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Boundary Element Method approach to forward problem solution in Diffusion Optical Tomography

***Abstract**. In this paper the capabilities of optical tomography (OT) applied to breast cancer detection have been discussed. The first step toward the creation of the model is forward problem solution. For this purpose BEM with isoparametric triangle element has been implemented, which improves the solution accuracy. The forward problem solution in OT is based on the solution of diffusion equations in the frequency domain. In the case where scattering and absorption are homogenous, the equation reduces to a Helmholtz equation with Robin boundary condition.*

***Streszczenie.** W pracy przedstawiono zastosowanie dyfuzyjnej tomografii optycznej (DTO) do detekcji raka piersi. W celu rozwiązania zagadnienia prostego w DTO należy rozwiązać równanie Helmholtza w dziedzinie częstotliwości z warunkami brzegowymi Robina. W tym celu zastosowano metodę elementów brzegowych z trójkątnym elementem izoparametrycznym, co wpływa na poprawę dokładności rozwiązania.(**Zastosowanie metody elementów brzegowych do rozwiązanie problemu prostego w dyfuzyjnej tomografii optycznej**).*

Keywords: Boundary Element Method, forward problem, Optical Tomography.
Słowa kluczowe: Metoda Elementów Brzegowych, zagadnienie proste, tomografia optyczna.

Introduction

Diffuse Optical Tomography (DOT) is a method of testing the interior of objects which have the ability to absorb and scatter the near infrared light [1, 2]. The application of this method may be examination the lesions of human tissue including the detection of cancer and examination of brain activity [3, 4]. Comparing to other diagnostic tomography-based methods like CT (x-ray tomography), PET (Positron Emission Tomography), ultrasound or MRI, DOT is an imaging technique, that allows displaying soft tissue contrast based on functional parameters (e.g. blood volume and oxygenation) as well as some structural differences, with use of portable and low cost medical monitor. Other advantages are use of harmless doses of non-ionising radiation, the possibility of simultaneous reconstruction of the entire three-dimensional volume of the examined organ and no requirement for compression of the organ.

Our previous research focused to Electrical Impedance Tomography application to tissue examination [5]. The method is reported to have similar desirable properties with exception of problems with the tissue conductivity. Currently, we adapted the numerical model of the EIT phenomena to DOT. In DOT, the light transport theory is used to explain photon propagation inside turbid medium [6, 7]. For a forward problem solution the Helmholtz partial differential equation with Robin boundary condition is applied, instead of Laplace equation in EIT.

As in the case of EIT, image reconstruction of the DOT image is tough, because the equations are non-linear and ill-posed. Even small errors in the modeling phase may give large bias in reconstruction, so it is crucial to develop and implement the accurate numerical method for the forward problem.

Boundary Element Method (BEM) is the one of the methods used to solve forward problems in biomedical engineering. It is an interesting alternative to commonly used Finite Element Method (FEM) [8, 9]. The principle advantage of using BEM is that the discretization is necessary only at the boundary of analyzed domain, what is connected with a significant reduction of the matrix dimension and number of unknowns from $O(N^2)$ to $O(N)$ [8]. The BEM uses Green's second theorem to express the potential distribution [8, 9, 10, 11]. In this method the partial differential equations (PDE) with adequate boundary conditions are transformed into an equivalent integral equation set defined only on the surface of the considered volume. The original integral equation governing surface potential can be approximated as a summation of surface integrals of each element. While the potential on each element remains to be calculated, it may be assumed to be a simple basis function of some unknowns. The potential on an element may be modeled as [8, 9, 10, 11]:

- a constant value (zero-order interpolation functions); the function and partial derivative are constant on each element,
- a linear functions, which attains a different value at each of the vertices of the element; the function and partial derivative are linear,
- a quadratic function; the function and partial derivative are described as a quadric function of the local variables.

Quadratic interpolation function

The factors, which restrict the accuracy and affect the time required to obtain a solution using the BEM are:

- the choice of basis function,
- density of elements,
- the shape of each element.

