Andrzej RUSEK, Andrzej POPENDA

Częstochowa University of Technology

Mathematical model of drive system for metallurgical roller table unit with rotation of rollers transmitted by chain transmission

Abstract. *The paper discusses the drive system with the chain transmission driven by induction motor through the multi-stage gear-reducer. The mathematical model is formulated taking into account the load at both endings of each roller of drive set. The equations describing the drive system are derived on the basis of variational Hamilton's principle and Lagrange's equation of the second type. The final system of equations was transformed to the linear differential equations. The aforementioned system of equations allows formulating the computer model for digital simulations.*

Streszczenie: *W artykule zaprezentowano układ napędowy z łańcuchowym napędem rolek z zastosowaniem wielostopniowego reduktora zębatego napędzanego silnikiem indukcyjnym. Model matematyczny sformułowano z uwzględnieniem obciążeń na obu końcach każdej rolki zestawu napędowego. W oparciu o wariacyjną zasadę Hamiltona oraz równanie Lagrane'a drugiego rodzaju wyprowadzono równania opisujące układ napędowy. Ostateczną postać układu równań sprowadzono do układu liniowych równań różniczkowych umożliwiającego sformułowanie modelu komputerowego do obliczeń symulacyjnych.* ***Układ napędowy z łańcuchowym napędem rolek z zastosowaniem wielostopniowego reduktora zębatego napędzanego silnikiem indukcyjnym***

Keywords: chain transmission, roller-table, mathematical modelling, technological loads.
Słowa kluczowe: napęd łańcuchowy, samotokowa linia transportowa, modelowanie matematyczne, obciążenia technologiczne.

Introduction

The drive systems for roller-table sets with rotation of the rollers transmitted by chain transmission are example of a constructional solution of the drive systems for roller-table transporting lines. The aforementioned drive systems are applied in transporting lines in which due to technological problems the instalment of gear-motors with self-transporting construction on each roller is not possible. It concerns also these cases where transmission of motion among rollers through the shaft with gears is not possible. These cases also appear in technological lines in which elements have sections transported perpendicular to the roller-table by single shifters or shifter sets apart from motion of these elements along the roller-table [3], [4].

The goal of the paper is to develop a mathematical model of drive system for roller-table set with chain transmission among rollers. The working torques have to be determined in the model. The chain transmission can have various solutions depending on technological requirements. In the paper an exemplary way of transmission of rollers' motion is presented. The drive consists of three gears fixed on rollers' shafts, two gears for chain tension and a driving gear fixed on the ending of the multistage gear-motor.

The kinematic diagram of the drive for three rollers of transporting roller-table with chain transmission covered rollers of the drive is depicted in Figure 1.

In the presented kinematic diagram of the abovementioned drive system the gears of the chain tension are shown as elements with inertia J_{17} and J_{19}. These gears in the other constructional solutions may be replaced by eight gears of the chain tension in such a way that the gears of the chain tension shown as elements with inertia J_{17} and J_{19} are not in the axis of the chain gears fixed on the endings of the transport rollers shown as elements with inertia J_{16}, J_{18}, J_{20}. The advantage of the presented drive system is the possibility of whichever forming of the rollers' span depending on the constructional requirements.

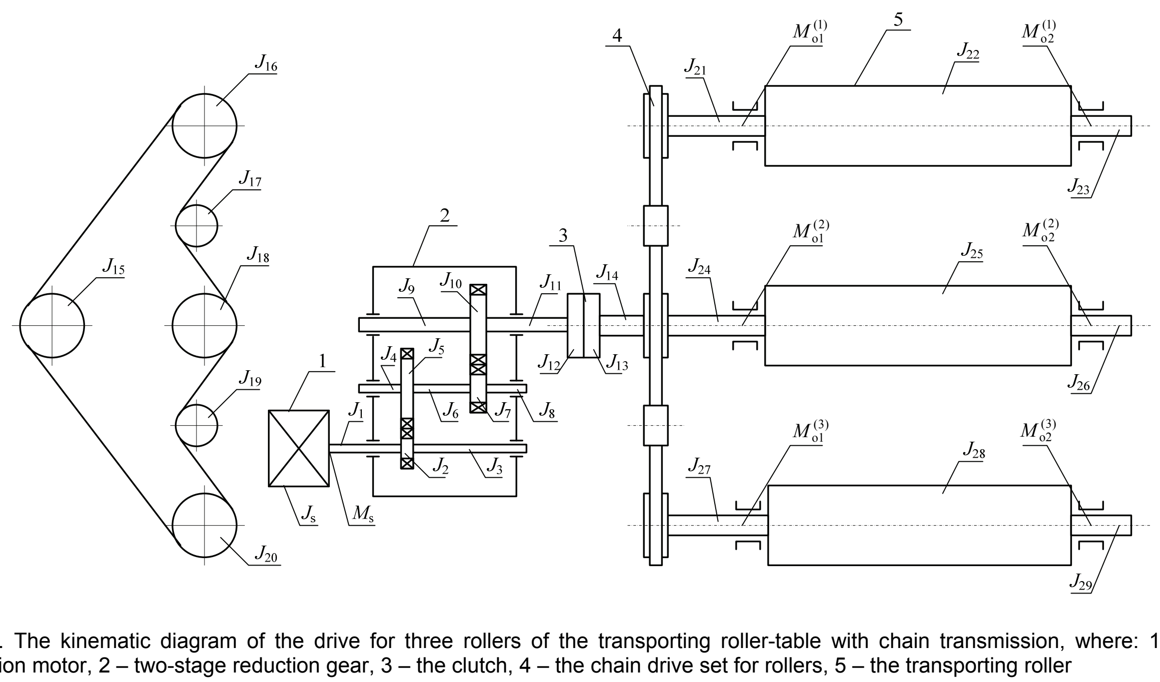

Fig. 1. The kinematic diagram of the drive for three rollers of the transporting roller-table with chain transmission, where: 1 – the induction motor, 2 – two-stage reduction gear, 3 – the clutch, 4 – the chain drive set for rollers, 5 – the transporting roller

