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New concept for control of linear and nonlinear plants

Abstract. *The idea of control of time-variable time-scale of plant output signal and way of its application to control of linear and nonlinear plants have been presented. This new control idea can be implemented on the basis of modified MFC control structure and allows to obtain high quality of control process. The presented results can be also used if other technical problems connected with necessity of time-scaling of signals in physical systems have to be solved.*

Streszczenie. *Przedstawiono koncepcję skalowania czasowego sygnału wyjściowego obiektu. Pokazano, że w oparciu o zmodyfikowaną strukturę MFC można ją wykorzystać do sterowania obiektami rzeczywistymi, liniowymi i nieliniowymi, uzyskując względnie prostą metodą wysoką jakość procesu sterowania. Otrzymane rezultaty mogą też służyć do rozwiązania innych problemów technicznych związanych z koniecznością skalowania czasowego sygnałów w układach fizycznych. (**Nowa koncepcja sterowania obiektami liniowymi i nieliniowymi**)*

Keywords: control algorithms for linear and nonlinear SISO plants, time-variable time-scale, MFC control structure, time-scaling of signals
Słowa kluczowe: algorytmy sterowania liniowymi i nieliniowymi obiektami SISO, zmienna skala czasu, struktura MFC, skalowanie czasowe sygnałów

Introduction

Let us assume that reference response of continuous-time SISO system to input signal $U(t)$ is $Y(t)$. Using algorithms given in [1,2] one can determine plant input signal $U_A(t)$ scaling system response to the form $Y(At)$, where A - constant number called "time scale coefficient". The synthesis of excitations scaling responses of MIMO plant is presented in [3]. The concept of time-scaling of responses of discrete-time SISO and MIMO systems is presented in [4]. Due to above possibilities one can control the SISO and MIMO plants in accordance with rules of time-scaling. Thus, the above-mentioned algorithms allow to speed-up or slow-down the system responses conserving their reference forms. Let us note, that properties of system after scaling, both in time and frequency domains, can be immediately defined on the basis of counterparts representing reference conditions. Such properties of control system after scaling as its stability, overshoot, static error are identical to those for reference conditions while transient state duration time is shortened A times (A>1) or lengthened A times (A<1). The frequency representations of signals and systems after scaling result from known property: if $\underline{y}(j\omega)$ is representation of $y(t)$, then $A^{-1}\underline{y}(jA^{-1}\omega)$ represents signal $y(At)$. It means, that form of frequency characteristic after scaling as well as other parameters defined in frequency domain (stability margin, etc.) are conected with reference counterparts by transparent relations.

The current paper deals with scaling of system output signal in case of time-variable time–scale coefficient $A(t)$. It is shown, that use of variable $A(t)$ yields quite new, very interesting concept of plant control and its implementation can be realized by modification of MFC (Model Following Control) structure. The further considerations do not refer to general theory of dynamic system defined on "time-scales" [5,6], because application of that theory to continuous-time systems recovers known, "old" results dedicated to those systems.

Forming of system response by means of time-variable time–scale coefficient

Let the real plant be represented by state equations:

$$
\begin{aligned}
\dot{X}_1 &= F_1(X_1, X_2, \ldots, X_n, U) \\
&\ldots\ldots\ldots \\
\dot{X}_{n-1} &= F_{n-1}(X_1, X_2, \ldots, X_n, U) \\
\dot{X}_n &= F_n(X_1, X_2, \ldots, X_n, U) \\
Y &= F(X_1, X_2, \ldots, X_n)
\end{aligned}
\tag{1}
$$

where: $X_1,\ldots,X_n$ – components of state vector $\boldsymbol{X}$, $F, F_1, \ldots, F_n$ - static nonlinear functions, U-input signal, Y-output signal. Let as introduce the "associated" form of model (1):

$$
\begin{aligned}
\dot{x}_1 &= A(t) f_1(x_1, x_2, \ldots, x_n, u) \\
&\ldots\ldots\ldots\ldots \\
\dot{x}_{n-1} &= A(t) f_{n-1}(x_1, x_2, \ldots, x_n, u) \\
\dot{x}_n &= A(t) f_n(x_1, x_2, \ldots, x_n, u) \\
y &= f(x_1, x_2, \ldots, x_n)
\end{aligned}
\tag{2}
$$