In [10] and [11] authors compare influence of these factors on the accuracy of the solution obtained with the BEM and conclude that linear- or quadratic-base functions are generally speaking superior to constant-base functions for the interpolation of the potential on plane triangular surface elements.

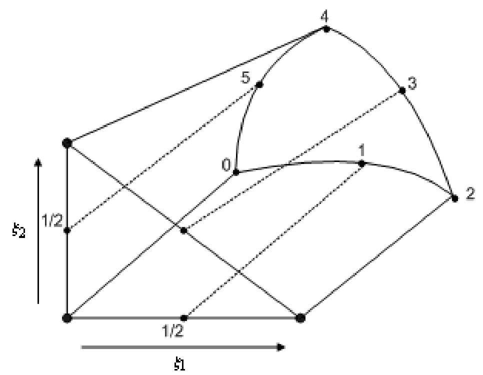

Fig. 1. Triangular surface element on the unitary triangle, which is parameterized in terms of ξ_1 and ξ_2

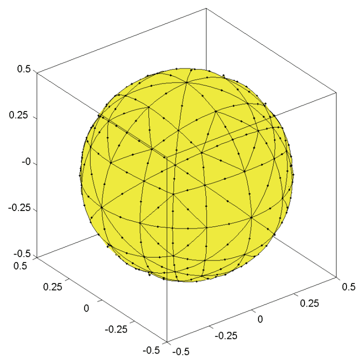

Fig. 2. Surface isoparametric element - sphere R=10mm

Regardless of the number of considered and modeled surfaces, using of BEM to realistic-shaped surfaces requires selection of available basis functions and using a specific grid pattern which determines element density and shape.

Assume a six-node triangular isoparametric element presented in Fig. 1. [5, 8, 9, 10, 11].

The use of isoparametric elements (Fig. 2) may significantly improve the way the mesh approximates the actual shapes in comparison to zero-order element without the need to introduce additional discretization points.

The shape functions are given by the following formulas [5, 8, 9, 10, 11]:

$$\begin{array}{ll} N_0(\xi_1,\xi_2) = -\xi_3(1-2\xi_3) & N_1(\xi_1,\xi_2) = 4\xi_1\xi_3 \\ N_2(\xi_1,\xi_2) = -\xi_1(1-2\xi_1) & N_3(\xi_1,\xi_2) = 4\xi_1\xi_2 \\ N_4(\xi_1,\xi_2) = -\xi_2(1-2\xi_2) & N_5(\xi_1,\xi_2) = 4\xi_2\xi_3 \end{array} \tag{1}$$

The first derivatives of the standard interpolation functions with respect to the ξ_1 and ξ_2 are given by:

$$\begin{array}{ll} \dfrac{\partial N_0(\xi_1,\xi_2)}{\partial \xi_1} = 1-4\xi_3 & \dfrac{\partial N_0(\xi_1,\xi_2)}{\partial \xi_2} = 1-4\xi_3 \\ \dfrac{\partial N_1(\xi_1,\xi_2)}{\partial \xi_1} = 4(\xi_3-\xi_1) & \dfrac{\partial N_1(\xi_1,\xi_2)}{\partial \xi_2} = -4\xi_1 \\ \dfrac{\partial N_2(\xi_1,\xi_2)}{\partial \xi_1} = -1+4\xi_1 & \dfrac{\partial N_2(\xi_1,\xi_2)}{\partial \xi_2} = 0 \\ \dfrac{\partial N_3(\xi_1,\xi_2)}{\partial \xi_1} = 4\xi_2 & \dfrac{\partial N_3(\xi_1,\xi_2)}{\partial \xi_2} = 4\xi_1 \\ \dfrac{\partial N_4(\xi_1,\xi_2)}{\partial \xi_1} = 0 & \dfrac{\partial N_4(\xi_1,\xi_2)}{\partial \xi_2} = -1+4\xi_2 \\ \dfrac{\partial N_5(\xi_1,\xi_2)}{\partial \xi_1} = -4\xi_2 & \dfrac{\partial N_5(\xi_1,\xi_2)}{\partial \xi_2} = 4(\xi_3-\xi_2) \end{array} \tag{2}$$