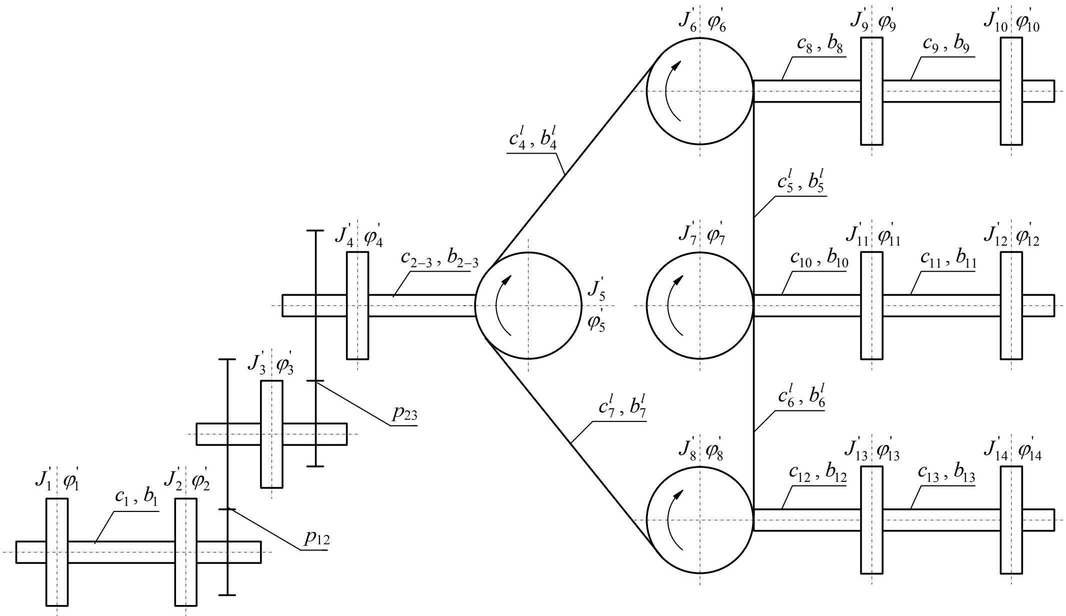

Fig. 2. The simplified computable kinematic diagram of the system depicted in Figure 1

Determination of the output quantities

Taking into consideration a kinematic diagram of the drive system illustrated in Figure 1 this system was transformed to the reduced computable kinematic diagram given in Figure 2.

The distributed load torques shown in Figure 1 for respective rollers appear in the simplified computable kinematic diagram (Fig. 2) as follows:

- for the first roller the load torques $M_{o1}^{(1)},\ M_{o2}^{(1)}$ at reduced shield with moments of inertia $J_9^{'},\ J_{10}^{'}$,
- for the second roller the load torques $M_{o1}^{(2)}, M_{o2}^{(2)}$ at reduced shield with moments of inertia $J_{11}^{'},\ J_{12}^{'}$,
- for the third roller the load torques $M_{o1}^{(3)},\ M_{o2}^{(3)}$ at reduced shield with moments of inertia $J_{13}^{'},\ J_{14}^{'}$.

The gear ratios of the gear transmissions shown in Figure 2 are determined by dependencies (1):

$$(1)\qquad p_{12}=\frac{d_2}{d_5},\qquad p_{23}=\frac{d_7}{d_{10}},$$

$$p_{34}=\frac{d_{15}}{d_{16}}=\frac{d_{15}}{d_{18}}=\frac{d_{15}}{d_{20}},\qquad p_{45}=\frac{d_{16}}{d_{17}}=\frac{d_{20}}{d_{19}}$$

The moments of inertia reduced to the shields shown in Fig. 2 for respective elements of the system are expressed by dependencies (2): for the driving motor J_1' for the reduction gear J_2' – J_4', for the chain set of the driven rollers J_5' – J_8', for the transporting rollers J_9' – J_{14}'

