

# Enhancement of instantaneous power method in the problems of estimation of electromechanical complexes power controllability

**Abstract.** A power approach to estimation of electromechanical complexes controllability using instantaneous power method has been offered. It has been determined that power controllability characterizes the quality of energy conversion processes in the system, reflects manifestation of nonlinear properties of the object. A mathematical procedure for determination of power losses for the whole power channel of electromechanical complex power conversion has been developed. It has been shown that formation of balance equations of separate instantaneous power components underlies the estimation of electromechanical complexes power controllability.

**Streszczenie.** Zaproponowano sposób oszacowania sterowności zespołów elektromechanicznych przy użyciu metody mocy chwilowej. Zostało określone, że sterowność mocy charakteryzuje jakość energii w procesie jej przekształcania oraz przedstawia własności nieliniowe obiektu. Rozwinięto procedurę matematyczną dla określenia start mocy w całym kanale mocy układów elektromechanicznych. Pokazano, że utworzenie równań równowagi dla składników mocy chwilowej leży u podłożu oszacowania sterowności układów elektromechanicznych. (**Użycie metody mocy chwilowej w problemach oszacowania sterowności mocy w układach elektromechanicznych**)

**Keywords:** electromechanical systems, instantaneous power, effective power, power controllability, nonlinear systems, power conditioning.  
**Słowa kluczowe:** systemy elektromechaniczne, moc chwilowa, moc skuteczna, sterowność mocy, systemy nieliniowe.

## Introduction

The problem of estimation of system controllability is rather extensive; it touches upon various spheres of activity – informational, social, technical ones, etc. One of the ways of controllability formalization consists in singling out the most significant practical activity sectors as possible objects of research, as well as development and application of formally new approaches to controllability estimation with this point in view.

Technical sector, including mechanical, electromechanical, hydrotransport and other types of systems, seems to be the most investigated from the point of view of mathematical description, theoretical research, and scientific experiments and directly or indirectly concerns particular features of energy conversion, control of energy flows in the considered object.

Electromechanical systems (EMS) of various technological complexes form a complicated power channel of energy transfer, conversion and consumption. During the process of functioning in the technological complex power channel there appear different energetic conditions quantitatively characterized by a set of electric, power and mechanical parameters (voltage, current, power, rotational frequency, moment) as well as technological indices (productivity, pressure). A single parameter characterizing any condition of an EMS of any complexity is power, expressed in the same metric units regardless of physical nature of the components forming parts of the expression for its definition. Power variables (electric power, mechanical, kinetic, hydraulic ones, etc.) provide a concrete characteristic of the current process of energy conversion and make it possible to estimate the efficiency of the power channel of the electromechanical complex, its power controllability (PC). Taking into consideration energy flow non-unidirectionality, presence of energy storage systems of various kinds, elements with nonlinear characteristics, complex character of energy processes changes in time, the problem of estimation of EMS PC deserves special attention.

To analyze power processes in an electromechanical complex (EMC) one usually uses active, reactive, total (apparent), interchange and other power components obtained using integral methods [1-3]. Such an approach is efficient in the cases when averaging at a certain time interval is admissible. A period of alternating voltage is usually accepted as such averaging. Integration process

results in a partial loss of information about real change of the signal [4]. Thus, mathematical interpretation of total power does not reflect real change of power processes in time domain.

Further development of the theory of power processes analysis led to the use of instantaneous power method [3-5] providing the possibility to characterize power change in time domain more completely, which makes it possible to solve the problems of EMC power controllability. The theory of instantaneous power operates with instantaneous values of voltages, currents, power, which are plotted on the axes of a spatial coordinate system.

The analysis revealed that most research based on the elements of instantaneous power theory is devoted to the issues of synthesis of control systems for power active filters, controlled rectifiers in the problems of compensation of reactive power, attenuation of current higher harmonics etc. [5-7]. However, the problems of the analysis of power conversion processes, determination of power controllability of technological complexes EMS characterized by different physical nature of considered signals, their form, periodicity, presence of nonlinear elements, storage devices etc., drop out of sight. In this case balance of harmonic components of instantaneous power of supply and EMC elements is the background for the theoretic basis of instantaneous power method. A power approach using instantaneous power method can be applied to direct and alternating current circuits in static and transient regimes with periodic and nonperiodic signals, for various media considering universality of the mechanism of approximation of power condition initial components with the use of trigonometric series.