Forward problem

The forward problem solution in Diffusion Optical Tomography is based on the solution of diffusion equations in the frequency domain [7, 12, 13, 14]:

$$\left(\nabla D \nabla - \mu_a + i\frac{i\omega}{c}\right)\Phi(\mathbf{r},\omega) = q_0(\mathbf{r},\omega) \quad \forall \mathbf{r} \in \Omega \setminus \Gamma \tag{3}$$

where: Ω is bounded edge Γ; Φ - photon density; $D = \dfrac{1}{3(\mu_a + \mu_s')}$ - diffusion coefficient; μ_a - absorbing coefficient; μ_s' - reduced scattering coefficient; $c(\mathbf{r}) = \dfrac{c_0}{v(\mathbf{r})}$ - the speed of light; $v(\mathbf{r})$ - refractive index; c_0 - the speed of light in a vacuum; q_0 - a source of light with modulation frequency ω.

In the case where scattering and absorption are homogenous, equation (3) reduces to a Helmholtz equation in frequency domain with complex wave number $k = \sqrt{\dfrac{\mu_a}{D} - i\dfrac{\omega}{cD}}$ [7, 12,13,14]:

$$\nabla^2\Phi(\mathbf{r},\omega) - k^2\Phi(\mathbf{r},\omega) = -\frac{q_0(\mathbf{r},\omega)}{D} \tag{4}$$

For the three dimensional space, fundamental solution of diffusion equation is:

$$G(|\mathbf{r}-\mathbf{r}'|,\omega) = \frac{1}{4\pi|\mathbf{r}-\mathbf{r}'|}e^{-k|\mathbf{r}-\mathbf{r}'|} \tag{5}$$

Forward problem solution in DOT for 3D

First step of the BEM is mesh generation. Meshing can be defined as the process of division a physical domain into smaller sub-domains (elements) in order to facilitate the numerical solution of a partial differential equation. In order to build the model of forward problem in DOT and create surface mesh we applied NETGEN [15].

For the description of input data (model geometry) two formats are used: CSG (Constructive Solid Geometry) and STL (stereolitography). The geometry in CSG format is described as a composition of basic solids (e.g. sphere, cylinder) using logical functions (e.g. AND, OR).