$$(2)\qquad J_1^{'}=J_s+0{,}5J_1,\ J_2^{'}=0{,}5J_1+J_2+J_3,$$

$$J_3^{'}=J_4+J_5+J_6+J_7+J_8,\ J_4^{'}=J_9+J_{10}+J_{11}+J_{12},$$

$$J_5^{'}=J_{13}+J_{14}+J_{15},\ J_6^{'}=J_{16}+0{,}5p_{45}^2J_{17}+0{,}5J_{21},$$

$$J_7^{'}=J_{18}+0{,}5p_{45}^2J_{17}+0{,}5p_{45}^2J_{19}+0{,}5J_{24},$$

$$J_8^{'}=J_{20}+0{,}5p_{45}^2J_{19}+0{,}5J_{27},$$

$$J_9^{'}=0{,}5J_{21}+0{,}5J_{22},\ J_{10}^{'}=0{,}5J_{22}+J_{23},$$

$$J_{11}^{'}=0{,}5J_{24}+0{,}5J_{25},\ J_{12}^{'}=0{,}5J_{25}+J_{26},$$

$$J_{13}^{'}=0{,}5J_{27}+0{,}5J_{28},\ J_{14}^{'}=0{,}5J_{28}+J_{29}$$

In the kinematic diagram (Fig. 2) the rotating masses are transformed to the shields with moments of inertia $J_i^{'}$ (i = 1,2,…,14). The position angles $\varphi_i^{'}$ (i = 1,2,…,14) and coefficients of elasticity $c_k=M_k/\Delta\varphi_k^{'}$ (k = 1,2,…,13) were assigned to the shields, where M_k is an internal torque resulting in torsion of any elastic element by angle $\Delta\varphi_k$. Assuming addition of the moments of inertia of the halves the clutch to the moments of inertia of the shafts with the gears as well as considering serial connection of the shafts with coefficients of elasticity c_2 and c_3 the equivalent coefficient of elasticity $c_{2\text{-}3}$ is defined as follows:

$$(3)\qquad c_{2-3}=\frac{c_2c_3}{c_2+c_3}$$

For the chains of the drive set the coefficient of elasticity $c_k^l=F_k/\Delta l_k$ (k = 4,…,7) is defined as the ratio of the force F_k and the chain elongation Δl_k. The following external torques are in the drive system:

- the electromagnetic torque of driving motor M_s,
- the load torques at the i-th transporting roller occurring in the places of its settlement $M_{o1}^{(i)},\ M_{o2}^{(i)}$.

In the drive system presented as the kinematic diagram (Fig. 2) the fourteen elements with the determined moments of inertia are indicated. There are 12 degrees of freedom determined as independent coordinates of angles of shafts' rotation in the system. These coordinates are expressed as the column matrix (4):

$$(4)\qquad \mathbf{q}^{\mathrm{T}}=[\varphi_1^{'}\ \varphi_2^{'}\ \varphi_5^{'}\ \varphi_6^{'}\ \varphi_7^{'}\ \varphi_8^{'}\ \varphi_9^{'}\ \varphi_{10}^{'}\ \varphi_{11}^{'}\ \varphi_{12}^{'}\ \varphi_{13}^{'}\ \varphi_{14}^{'}]$$

In the drive system presented as the kinematic diagram (Fig. 2) the transformed moments of inertia given by the diagonal matrix (5) correspond with the considered independent coordinates given by dependency (4):

$$(5)\qquad \mathbf{D}=\mathrm{diagon}[J_1^*\ J_2^*\ J_5^*\ J_6^*\ J_7^*\ J_8^*\ J_9^*\ J_{10}^*\ J_{11}^*\ J_{12}^*\ J_{13}^*\ J_{14}^*],$$

where: $J_1^*=J_1^{'}$, $J_2^*=J_2^{'}+p_{12}^2J_3^{'}+p_{12}^2p_{23}^2J_4^{'}$, $J_k^*=J_k^{'}$, $k=5,...,14$.

Mathematical model

The mathematical model of the drive system (Fig. 2) is based on Lagrange's equation of a second type taking into consideration the Rayleigh's dissipation function [1], [2]. In general form the Lagrange's equation of a second type is expressed by the following dependency:

$$\text{(6)}\quad \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial}{\partial \dot{q}_k}T\right)-\frac{\partial}{\partial q_k}T+\frac{\partial}{\partial q_k}V+\frac{\partial}{\partial \dot{q}_k}\Phi=Q_k\ ,\quad k=1,\ldots,n$$

where: T is total kinetic energy of the system, V is total potential energy, Φ is total power of energy dissipation in the system (Rayleigh's dissipation function), q_k is the k-th independent generalized coordinate, Q_k is the non-potential generalized force affecting on a part of the system with k-th coordinate, n is a number of independent generalized coordinates of motion.

The total kinetic energy of the analyzed drive (Fig. 2) is given as follows:

$$\text{(7)}\quad T=0{,}5\left(J_1^*\frac{\mathrm{d}^2\varphi_1^{'}}{\mathrm{d}t^2}+J_2^*\frac{\mathrm{d}^2\varphi_2^{'}}{\mathrm{d}t^2}+\sum_{j=5}^{14}J_j^*\frac{\mathrm{d}^2\varphi_j^{'}}{\mathrm{d}t^2}\right)$$

In the chain drive set for the rollers the deformation potential energy of the stretched chain between driving chain gear and driven chain gear is collected. The deformation between these gears may by expressed as follows:

$$\text{(8)}\quad \Delta l_{i-(i-1)}=0{,}5\left(d_i\varphi_i-d_{i-1}\varphi_{i-1}\right)>0$$

The value of the potential energy collected in the stretched chain between the driving chain gear and driven chain gear is given by the dependency (9):

$$\text{(9)}\quad V_{i-(i-1)}=\begin{cases}0{,}5c_{i-(i-1)}^{l}\Delta l_{i-(i-1)}^{2}\,, & \text{dla }\ \Delta l_{i-(i-1)}>0\\ 0\ , & \text{dla }\ \Delta l_{i-(i-1)}\le 0\end{cases}$$

