Wheeler, "Inductance Formulas for Circular and Square Coils," *Proc. IEEE*, vol. 70, no. 12, pp. 1449-1450, Dec. 1982
[5] Jae-Wook Kim, Myung-Hee Jung, Nho Kyung Park, Eui-Jung Yun, „Microfabrication of solenoid-type RF SMD chip inductors with an Al2O3 core”, *Current Applied Physics 8* (2008) 631–636
[6] Marian K. Kazimierczuk, “High-Frequency Magnetic Components”, John Wiley & Sons, Ltd. 2009
[7] Alberto Reatti and Marian K. Kazimierczuk, „Comparison of Various Methods for Calculating the AC Resistance of Inductors”, *IEEE Transactions on Magnetics*, Vol. 38, No. 3, May 2002
[8] Luca Dalessandro, Fabiana da Silveira Cavalcante, and Johann W. Kolar, „Self-Capacitance of High-Voltage Transformers”, *IEEE Transactions on Power Electronics*, Vol. 22, No. 5, September 2007
[9] Gabriele Grandi, Marian K. Kazimierczuk, Antonio Massarini, and Ugo Reggiani, „Stray Capacitances of Single-Layer Solenoid Air-Core Inductors”, *IEEE Transactions on Industry Application,* Vol. 35, No.5, September/October 1999
[10] Philips Magnetic Products Application Notes, “Ferrite Rod Antennas for RF-identification Transponders”, http://www.elnamagnetics.com/library/rfant.pdf

***Authors**:*
Peter Csurgai, “Széchenyi István” University, Lab. of Electromagnetic Field, Dept. of Telecommunications, Hungary, E-mail: peter.csurgai@yahoo.com