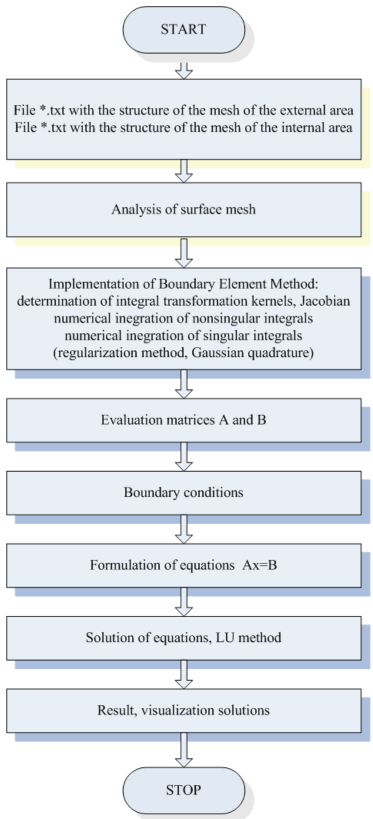

Fig. 3. Flowchart of BEM

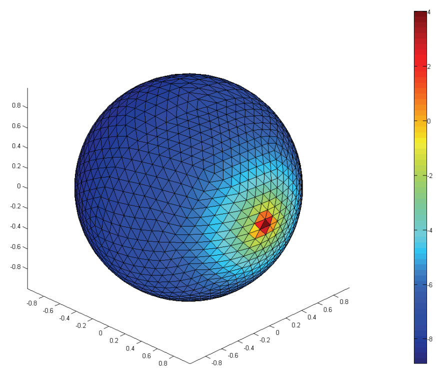

Fig. 4. Solution by BEM: $\frac{d\Phi}{dn}$ on the surface

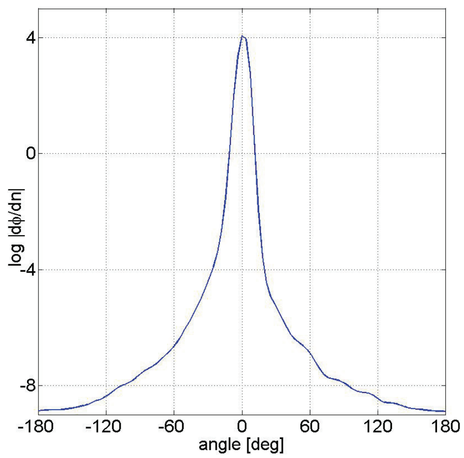

Fig. 5. Solution by BEM:on the circumference of the sphere. The source is located at 0 deg.

The problem of mesh generation for CSG model is divided into stages:

- geometrical model creation,
- characteristic points calculation,
- edges calculation,
- mesh generation on the delimiting surfaces.

The use of isoparametric elements (Fig. 2) may significantly improve the way the mesh approximates the actual shapes eg. in comparison to zero-order element, without the need to introduce additional discretization points.

The simulations have been performed for spherical model (a homogenous phantom with the same background properties) with following coefficients: a refractive index of 1.0 was assumed, absorbing coefficient was equal 0.007 mm^{-1} and reduced scattering coefficient was equal 0.8 mm^{-1} [14]. These values are representative for breast tissue [13,14].

The numerical methods – BEM with flat triangular elements and BEM with isoparametric triangle elements have been implemented in C++.

In the first simulation for discretization of sphere surface, the flat triangular elements were used. The mesh had 640 boundary elements and 322 nodes.

Figs. 6 and 7 present solution of Helmholz equation in frequency domain using BEM. The spheres was discretised by 384 isoprarametric triangle elements.

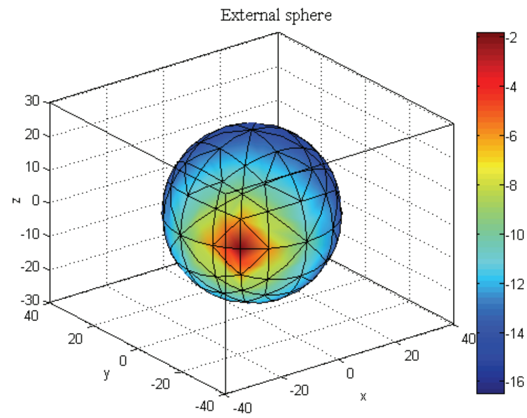

Fig. 6. External surface – photon density

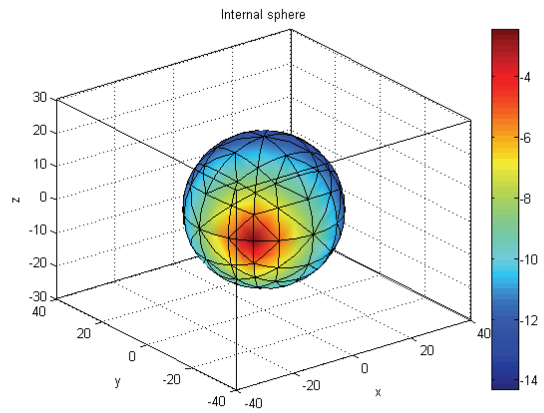

Fig. 7. Internal surface – photon density

Conclusion

We have successfully adapted BEM for the modeling phase from EIT to DOT. The process required replacing Laplace with Dirichlet and Neumann boundary condition by Helmholtz equation with Robin boundary condition. The Helmholtz equation caused extension of the computational space from floating-point with double precision to complex data types. No changes needed both mesh generation and data visualization phases.

The main advantages of using BEM in DOT are:

- only the surface of the analyzed domain is discretized, which is connected with reduction the dimension of the problem and total computational time,
- finding the unknown function in the inside points is possible, without necessity of discretization of this domain,
- this method allows analyzing boundless domains.

In the future we plan to develop a breast hemispherical model with infinite elements in the bottom. This will prevent creation of the artificial boundary between breast and the chest wall.

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