Differentiation of the potential energy given by (9) over generalized coordinate results in the dependency (10):

$$\frac{\partial V_{i-(i-1)}}{\partial\varphi_i}=\frac{\partial V_{i-(i-1)}}{\partial\Delta l_{i-(i-1)}}\,\frac{\partial\Delta l_{i-(i-1)}}{\partial\varphi_i}=$$

$$\text{(10)}\quad =\left\{\begin{matrix}0{,}25d_ic_{i-(i-1)}^{l}\left(d_i\varphi_i-d_{i-1}\varphi_{i-1}\right), & \text{for }\ \Delta l_{i-(i-1)}>0\\ 0\ , & \text{for }\ \Delta l_{i-(i-1)}\le 0\end{matrix}\right\}=$$

$$=0{,}25\cdot\mathbf{1}\left(\Delta l_{i-(i-1)}\right)d_ic_{i-(i-1)}^{l}\left(d_i\varphi_i-d_{i-1}\varphi_{i-1}\right)$$

where the unit step function: $\mathbf{1}\left(\Delta l_{i-(i-1)}\right)=\begin{cases}1, \text{ for } \Delta l_{i-(i-1)}>0\\ 0, \text{ for } \Delta l_{i-(i-1)}\le 0\end{cases}$

The torque which deforms (stretches) the chain in the chain drive set for the rollers transformed to the shaft of the driving chain gear may be expressed by dependency (11):

$$\text{(11)}\quad M_i=\begin{cases}0{,}5d_i\Big[c_{i-(i-1)}^{l}\Delta l_{i-(i-1)}+ & \\ +b_i^{l}\left(d_i\dot{\varphi}_i-d_{i-1}\dot{\varphi}_{i-1}\right)\Big], & \text{for }\ \Delta l_{i-(i-1)}>0\\ 0\ , & \text{for }\ \Delta l_{i-(i-1)}\le 0\end{cases}$$

where: b_i^l is the general damping coefficient of deformed chain.

Considering the value of potential energy collected in the deformed chain, the total potential energy of the analyzed drive system (Fig. 2) may be expressed by dependency (12):

$$\text{(12)}\quad \begin{aligned}V=&\,0{,}5\Big[c_1\left(\varphi_1^{'}-\varphi_2^{'}\right)^2+c_{2-3}\left(p_{12}p_{23}\varphi_2^{'}-\varphi_5^{'}\right)^2+\\&+V_{5-6}+V_{6-7}+V_{7-8}+V_{8-5}+0{,}5\Big[c_8\left(\varphi_6^{'}-\varphi_9^{'}\right)^2+\\&+c_9\left(\varphi_9^{'}-\varphi_{10}^{'}\right)^2+c_{10}\left(\varphi_7^{'}-\varphi_{11}^{'}\right)^2+c_{11}\left(\varphi_{11}^{'}-\varphi_{12}^{'}\right)^2\Big]+\\&+0{,}5\Big[c_{12}\left(\varphi_8^{'}-\varphi_{13}^{'}\right)^2+c_{13}\left(\varphi_{13}^{'}-\varphi_{14}^{'}\right)^2\Big]\end{aligned}$$

The Rayleigh's dissipation function of the analyzed drive system (Fig. 2) is given by the dependency (13):

$$\text{(13)}\quad \begin{aligned}\Phi=&\,0{,}5\Big[b_1\left(\dot{\varphi}_1^{'}-\dot{\varphi}_2^{'}\right)^2+b_{2-3}\left(p_{12}p_{23}\dot{\varphi}_2^{'}-\dot{\varphi}_5^{'}\right)^2\Big]+\\&+0{,}5\Big[0{,}25\cdot\mathbf{1}(\Delta l_{5-6})\,b_4^l\left(d_5\dot{\varphi}_5^{'}-d_6\dot{\varphi}_6^{'}\right)^2+\\&+0{,}25\cdot\mathbf{1}(\Delta l_{6-7})\,b_5^l\left(d_6\dot{\varphi}_6^{'}-d_7\dot{\varphi}_7^{'}\right)^2\Big]+\\&+0{,}5\Big[0{,}25\cdot\mathbf{1}(\Delta l_{7-8})\,b_6^l\left(d_7\dot{\varphi}_7^{'}-d_8\dot{\varphi}_8^{'}\right)^2+\\&+0{,}25\cdot\mathbf{1}(\Delta l_{8-5})\,b_7^l\left(d_8\dot{\varphi}_8^{'}-d_5\dot{\varphi}_5^{'}\right)^2\Big]+\\&+0{,}5\Big[b_8\left(\dot{\varphi}_6^{'}-\dot{\varphi}_9^{'}\right)^2+b_9\left(\dot{\varphi}_9^{'}-\dot{\varphi}_{10}^{'}\right)^2+b_{10}\left(\dot{\varphi}_7^{'}-\dot{\varphi}_{11}^{'}\right)^2+\\&+b_{11}\left(\dot{\varphi}_{11}^{'}-\dot{\varphi}_{12}^{'}\right)^2\Big]+0{,}5\Big[b_{12}\left(\dot{\varphi}_8^{'}-\dot{\varphi}_{13}^{'}\right)^2+b_{13}\left(\dot{\varphi}_{13}^{'}-\dot{\varphi}_{14}^{'}\right)^2\Big]\end{aligned}$$

where: $b_{2-3}=\dfrac{b_2b_3}{b_2+b_3}$ – the generalized damping coefficient.