## Problem statement

Formalization of the notion of electromechanical complexes controllability by means of the analysis of power conversion processes in all the power channel elements on the basis of instantaneous power components.

## Research method

Each one of  $i$ -th elements of EMC is characterized by energy regime parameters  $U_i(t)$  and  $I_i(t)$ , and they are not necessarily voltage and current, but components describing physical process of energy conversion. Product of these components gives power  $P_i(t) = U_i(t)I_i(t)$  – the

single parameter characterizing any regime of an EMS of any complexity. Having determined power  $P_i(t)$  for every EMC element, one can make conclusion about the pattern of power conversion in the whole system.

Henceforward, let us present instantaneous values of voltage and current signals consisting of a number of harmonic components:

$$(1) \quad U(t) = \sum_{n=0}^N U_{na} \cos(n\Omega t - \varphi_n) = \\ = \sum_{n=0}^N U_{na} \cos(n\Omega t) + \sum_{n=0}^N U_{nb} \sin(n\Omega t),$$

$$(2) \quad I(t) = \sum_{m=0}^M I_{ma} \cos(m\Omega t - \psi_m) = \\ = \sum_{m=0}^M I_{ma} \cos(m\Omega t) + \sum_{m=0}^M I_{mb} \sin(m\Omega t)$$

where:  $n, m$  – are voltage and current harmonic numbers, correspondingly;  $N, M$  – number of voltage and current components;  $\varphi, \psi$  – phase angles;  $\Omega = 2\pi f$  – circular frequency of the signal of voltage or current;  $f$  – signal change frequency;  $t$  – time of signal change.

Then instantaneous power in the system:

$$(3) \quad P(t) = U(t)I(t) = \sum_{n=1}^N U_{na} \cos(n\Omega t - \varphi_n) \times \\ \times \sum_{m=1}^M I_{ma} \cos(m\Omega t - \psi_m) = \\ = P_{0\Sigma} + \sum_{k=1}^K P_{ka} \cos(k\Omega t) + \sum_{k=1}^K P_{kb} \sin(k\Omega t)$$

where:  $P_{0\Sigma}$  – is instantaneous power total constant component;  $P_{ka}$  – amplitude value of the  $k$ -th instantaneous power cosine component;  $P_{kb}$  – amplitude value of the  $k$ -th instantaneous power sine component;  $k$  – power harmonic number ( $k = |n \pm m|$ );  $K$  – power components number.

Analysis (3) showed that instantaneous power includes the sum of constant and variable cosine and sine components. It should be noted that time variable component of power condition characterizes energy exchange process between the network and consumer, technological mechanism and motor. Presence of power variable component decreases power efficiency of conversion process and consequently results in reduction of power system controllability, which is conditioned by the presence of storage devices, nonlinearity, reflecting the specific character of technological mechanism electric drive operation.

Further transformations of expression (3) allowed one to single out the following components groups of instantaneous power: the first group is determined by multiplication of single-frequency ( $n = m$ ) voltage and current components and forms constant and canonic power components; the second group of components is determined by the product of various-frequency ( $n \neq m$ ) voltage and current components and presents non-canonic

power components. Thus, instantaneous power consists of the sum of constant component and canonic and non-canonic power components:

$$(4) \quad P(t) = P_0 + P_{ka\,c}(t) + P_{ka\,s}(t) + P_{kb\,c}(t) + P_{kb\,s}(t)$$

where:  $P_0$  – instantaneous power constant component;  $P_{ka\,c}(t)$  – instantaneous power canonic order cosine component;  $P_{kb\,c}(t)$  – instantaneous power canonic order sine component;  $P_{ka\,s}(t)$  – instantaneous power non-canonic order cosine component;  $P_{kb\,s}(t)$  – instantaneous power non-canonic order sine component.