where: $x_1,\ldots,x_n$ – components of state vector $\boldsymbol{x}$, $f_1,\ldots,f_n, f$ - static nonlinear functions, u - model input, y - model output, $A(t)$ - time–variable time-scale coefficient. If $A(t)=1$, $F(..)=f(..)$, $F_l(..)=f_l(..)$ for $l=1,..,n$, then model (2) is identical to plant representation (1). The exemplary scheme of system of type (2), for $f_l(..)=x_{l+1}$, $l=1,..,n-1$, $f_n=(\phi(x_1,..,x_n)+u)$, is shown in Fig. 1. If plant response to input signal $U(t)$ is $Y(t)$ then, using model (2) and putting to it $f(..)=F(..)$, $f_l(..)=F_l(..)$, where $l=1,..,n$, one can generate signal y equal to scaled plant output $Y(T)$, as the response of model (2) to input $u=U(T)$. The "new" time T is defined by relation $dT=(A(t))dt$, hence $T=\int_0^t (A(t))dt$.

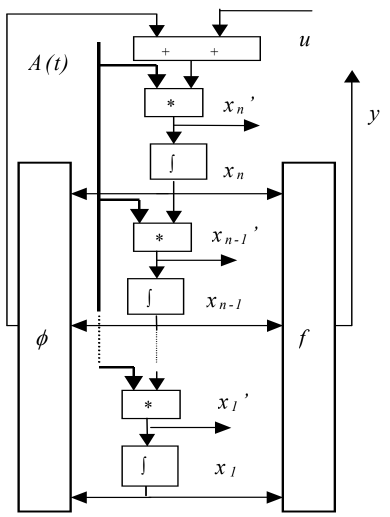

Fig. 1. The exemplary scheme for scaling of output y by means of time-variable scale coefficient $A(t)$

To indicate the signal $A(t)$ as "scaling subject" let us introduce notation $y(A(t))$ for $y(t)$ scaled with $A(t)$, although $T \neq A(t)$. The output $y(t)$ of model (2) can be formed by pair of independent signals $u(t)$ and $A(t)$.

Example 1. The responses of system shown in Fig. 1, for $n=3$, $f(.)= -(x_1+|x_1|+x_2+x_3)^3$, $\phi(.)=|x_1+x_2+x_3|$ and $u(t)=2*1(t)$, are presented in Fig. 2. The "reference" response has been obtained for $A(t)=const.=1$. The effect of scaling has been illustrated for $A(t)=2*1(t)-1(t-2)-1(t-5)-1(t-7)$. For $t<2$ we observe faster rate of generation of scaled response in comparison with rate for "reference" response. For $2<t<5$ the both rates of generation of signals under comparison are identical, for $5<t<7$ state variables and scaled response do not change their values ($A(t)=0$) and for $t>7$ the scaled response is generated backwards with rate identical to this one for "reference" response (because $A(t)= -1$).

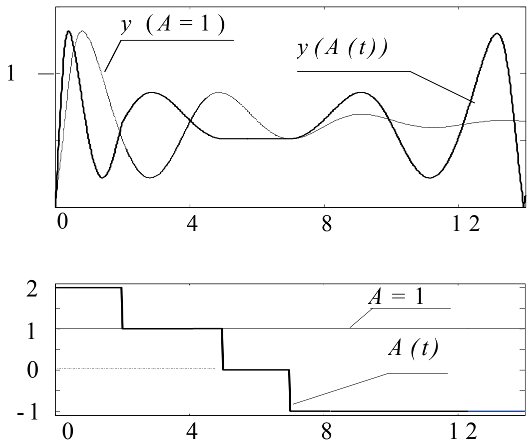

Fig. 2. The step responses of system in Fig. 1 for $A=1$ and $A(t)$ as shown beneath