Substitution in the equation (6) T, M_i and V equal (7), (11) and (12), respectively, as well as differentiation of the equation over all independent coordinates determined by the column matrix (4) and over generalized velocities corresponding with these coordinates result in the system of the 14 differential equations describing the drive system (Fig. 2).

$$\text{(14)}\quad J_1^*\ddot{\varphi}_1^{'}+b_1\left(\dot{\varphi}_1^{'}-\dot{\varphi}_2^{'}\right)+c_1\left(\varphi_1^{'}-\varphi_2^{'}\right)=M_s\ ,$$

$$\begin{aligned}&J_2^*\ddot{\varphi}_2^{'}-b_1\left(\dot{\varphi}_1^{'}-\dot{\varphi}_2^{'}\right)+p_{12}p_{23}b_{2-3}\left(p_{12}p_{23}\dot{\varphi}_2^{'}-\dot{\varphi}_5^{'}\right)+\\&-c_1\left(\varphi_1^{'}-\varphi_2^{'}\right)+p_{12}p_{23}c_{2-3}\left(p_{12}p_{23}\varphi_2^{'}-\varphi_5^{'}\right)=0,\end{aligned}$$

$$\begin{aligned}&J_5^*\ddot{\varphi}_5^{'}-b_{2-3}\left(p_{12}p_{23}\dot{\varphi}_2^{'}-\dot{\varphi}_5^{'}\right)+\\&+0{,}25\cdot\mathbf{1}(\Delta l_{5-6})\,d_5b_4^l\left(d_5\dot{\varphi}_5^{'}-d_6\dot{\varphi}_6^{'}\right)-\\&-0{,}25\cdot\mathbf{1}(\Delta l_{8-5})\,d_5b_7^l\left(d_8\dot{\varphi}_8^{'}-d_5\dot{\varphi}_5^{'}\right)-\\&-c_{2-3}\left(p_{12}p_{23}\varphi_2^{'}-\varphi_5^{'}\right)+\\&+0{,}25\cdot\mathbf{1}(\Delta l_{5-6})\,d_5c_4^l\left(d_5\varphi_5^{'}-d_6\varphi_6^{'}\right)-\\&-0{,}25\cdot\mathbf{1}(\Delta l_{8-5})\,d_5c_7^l\left(d_8\varphi_8^{'}-d_5\varphi_5^{'}\right)=0,\end{aligned}$$

$$J_6^*\ddot{\varphi}'_6 - 0{,}25\cdot\mathbf{1}(\Delta l_{5-6})\,d_6 b_4^l\left(d_5\dot{\varphi}'_5 - d_6\dot{\varphi}'_6\right) + 0{,}25\cdot\mathbf{1}(\Delta l_{6-7})\,d_6 b_5^l\left(d_6\dot{\varphi}'_6 - d_7\dot{\varphi}'_7\right) + b_8\left(\dot{\varphi}'_6 - \dot{\varphi}'_9\right) - 0{,}25\cdot\mathbf{1}(\Delta l_{5-6})\,d_6 c_4^l\left(d_5\varphi'_5 - d_6\varphi'_6\right) + 0{,}25\cdot\mathbf{1}(\Delta l_{6-7})\,d_6 c_5^l\left(d_6\varphi'_6 - d_7\varphi'_7\right) + c_8\left(\varphi'_6 - \varphi'_9\right) = 0,$$

$$J_7^*\ddot{\varphi}'_7 - 0{,}25\cdot\mathbf{1}(\Delta l_{6-7})\,d_7 b_5^l\left(d_6\dot{\varphi}'_6 - d_7\dot{\varphi}'_7\right) + 0{,}25\cdot\mathbf{1}(\Delta l_{7-8})\,d_7 b_6^l\left(d_7\dot{\varphi}'_7 - d_8\dot{\varphi}'_8\right) + b_{10}\left(\dot{\varphi}'_7 - \dot{\varphi}'_{11}\right) - 0{,}25\cdot\mathbf{1}(\Delta l_{6-7})\,d_7 c_5^l\left(d_6\varphi'_6 - d_7\varphi'_7\right) + 0{,}25\cdot\mathbf{1}(\Delta l_{7-8})\,d_7 c_6^l\left(d_7\varphi'_7 - d_8\varphi'_8\right) + c_{10}\left(\varphi'_7 - \varphi'_{11}\right) = 0,$$

$$J_8^*\ddot{\varphi}'_8 - 0{,}25\cdot\mathbf{1}(\Delta l_{7-8})\,d_8 b_6^l\left(d_7\dot{\varphi}'_7 - d_8\dot{\varphi}'_8\right) + 0{,}25\cdot\mathbf{1}(\Delta l_{8-5})\,d_8 b_7^l\left(d_8\dot{\varphi}'_8 - d_5\dot{\varphi}'_5\right) + b_{12}\left(\dot{\varphi}'_8 - \dot{\varphi}'_{13}\right) - 0{,}25\cdot\mathbf{1}(\Delta l_{7-8})\,d_8 c_6^l\left(d_7\varphi'_7 - d_8\varphi'_8\right) + 0{,}25\cdot(\Delta l_{8-5})\,d_8 c_7^l\left(d_8\varphi'_8 - d_5\varphi'_5\right) + c_{12}\left(\varphi'_8 - \varphi'_{13}\right) = 0,$$