### Research results

The main expressions reflecting power conversion processes in EMC are presented by the equations of instantaneous power balance of power supply  $P_{is}(t)$  and instantaneous power components  $P_i(t)$  on the elements of elementary consumers included in the complex  $P_{is}(t) = \sum_{i=1}^p P_i(t)$ , where  $p$  – index of the correspondent elementary consumer.

With reference to a drive electric motor, it is possible to write down:

$$(5) \quad P_{ed}(t) = P_{is}(t) - \Delta P_{\Sigma ed}(t)$$

where:  $P_{is}(t)$  – power on the motor supply terminals;  $\Delta P_{\Sigma ed}(t)$  – power of motor losses.

Power on the motor shaft:

$$(6) \quad P_{ed}(t) = M_{ed}(t)\omega(t)$$

where:  $M_{ed}(t) = M_s(t) + M_d(t)$  – electric motor torque;  $M_s(t)$  – resistance static moment;

$M_d(t) = \frac{d}{dt} \left( J(t) \frac{\omega^2(t)}{2} \right) / \omega(t)$  – electric motor dynamic moment;  $\omega(t)$  – instantaneous value of angular velocity of the motor;  $J(t)$  – motor inertia moment.

Then the expression of power balance for rotating masses with a constant inertia moment of the motor will be of the form:

$$(7) \quad P_{ed}(t) = M_s(t)\omega(t) + J(t)\omega(t) \frac{d\omega(t)}{dt}.$$

For the system with a variable inertia moment:

$$(8) \quad P_{ed}(t) = M_s(t)\omega(t) + J(t)\omega(t) \frac{d\omega(t)}{dt} + \\ + \frac{\omega^2(t) dJ(t)}{2}.$$

Determination of parameter  $\Delta P_{\Sigma ed}(t)$  and expression (5) is based on formation of equations of instantaneous power balance on the elements of electric motor equivalent circuit.

A similar result can be obtained not only for power on the motor shaft, as it was shown above, but also for power of technological load.

So, hydraulic power of a turbomechanism:

$$(9) \quad P_{hydro\ tm}(t) = \rho g H(t) Q(t)$$

where:  $H(t)$  – water head at the pump output;  $Q(t)$  – turbomechanism productivity (discharge);  $\rho$  – density of the pumped medium;  $g = 9.81 \text{ m} / \text{s}^2$  – gravitational acceleration.

Turbomechanism consumed power:

$$(10) \quad P_{tm}(t) = A_3 v^2(t) Q(t) + B_3 v(t) Q^2(t) + D_3 v^3(t)$$

where:  $A_3, B_3, D_3$  – approximation coefficients determined according to the catalog power characteristic of the pump;  $v(t) = \omega_i(t)/\omega_n(t)$  – relative rotational frequency of the pump impeller;  $\omega_i(t), \omega_n(t)$  – current and nominal angular velocities of the pump impeller.

Then, power losses in turbomechanism:

$$(11) \quad \Delta P_{tm}(t) = P_{tm}(t) - P_{hydro\ tm}(t) = (A'_3 - A'_2) v^2(t) \times Q(t) + (B'_3 - B'_2) v(t) Q^2(t) + D'_3 v^3(t) - C'_2 Q^3(t)$$

where:  $A'_2 = \rho g A_2$ ;  $B'_2 = \rho g B_2$ ;  $C'_2 = \rho g C_2$ .

Power losses in a pipeline system:

$$(12) \quad \Delta P_{tr}(t) = \Delta h(t) Q(t) = \frac{\lambda L}{2gDS^2} Q^3(t)$$

where:  $\Delta h(t) = \lambda \frac{L}{D} \frac{v^2(t)}{2g}$  – head losses in the pipeline;

$9(t) = Q(t)/S$  – liquid motion velocity;  $h(t) = m(t)/(ρS)$  – liquid rise piezometric head;  $λ$  – hydraulic resistance coefficient;  $S$  – pipeline cross-section area;  $L, D$  – pipeline length and diameter, correspondingly.