The *Example* 1 deals with "externally" generated signal $A(t)$, which modulates system output. This seems to be interesting possibility, if control purposes are taken into account. However, the other extremely interesting possibility appears. Let us note, that for time ranges where $A(t)=0$, the state variable structure of the form (2) is decoupled (see also Fig.1) and values of their state variables $x_1,...,x_n$ are conserved. Thus, the value of output y is conserved too (see Fig. 2). This interesting property of model (2) yields immediately the following concept of model output control: taking into account the desired form of output signal, i.e. set of available values of model output, relations between consecutive values of output, etc., one can chose the form of signal $u(t)$. The simplest choice $u(t)=1(t)$ makes, that the model step response will be "modulated" by $A(t)$. Now the time-scale $A(t)$ can be treated as main signal controlling the model output, i.e. $A(t)$ should be dependant on error of control process $e(t)= y_o(t)-y(t)$, where $y_o(t)$ - reference signal. The control algorithm should base on fundamental control rule: if error signal in control process attains value $e(t)=0$, then should be $A(t)=0$ too. Thus, even easiest control algorithm of type "P", i.e. $\pm A(t)= K\, e(t)$, where K – constant gain, can guarantee the perfect quality of control, providing that sign of $A(t)$ is properly chosen. In order to influence the control process transient state one can propose the other control algorithms. Those algorithms can be represented by general formula $A(t)=q(e,y_o,u)$, where $q(0,y_o,u)=0$, $q(.)$ - static nonlinear function. Theoretically, we do not need to modify control algorithm taking into account stability of system (2). Nevertheless, the solutions of unstable differential equations can occur very sensitive to inaccuracies and for those "unfortunate" cases the proposed simple control algorithm, realized even in "computer environment" can be insufficient.

Dealing with the real, physical plant one can not manipulate in its "interior" by equipping it with elements realizing multiplication of signals or other necessary operations on signals. Furthermore, the structure of real plant is usually completely different in comparison with the structure of its model representing input-output relation. Additionally, the control of SISO plant has to be carried out throughout the single, accessible input U. Hence the following question arises: can we modify and adjust the algorithm for control of output of model (2) to the form enabling control of output Y of plant (1)?

Control of real plants

Let us assume that form of (1) allows to express the input-output relation for plant (1) by differential equation:

$$(3)\quad \frac{d^nY}{dt^n} + g(Y,\frac{dY}{dt},..,\frac{d^{n-1}Y}{dt^{n-1}}) = h(U,\frac{dU}{dt},...,\frac{d^{k-1}U}{dt^{k-1}})$$

where $g(.)$, $h(.)$ - static nonlinear functions, $k \leq n$. Let us assume additionally that input-output relation for model (2) can be brought to the form:

$$(4)\quad \frac{d^ny}{dt^n} + g(y,\frac{dy}{dt},...,\frac{d^{n-1}y}{dt^{n-1}}) = g(y,\frac{dy}{dt},..,\frac{d^{n-1}y}{dt^{n-1}}) + p(...)$$

where $p(..)$ is static, nonlinear function of variables:

$$y,\frac{dy}{dt},...,\frac{d^{n-1}y}{dt^{n-1}},x_1,..,x_n,\frac{dx_1}{dt},..,\frac{dx_n}{dt},A,\frac{dA}{dt},...,\frac{d^{n-1}A}{dt^{n-1}},$$
$$u,\frac{du}{dt},...,\frac{d^{k-1}u}{dt^{k-1}}$$

Assuming, that arguments of $g(.)$ and $p(.)$ in (4) are directly or indirectly accessible one can create the differential equation of the form:

(5)
$$h(U,\frac{dU}{dt},..,\frac{d^{k-1}U}{dt^{k-1}}) = g(y,\frac{dy}{dt},..,\frac{d^{n-1}y}{dt^{n-1}}) + p(..)$$

The real time solution $U(t)$ of equation (5) transformed to physical signal can be treated as input signal of plant described by (3). The input signal $U(t)$ obtained from (5) makes, that plant output $Y(t)$ is identical to model output $y(t)$. This conclusion can be easily drawn due to analysis of similarity of forms of equations (3) and (4). Of course, the identical forms of y and Y can be obtained, if all necessary operations on signals defined by (5) are realizable with satisfying accuracies. The components of vectors $\mathbf{x}$, $\mathbf{x'}$ are accessible in structure representing model state variables (2) – see the exemplary scheme in Fig. 1. The consecutive derivatives of y can be obtained on the basis of operations on components of $\mathbf{x}$ and $\mathbf{x'}$ defined by results of differentiation of y. For example:

$$\frac{dy}{dt} = \frac{\partial f}{\partial x_1}\frac{dx_1}{dt} + ... + \frac{\partial f}{\partial x_n}\frac{dx_n}{dt}$$