$$J_9^*\ddot{\varphi}'_9 - b_8\left(\dot{\varphi}'_6 - \dot{\varphi}'_9\right) + b_9\left(\dot{\varphi}'_9 - \dot{\varphi}'_{10}\right) - c_8\left(\varphi'_6 - \varphi'_9\right) + c_9\left(\varphi'_9 - \varphi'_{10}\right) = M_{o1}^{(1)},$$

$$J_{10}^*\ddot{\varphi}'_{10} - b_9\left(\dot{\varphi}'_9 - \dot{\varphi}'_{10}\right) - c_9\left(\varphi'_9 - \varphi'_{10}\right) = M_{o2}^{(1)},$$

$$J_{11}^*\ddot{\varphi}'_{11} - b_{10}\left(\dot{\varphi}'_7 - \dot{\varphi}'_{11}\right) + b_{11}\left(\dot{\varphi}'_{11} - \dot{\varphi}'_{12}\right) - c_{10}\left(\varphi'_7 - \varphi'_{11}\right) + c_{11}\left(\varphi'_{11} - \varphi'_{12}\right) = M_{o1}^{(2)},$$

$$J_{12}^*\ddot{\varphi}'_{12} - b_{11}\left(\dot{\varphi}'_{11} - \dot{\varphi}'_{12}\right) - c_{11}\left(\varphi'_{11} - \varphi'_{12}\right) = M_{o2}^{(2)},$$

$$J_{13}^*\ddot{\varphi}'_{13} - b_{12}\left(\dot{\varphi}'_8 - \dot{\varphi}'_{13}\right) + b_{13}\left(\dot{\varphi}'_{13} - \dot{\varphi}'_{14}\right) - c_{12}\left(\varphi'_8 - \varphi'_{13}\right) + c_{13}\left(\varphi'_{13} - \varphi'_{14}\right) = M_{o1}^{(3)},$$

$$J_{14}^*\ddot{\varphi}'_{14} - b_{13}\left(\dot{\varphi}'_{13} - \dot{\varphi}'_{14}\right) - c_{13}\left(\varphi'_{13} - \varphi'_{14}\right) = M_{o2}^{(3)}.$$

The final linear equations given by the dependencies (15) were obtained as a result of the successive transformations of the equations given by (14).

$$\text{(15)}\qquad \ddot{\varphi}'_1 + \frac{b_1}{J_1^*}\dot{\varphi}'_1 - \frac{b_1}{J_1^*}\dot{\varphi}'_2 + \frac{c_1}{J_1^*}\varphi'_1 - \frac{c_1}{J_1^*}\varphi'_2 = \frac{M_s}{J_1^*}.$$

$$\ddot{\varphi}'_2 - \frac{b_1}{J_2^*}\dot{\varphi}'_1 + \frac{b_1 + p_{12}^2 p_{23}^2 b_{2-3}}{J_2^*}\dot{\varphi}'_2 - \frac{p_{12}p_{23}b_{2-3}}{J_2^*}\dot{\varphi}'_5 - \frac{c_1}{J_2^*}\varphi'_1 + \frac{c_1 + p_{12}^2 p_{23}^2 c_{2-3}}{J_2^*}\varphi'_2 - \frac{p_{12}p_{23}c_{2-3}}{J_2^*}\varphi'_5 = 0,$$

$$\ddot{\varphi}'_5 - \frac{p_{12}p_{23}b_{2-3}}{J_5^*}\dot{\varphi}'_2 - \frac{p_{12}p_{23}c_{2-3}}{J_5^*}\varphi'_2 + \frac{b_{2-3} + 0{,}25d_5^2\left[\mathbf{1}(\Delta l_{5-6})\,b_4^l + \mathbf{1}(\Delta l_{8-5})\,b_7^l\right]}{J_5^*}\dot{\varphi}'_5 - \frac{0{,}25\,d_5 d_6\,\mathbf{1}(\Delta l_{5-6})\,b_4^l}{J_5^*}\dot{\varphi}'_6 - \frac{0{,}25\,d_5 d_8\,\mathbf{1}(\Delta l_{8-5})\,b_7^l}{J_5^*}\dot{\varphi}'_8 + \frac{c_{2-3} + 0{,}25d_5^2\left[\mathbf{1}(\Delta l_{5-6})\,c_4^l + \mathbf{1}(\Delta l_{8-5})\,c_7^l\right]}{J_5^*}\varphi'_5 - \frac{0{,}25\,d_5 d_6\,\mathbf{1}(\Delta l_{5-6})\,c_4^l}{J_5^*}\varphi'_6 - \frac{0{,}25\,d_5 d_8\,\mathbf{1}(\Delta l_{8-5})\,c_7^l}{J_5^*}\varphi'_8 = 0,$$