In accordance with the above done analyses, Fig. 1 presents schemes of EMC energy losses allocation with different degrees of detailed elaboration. Each of them can be taken as the basis for research of the problem of power controllability estimation or solution of any other problems.

Fig.1a shows a diagram of the electric part of the system. Fig.1b shows a diagram of distribution of power parameters directly on the motor shaft. Fig. 1c shows a variant of energy balance diagram taking into consideration the technological parameters of the turbomechanism and losses in it:

$$(13) \quad P_{is}(t) = R(I(t)) I^2(t) + L(I(t)) I(t) \frac{dI(t)}{dt} + \Delta P_{tm}(t) + P_{hydro\ tm}(t).$$

Instantaneous power effective value is a measure of EMC power processes quality estimation, and instantaneous power components present initial parameters for such estimation. Taking the above said into consideration, power effective value:

$$(14) \quad P_e = \sqrt{\frac{1}{T} \int_0^T P^2(t) dt} = \sqrt{\frac{1}{T} \int_0^T (P_0 + P_{kac}(t) + P_{kas}(t) + P_{kbc}(t) + P_{kbs}(t))^2 dt} = \sqrt{P_0^2 + \frac{1}{2} \sum_{k=1}^K (P_{kac} + P_{kas})^2 + \frac{1}{2} \sum_{k=1}^K (P_{kbc} + P_{kbs})^2}.$$

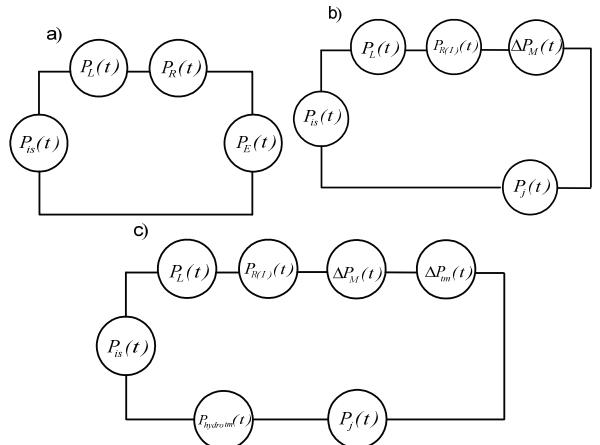


Fig.1. Power diagrams of EMC:  $P_{is}(t)$  – supply instantaneous power;  $P_L(t)$  – inductance instantaneous power;  $P_R(t)$  – active resistance instantaneous power in the power circuit;  $P_E(t)$  – motor electromagnetic power;  $ΔP_M(t)$  – motor mechanical losses power;  $P_j(t)$  – rotating masses instantaneous power;  $ΔP_{tm}(t)$  – turbomechanism losses instantaneous power;  $P_{hydro\ tm}(t)$  – hydraulic power at the turbomechanism output

Knowing effective power in ideal system  $P_{eid}$  (in the absence of most typical nonlinearities) and in real system  $P_{er}$  (in the presence of nonlinearities reflecting the specific character of technological mechanism electric drive operation), one can determine EMC PC index:

$$(15) \quad k_c = P_{eid} / P_{er}.$$

In this case the master control in ideal and real systems should be equal as to the constant component amplitude as well as the variable component amplitude and frequency. If the system is completely controllable,  $k_c = 1$ ; if the system is uncontrollable,  $k_c$  tends to zero.

Thus, power controllability is the most important index of system functioning efficiency. It characterizes the object energetic reaction to the change of controlling or disturbing effects. Obviously, the presence of nonlinearities in the analyzed system results in decrease of power controllability, which is accompanied by power processes with higher harmonics in power spectrum and increase of EMS effective power value.

### Experimental verification

To confirm the obtained results we created an experimental hydrotransport unit laboratory complex (Fig. 2) including two identical, as to their parameters, centrifugal pumps with induction motors on the same shaft; a manifold pipe network with installed shut-off-and-regulating fittings and receiving tanks; a regulated rotary shutter with an electric drive; frequency transformers for changing rotation frequency of pump electric motors; a control instrumentation (current, voltage, rotation frequency, pressure and discharge sensors). Engineering performance of the used equipment is shown in Tables 1, 2.