$$\frac{d^2y}{dt^2} = \frac{\partial^2 f}{\partial x_1^2}(\frac{dx_1}{dt})^2 + \frac{\partial f}{\partial x_1}\frac{d^2x_1}{dt^2} + ... + \frac{\partial^2 f}{\partial x_n^2}(\frac{dx_n}{dt})^2 + \frac{\partial f}{\partial x_n}\frac{d^2x_n}{dt^2}$$

where, for $i=1,...,n$ is

$$\frac{d^2x_i}{dt^2} = (A(t)f_i(.))' = \frac{dA}{dt}f_i(x_1,..,x_n,u) +$$
$$+ A(t)(\frac{\partial f_i}{\partial x_1}\frac{dx_1}{dt} + ... + \frac{\partial f_i}{\partial x_n}\frac{dx_n}{dt} + \frac{\partial f_i}{\partial u}\frac{du}{dt})$$

and so on for derivatives of higher orders.

Example 2: Let plant (1) be represented by transfer function:

$$G(s) = \frac{Y(s)}{U(s)} = \frac{s+1}{s^3 + 3s^2 + 2s + 1} \quad (6)$$

Hence $n=3$, $F(.)=X_1+X_2$, $F_1(.)=X_2$, $F_2(.)=X_3$, $F_3(.)=-X_1-2X_2-3X_3+U$. Let us define the "associated" model of (6) by relations: $n=3$, $f(.)=x_1+x_2$, $f_1(.)=A(t)x_2$, $f_2(.)=A(t)x_3$, $f_3(.)=-A(t)(x_1+2x_2+3x_3))+u$ (see (2)). Putting those expressions to (2) we can observe, that for $A(t)=1$ the mathematical representations of plant (1) and its model (2) are identical. This makes, that reference output signals of plant and assumed model are identical too. If the above assumptions do not hold, then plant output $Y(t)$ follows scaled reference response of assumed model, though its own reference response is different one. The plant (6) is described by differential equation of type (3), where:

$$g(.) = Y + 2\frac{dY}{dt} + 3\frac{d^2Y}{dt^2}; \quad h(.) = U + \frac{dU}{dt} \quad (7)$$

To obtain $p(.)$ we have to calculate derivatives of y:

$$y = x_1 + x_2, \quad \frac{dy}{dt} = A(x_2 + x_3), \quad (8)$$

$$\frac{d^2y}{dt^2} = \frac{dA}{dt}(x_2 + x_3) + A(\frac{dx_2}{dt} + \frac{dx_3}{dt}),$$

$$\frac{d^3y}{dt^3} = r(.) = -A^2(\frac{dx_1}{dt} + 2\frac{dx_2}{dt} + 2\frac{dx_3}{dt} - \frac{du}{dt}) -$$

$$- A\frac{dA}{dt}(x_1 + 2x_2 + 2x_3 - u) + 2\frac{dA}{dt}(\frac{dx_2}{dt} + \frac{dx_3}{dt}) +$$

$$+ \frac{d^2A}{dt^2}(x_2 + x_3)$$

The equation (5) obtains the form:

$$\frac{dU}{dt} + U = y + 2\frac{dy}{dt} + 3\frac{d^2y}{dt^2} + r(.) \quad (9)$$

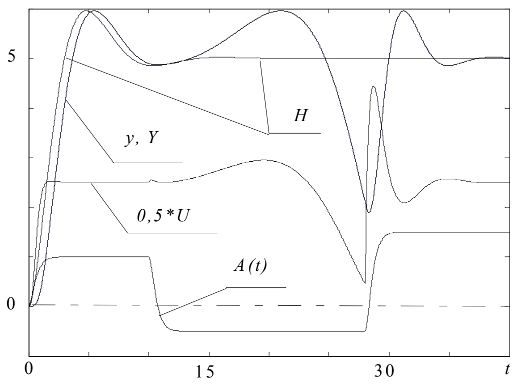

Fig.3. The time-scaling of response of system (6) to signal $u=5(1-exp(-10t))$ which is differentiable approximation of $u=5*1(t)$: H – reference response for $A=1$, $A(t)$ – time scale, U - input of plant (6), y – output of plant model shown in Fig.1, Y – output of plant (6). Signals y and Y are almost identical

The obvious interpretation of result (9) leads us to conclusion, that plant input signal $U(t)$ can be generated as response of first order inertia system (with gain and time constant equal to 1) to input signal composed in accordance with expression representing the right member in equation (9). We can observe, that state variables of model, theirs derivatives (i.e. input signals of integrators), input u, output y, time-scale $A(t)$ are directly accessible in the "associated" state variable model, that shown in Fig.1. Thus, we can use them in operations necessary for forming of inertia system excitation (right member of equation (9)). The way of organizing of structures generating signals $u(t)$ and $A(t)$ should guarantee access to derivatives $u'(t)$, $A'(t)$, $A''(t)$ – see (8). The referring simulation results are shown in Fig. 3. The reference response denoted by H, which is identical for plant and "associated" model, determines the range of available values of output Y.