$$\ddot{\varphi}'_6 - \frac{0{,}25\,d_5 d_6\,\mathbf{1}(\Delta l_{5-6})\,b_4^l}{J_6^*}\dot{\varphi}'_5 + \frac{R_8 + (0{,}5d_6)^2\left[\mathbf{1}(\Delta l_{5-6})\,b_4^l + \mathbf{1}(\Delta l_{6-7})\,b_5^l\right]}{J_6^*}\dot{\varphi}'_6 - \frac{0{,}25\,d_6 d_7\,\mathbf{1}(\Delta l_{6-7})\,b_5^l}{J_6^*}\dot{\varphi}'_7 - \frac{b_8}{J_6^*}\dot{\varphi}'_9 - \frac{0{,}25\,d_5 d_6\,\mathbf{1}(\Delta l_{5-6})\,c_4^l}{J_6^*}\varphi'_5 + \frac{c_8 + (0{,}5d_6)^2\left[\mathbf{1}(\Delta l_{5-6})\,c_4^l + \mathbf{1}(\Delta l_{6-7})\,c_5^l\right]}{J_6^*}\varphi'_6 - \frac{0{,}25\,d_6 d_7\,\mathbf{1}(\Delta l_{6-7})\,c_5^l}{J_6^*}\varphi'_7 - \frac{c_8}{J_6^*}\varphi'_9 = 0,$$

$$\ddot{\varphi}'_7 - \frac{0{,}25\,d_6 d_7\,\mathbf{1}(\Delta l_{6-7})\,b_5^l}{J_7^*}\dot{\varphi}'_6 + \frac{b_{10} + 0{,}25d_7^2\left[\mathbf{1}(\Delta l_{5-6})\,b_5^l + \mathbf{1}(\Delta l_{7-8})\,b_6^l\right]}{J_7^*}\dot{\varphi}'_7 - \frac{0{,}25\,d_7 d_8\,\mathbf{1}(\Delta l_{7-8})\,b_6^l}{J_7^*}\dot{\varphi}'_8 - \frac{b_{10}}{J_7^*}\dot{\varphi}'_{11} - \frac{0{,}25\,d_6 d_7\,\mathbf{1}(\Delta l_{6-7})\,c_5^l}{J_7^*}\varphi'_6 + \frac{c_{10} + 0{,}25d_7^2\left[\mathbf{1}(\Delta l_{5-6})\,c_5^l + \mathbf{1}(\Delta l_{7-8})\,c_6^l\right]}{J_7^*}\varphi'_7 - \frac{0{,}25\,d_7 d_8\,\mathbf{1}(\Delta l_{7-8})\,c_6^l}{J_7^*}\varphi'_8 - \frac{c_{10}}{J_7^*}\varphi'_{11} = 0,$$

$$\ddot{\varphi}'_8 - \frac{0{,}25\,d_5 d_8\,\mathbf{1}(\Delta l_{8-5})\,b_7^l}{J_8^*}\dot{\varphi}'_5 - \frac{0{,}25\,d_7 d_8\,\mathbf{1}(\Delta l_{7-8})\,b_6^l}{J_8^*}\dot{\varphi}'_7 + \frac{b_{12} + 0{,}25d_8^2\left[\mathbf{1}(\Delta l_{7-8})\,b_6^l + \mathbf{1}(\Delta l_{8-5})\,b_7^l\right]}{J_8^*}\dot{\varphi}'_8 - \frac{b_{12}}{J_8^*}\dot{\varphi}'_{13} - \frac{0{,}25\,d_5 d_8\,\mathbf{1}(\Delta l_{8-5})\,c_7^l}{J_8^*}\varphi'_5 - \frac{0{,}25\,d_7 d_8\,\mathbf{1}(\Delta l_{7-8})\,c_6^l}{J_8^*}\varphi'_7 + \frac{c_{12} + 0{,}25d_8^2\left[\mathbf{1}(\Delta l_{7-8})\,c_6^l + \mathbf{1}(\Delta l_{8-5})\,c_7^l\right]}{J_8^*}\varphi'_8 - \frac{c_{13}}{J_8^*}\varphi'_{13} = 0,$$

$$\ddot{\varphi}'_9 - \frac{b_8}{J_9^*}\dot{\varphi}'_6 + \frac{b_8 + b_9}{J_9^*}\dot{\varphi}'_9 - \frac{b_9}{J_9^*}\dot{\varphi}'_{10} - \frac{c_8}{J_9^*}\varphi'_6 + \frac{c_8 + c_9}{J_9^*}\varphi'_9 - \frac{c_9}{J_9^*}\varphi'_{10} = \frac{M_{o1}^{(1)}}{J_9^*},$$

$$\ddot{\varphi}_{10}^{'} - \frac{b_9}{J_{10}^*}\dot{\varphi}_9^{'} + \frac{b_9}{J_{10}^*}\dot{\varphi}_{10}^{'} - \frac{c_9}{J_{10}^*}\varphi_9^{'} + \frac{c_9}{J_{10}^*}\varphi_{10}^{'} = \frac{M_{o2}^{(1)}}{J_{10}^*},$$

$$\ddot{\varphi}_{11}^{'} - \frac{b_{10}}{J_{11}^*}\dot{\varphi}_7^{'} + \frac{b_{10}+b_{11}}{J_{11}^*}\dot{\varphi}_{11}^{'} - \frac{b_{11}}{J_{11}^*}\dot{\varphi}_{12}^{'} -$$
$$- \frac{c_{10}}{J_{11}^*}\varphi_7^{'} + \frac{c_{10}+c_{11}}{J_{11}^*}\varphi_{11}^{'} - \frac{c_{11}}{J_{11}^*}\varphi_{12}^{'} = \frac{M_{o1}^{(2)}}{J_{11}^*},$$