Table 1. Motor engineering performance

Parameter name	Parameter value
Rated power [W]	830
Rated voltage [V]	380
Mains frequency [Hz]	50
Rotation frequency [rpm]	2900

Table 2. Engineering performance of MNI-402 centrifugal pump

Parameter name	Parameter value
Maximum discharge [m <sup>3</sup> /hour]	8
Maximum head [m]	22
Rated power [W]	550

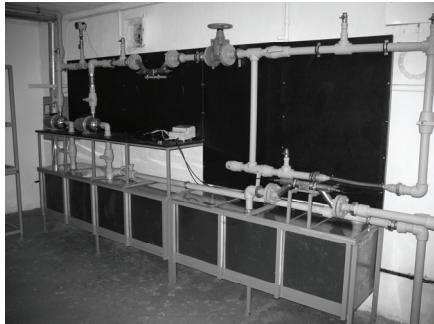


Fig. 2. Physical configuration of the experimental hydrotransport unit complex

The developed experimental complex embraces a whole spectrum of research and scientific applied problems. One of the main items comprises research of processes of energy transformation in all the links of hydrotransport unit presenting a complicated interconnected complex of different, as to its physical nature, equipment: electromechanical and hydraulic. Characteristics of pumping units, hydrodynamic network, pipe fittings present nonlinear dependences. Their analysis revealed that when regulation is made by a stopcock, harmonic components of higher orders (2, 3, 4 etc.) are present in the power spectrum (Fig. 3).

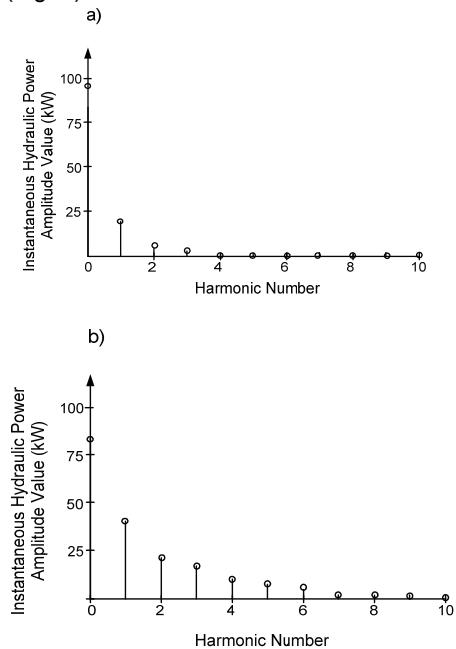


Fig. 3. Instantaneous hydraulic power amplitude spectra when productivity capacity is regulated by change of rotational frequency (a) and by a stopcock (b)

The method of pump productivity regulation by means of a stopcock is characterized by power nonproductive losses at the pipeline valves actuator. Obviously, the use of the regulated electric drive device in a hydrotransport unit results in improvement of power controllability of the considered system as compared with regulation of technological parameters by a stopcock. As decrease of hydrotransport complex EMS PC is accompanied by power

processes with the high harmonic in power spectra, in order to change the amplitudes of these components, it is necessary to form control and master actions of a certain kind by using existing or created techniques of both the electric drive itself and the elements of communication network.

## Conclusions

It is shown that analysis of the processes of power conversion in all the links of power channel, using instantaneous power components, underlies the estimation of electromechanical complex controllability. Presence of a nonlinear element in the electromechanical system results in appearance of higher harmonics in power amplitude spectrum and is accompanied by increase of the system effective power. Power controllability is the most important system functioning efficiency index characterizing energetic reaction of the considered object to manifestation of nonlinear properties or processes in it. Power controllability index depends on the change of controlling and disturbing effects in the system. Universality of the offered approach has been proved. It consists in the possibility of the analysis of power processes in both electric and other systems: mechanic, electromechanical, hydraulic, etc., where initial signals, forming instantaneous power, may be of a complicated character (periodic and nonperiodic).

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