The *Example 2* confirms possibility of forming of plant input U which makes, that plant output Y is identical to associated model output y, although this perfect result can be obtained, if we can overcome some limitations connected with existence and accessibility for derivatives of u and A. Considering the effect of decoupling of state variable scheme and the associated concept of control by use of time-variable time-scale $A(t)$ depending on control error e (see previous section) one can propose the modification of classic MFC structure to the form in Fig. 4, if real plant output Y, which follows model output y, has to be controlled. The model output y is controlled by static operation $q(.)$ which decouples the model structure for $e(t)=0$, i.e. for $y_o(t) = y(t)$, like it has been described in previous section (see also *Example 1*). The real-time solver of equation (5) generates plant input U on the basis of model state variables, their derivatives, model reference input u and reference signal y_o. The tasks of controller in auxiliary control loop are common for all MFC structures: it corrects inaccuracies in calculation of U caused by inaccurate identification of plant (1) and damps disturbances influencing the plant output Y.

Example 3. The structure in Fig. 4 has been simulated for plant (6) and the simplest control algorithm $A(t)=e(t)$. The signals $u(t)$ and $y_o(t)$ have been generated outside of system shown in Fig.2 in structures with direct access to derivatives of those signals. The derivatives of $A(t)= y_o(t)-y(t)$ have been determined indirectly from accessible derivatives of $y_o(t)$ and $y(t)$. Derivatives of $y(t)$ were composed accordingly to (8). The results of simulation are shown in Fig. 5. For identical parameters of plant and associated model (like in *Example 2*) the curve U overlaps U_I and y overlaps Y – controller in auxiliary loop does not influence the output Y. If values of $y_o(t)$ are smaller than maximum of plant and model reference response H determining the range of available values of y, then errors of follow-up action are small, provided that $A(t)$ for current $e(t)$ is sufficiently big ($A(t)$ controls the rate of generating of consecutive values belonging to H). We can observe lack of overshoots (result of effect of decoupling for $e(t)=0$) which is very advantageous feature of algorithm. The results in Fig. 6 proves that proposed control idea conserves all advantageous properties of MFC structure in case of calculation of signal U on the basis of inaccurate model of plant.

Conclusions

1. The responses of linear and nonlinear plants can be scaled by means of time-variable time-scale $A(t)$, if plant input U can be formed in accordance with (5).

The crucial components necessary for generating of U can be obtained directly or indirectly on the basis of real-time signals in scaled model of plant state variables (2). Due to the above possibility one can control plant output Y using modified MFC control structure (Fig.4), where model, model output and simultaneously plant output are scaled with $A(t)$ depending on error e (if $e(t)=0$ then $A(t)=0$). The high quality of control process can often be obtained even for simplest control algorithms, like $A(t)=q(.)=K\ e(t)$ – see <u>*Example 3*</u>.

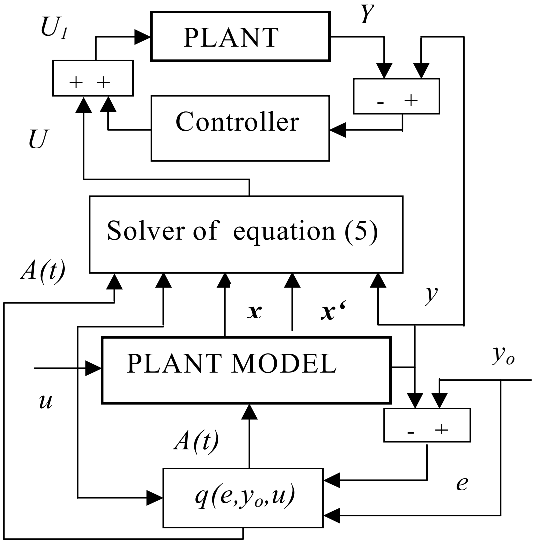

Fig. 4. Idea of control: if $e=0$ then $A(t)=0$ and $y=y_o=Y$. The input U is formed with signals accessible in state variable scheme representing (2) – see exemplary case shown in Fig.1