$$\ddot{\varphi}_{12}^{'} - \frac{b_{11}}{J_{12}^*}\dot{\varphi}_{11}^{'} + \frac{b_{11}}{J_{12}^*}\dot{\varphi}_{12}^{'} - \frac{c_{11}}{J_{12}^*}\varphi_{11}^{'} + \frac{c_{11}}{J_{12}^*}\varphi_{12}^{'} = \frac{M_{o2}^{(2)}}{J_{12}^*},$$

$$\ddot{\varphi}_{13}^{'} - \frac{b_{12}}{J_{13}^*}\dot{\varphi}_8^{'} + \frac{b_{12}+b_{13}}{J_{13}^*}\dot{\varphi}_{13}^{'} - \frac{b_{13}}{J_{13}^*}\dot{\varphi}_{14}^{'} -$$
$$- \frac{c_{12}}{J_{13}^*}\varphi_8^{'} + \frac{c_{12}+c_{13}}{J_{13}^*}\varphi_{13}^{'} - \frac{c_{13}}{J_{13}^*}\varphi_{14}^{'} = \frac{M_{o1}^{(3)}}{J_{13}^*},$$

$$\ddot{\varphi}_{14}^{'} - \frac{b_{13}}{J_{14}^*}\dot{\varphi}_{13}^{'} + \frac{b_{13}}{J_{14}^*}\dot{\varphi}_{14}^{'} - \frac{c_{13}}{J_{14}^*}\varphi_{13}^{'} + \frac{c_{13}}{J_{14}^*}\varphi_{14}^{'} = \frac{M_{o2}^{(3)}}{J_{14}^*}.$$

where: Δl_{5-6}, Δl_{6-7}, Δl_{7-8} are length increments of the chain in the drive system among the chain gears with the moments of inertia $J_5^{'}$, $J_6^{'}$, $J_7^{'}$, $J_8^{'}$.

The system of equations expressed by dependency (15) allows setting the true loads on the respective rollers working in the transport line. In many studies the load of drive systems is assumed to be a technological torque determined as a ratio of the rated torque of the motor in the drive system.

The formulated system of equations allows for consideration of the drive system consisting of the assumed chain gears at determined number of rollers for various constructional solutions and dimensions of driving chain applied in the chain drive set. Various solutions with regard to: the distance between rollers as well as the shape of the chain route may be assumed for this drive set dependently on constructional requirements.

Additionally, the system of equations expressed by dependencies (15) allows for consideration of the meaningful technological case concerning irregular placement of the transported elements against the supports of the transporting rollers. Irregular placement of the transported element may be considered in the system of equations (15) using the symmetry factor determined by dependency (16):

$$w_p = \frac{2a_x}{L-a} \tag{16}$$

where: a is the width of transported element measured along the roller axis, a_x is the distance between the edge of transported element and the roller ending, L is the roller length.

The system of equations (15) takes into account the dependencies dealing with free damping of elastic elements of the analyzed drive system and allows for analysis of this system for different materials applied in true drive systems. It expands significantly the possibility of practical application of the mathematical model in analysis of the drive systems at the stage of their designing or setting up the prototypical systems. It was not considered in previous mathematical models [5], [6]. The significant practical meaning has the proper selection of the driving chain in the systems with precise positioning of the transported element which is the most damage element of the system and it is exposed to the significant deformation during the operation of the drive system in frequent dynamical states.

Conclusions

The analysis of the drive system presented as computable kinematic diagram (Fig. 2) results in the system of equations (15). This system of equations allows for the wide and optional analysis of the dynamical states of the drive system. The obtained mathematical model allows analyzing, inter alia, the following cases:
- the computable analysis of the system in dynamical states considering or not the dissipation energy,
- the computable analysis of the drive system for various configuration of the chain gears contained in the chain drive set for the rollers,
- the computable analysis of dynamical states of the drive system for different sorts and lengths of the chain working in the chain drive set for the rollers.

The analysis presented in the paper and developed mathematical model does not consider an influence of the placement and elasticity of the transported element on the operation of the transporting rollers in analyzed section of operation of the transporting lines.

REFERENCES

[1] Puchała A., *Dynamics of the machines and electro-dynamic systems* (in Polish), Warszawa: PWN, 1977

[2] Szklarski L., Jaracz K., *Selected problems of dynamics of electric drives* (in Polish), Warszawa: PWN, 1986

[3] Rusek A., Mathematical model of the gear-motor based drive system for the roller-table transporting line considering the phenomenon of clearances (in Polish), *Electrotechnical Review*, 5s (2002), 255-259

[4] Rusek A., Flasza J., The influence of the true load torque on the variability of electromagnetic torque in roller-table drive system (in Polish), *Electrotechnical Review*, 11 (2003), 844-846

[5] Flasza J., *The dynamical states of the roller-table drive system with the specially designed gear-motors considering the true loads* (in Polish), doctoral thesis, Częstochowa: Technical University of Częstochowa, 2002

[6] Popenda A., Rusek A., Mathematical model and selected transient states of the main drive for polymerization reactor considering operating parameters of the mixer chamber (in Polish), *Electrotechnical Review* 5 (2008), 84-87

***Authors**: prof. dr hab. Andrzej Rusek,*
E -mail: rusek@el.pcz.czest.pl;
dr Andrzej Popenda, E-mail: popenda@el.pcz.czest.pl
Politechnika Częstochowska, Instytut Elektrotechniki Przemysłowej, Al. Armii Krajowej 17, 42-200 Częstochowa, Poland