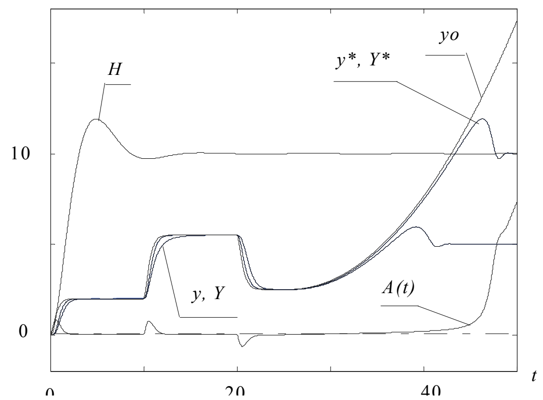

Fig. 5. The control of output of plant (6) according to scheme in Fig.4 for $A(t)=e(t)$: y_o-reference signal, y,Y – output signals of model and plant for $u(t)=[5(1\text{-}exp(\text{-}10t)]$ and scope of reachable outputs like in Fig. 3, y^*,Y^*-model and plant outputs for enlarged scope of reachable outputs, i.e. for reference response H, if model input is $u(t)=[10(1\text{-}exp(\text{-}10t)]$

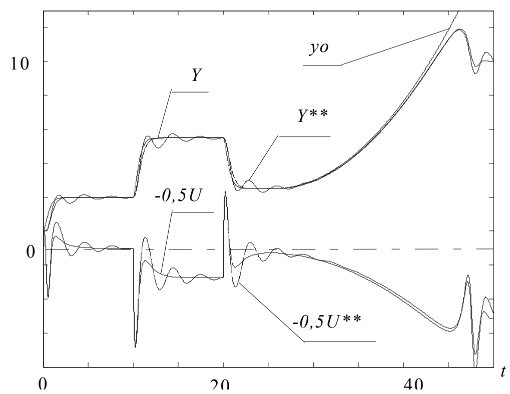

Fig. 6. The control of plant output for big differences between parameters of plant and its model, if $u(t)=[10(1\text{-}exp(\text{-}10t)]$: U,Y-plant input and output for exact model of plant, U^{**},Y^{**}- plant input and output if plant is described by transfer function $[(s+1)(s^3+2s^2+1.5s+1)^{-1}]$ while its inaccurate model is given by (6) and PI controller transmittance is $(4+s^{-1})$

2. To implement the considered control idea for plant of n-order one needs $(n\text{-}1)$ subsequent derivatives of $A(t)$ and some derivatives of u and y_o. Taking into account that calculations of U are carried out in computer environment on the basis of mathematical models the direct or indirect access to necessary derivatives does not seem to be serious problem – see <u>*Example 3*</u>. For follow-up mode of control action the unknown, external reference signal y_o can be "slightly" smoothed. Then approximate derivatives of y_o can be available as state variables of smoothing filter.

3. The values $Y(A(t))$ during control process are fully predictable, because they can be obtained by putting $t=T$ to reference response $H=Y(t)$. Note, that point mapping the plant output moves along the reference response H, whereas rate and direction of movement is governed by $A(t)$. That property extremely simplifies the analysis of system stability. The problems associated with minimizing of error e can be successfully overcome by choice of model input u and function $q(.)$ – see Fig.4. The "clever" choice of $q(.)$ and u yields output Y without overshoots and guarantees the short setting times. Furthermore, application of proposed control rules allows to utilize the well-known common benefits associated with use of classic MFC control.

4. The calculations of U base on computational model. That is why there are flexible possibilities of modifying the calculation procedure to adjust U to physical constraints imposed on plant input, like range of acceptable values of U, acceptable ranges for derivatives of U, etc.

5. The considerations in current paper were concentrated on synthesis of $A(t)$ for control purposes. However input $A(t)$ of plant model in Fig. 4 can be also formed on the basis of quite different assumptions. Thus, one can easily adjust presented results in order to solve other technical problems connected with necessity of time-scaling of signals in physical systems by proper forming of input signals.

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