Paweł JABŁOŃSKI

Częstochowa University of Technology

Approximate BEM analysis of a thin magnetic shield of variable thickness

***Abstract**. The paper presents an approximate method of analysis of magnetic field in a thin magnetic shield of variable thickness. The method of analysis is a hybrid method based on the boundary element method (BEM) and an approximate analytical solution for the shield. Such an approach allows avoiding significant numerical errors and reducing the computation time as well as the memory used.*

***Streszczenie.** Praca przedstawia przybliżoną metodę analizy pola magnetycznego cienkościennego ekranu magnetycznego o zmiennej grubości. Metoda analizy jest metodą hybrydową wykorzystującą metodę elementów brzegowych oraz przybliżone rozwiązanie analityczne w obszarze ekranu. Proponowane podejście pozwala uniknąć znacznych błędów numerycznych oraz zmniejsza czas obliczeń i obciążenie pamięci operacyjnej. (**Aproksymacyjna metoda elementów brzegowych w analizie cienkich warstw magnetycznych o zmiennej grubości**)*

Keywords: boundary element method, magnetic shielding, thin shells.
Słowa kluczowe: metoda elementów brzegowych, ekranowanie magnetyczne, cienkie warstwy.

Introduction

Thin layers whose material properties differ from the properties of surrounding media are met in many real configurations, e.g. magnetic or electromagnetic shields, walls of biological cells, human skin, painting shells, dirty or dusted surfaces. Modeling the electromagnetic field in such configurations often requires using a numerical method due to geometrical complexity or other aspects. However, thin layers are always troublesome in numerical analysis and often require special treatment, e.g.

- very fine discretization of the layer and its surroundings,
- using a special kind of elements (e.g. air gap elements in the Finite Element Method),
- transforming the governing equations to more applicable in the considered case,
- using a hybrid method.

Many of the considered types of problems could be efficiently solved with use of the Boundary Element Method (BEM) [1, 2]. Although from the mathematical point of view the resulting system of BEM equations is not singular, yet from the numerical point of view it is ill conditioned [3]. This is because BEM equations for corresponding points lying on the opposite surfaces of the layer contain coefficients of similar values, resulting in nearly linear dependence of some rows of the main matrix. Moreover, since the elements of the matrix are integrals which must be often evaluated numerically, they may be very inaccurate, especially if the integrals are nearly singular. In fact, this is the case which occurs in a BEM equation for a thin layer, because the observation and source points can be very close. Small changes in small distance result in large changes in its reciprocal, what finds its reflection in abrupt changes in the integrand – a function of the reciprocal of the distance. Numerical integration of such functions may require special treatment and can lead to very inaccurate results.

As a conclusion, modeling electromagnetic field in thin layers with use of BEM requires some kind of special treatment. Some approaches were proposed in [3-6]. In this paper one of them is developed and used for a thin magnetic shield of variable thickness. It may be regarded as a generalization of the approach described in [6]. For simplicity, the considerations are limited to 2D problems.

Problem description

A closed magnetic shield, Ω_1, is placed in free space, Ω_0, and encloses a protected region, Ω_2 – Fig. 1. The external and internal surfaces of the shield are referred to as S_1 and S_2, respectively. The magnetic region is considered to be very thin, whose thickness, d, can vary from point to point. The relative permeability of the shield is assumed to be μ_r = const, while the permeability of the protected region and the free space equals μ_0.

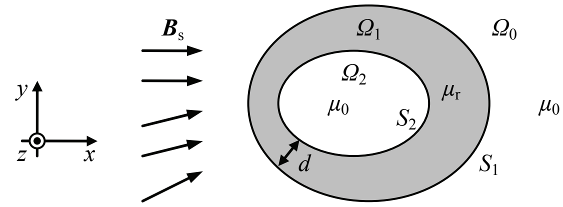

Fig. 1. Problem diagram – magnetic shell Ω_1 under an external transverse magnetic field $\boldsymbol{B}_s$

Although this problem can be solved by means of scalar magnetic potential, the vector magnetic potential can be used as well. In 2D problems, these two approaches have the same level of complexity, because the vector magnetic potential has only a z-component ($\boldsymbol{A} = A\boldsymbol{1}_z$). The equations describing the field are as follows:

$$\nabla^2 A^{(m)} = 0 \qquad \text{for } \Omega_m,\ m = 0, 1, 2\,, \tag{1}$$

with $A^{(0)} = A^{(1)}$ on boundary S_1, $A^{(1)} = A^{(2)}$ on boundary S_2, and

$$\left.\frac{\partial A^{(0)}}{\partial n}\right|_{S_1} = -\frac{1}{\mu_r}\left.\frac{\partial A^{(1)}}{\partial n}\right|_{S_1}, \quad \left.\frac{\partial A^{(2)}}{\partial n}\right|_{S_2} = -\frac{1}{\mu_r}\left.\frac{\partial A^{(1)}}{\partial n}\right|_{S_2}, \tag{2}$$

and $A^{(0)} \to A_s$ far from the magnetic screen, where A_s is the z-th component of the magnetic vector potential of externally applied magnetic field $\boldsymbol{B}_s$ (i.e. $\boldsymbol{B}_s = \mathrm{curl}(A_s\boldsymbol{1}_z)$).

Standard BEM model

The standard BEM procedure applied to this problem, after using the continuity conditions, leads to the following system of equations

$$\begin{bmatrix} \boldsymbol{H}_1^{(0)} & \frac{1}{\mu_r}\boldsymbol{G}_1^{(0)} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{H}_1^{(1)} & -\boldsymbol{G}_1^{(1)} & \boldsymbol{H}_2^{(1)} & -\boldsymbol{G}_2^{(1)} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{H}_2^{(2)} & \frac{1}{\mu_r}\boldsymbol{G}_2^{(2)} \end{bmatrix} \begin{Bmatrix} \boldsymbol{A}_1 \\ \boldsymbol{Q}_1^{(1)} \\ \boldsymbol{A}_2 \\ \boldsymbol{Q}_2^{(1)} \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{A}_s \\ \boldsymbol{0} \\ \boldsymbol{0} \end{Bmatrix}, \tag{3}$$

where $\boldsymbol{H}_l^{(m)}$ and $\boldsymbol{G}_l^{(m)}$ – the BEM matrices corresponding to boundary S_l connected with domain Ω_m, $\boldsymbol{A}_l$ – vector of nodal values of magnetic vector potential (**z**-th component) A on boundary S_l, $\boldsymbol{Q}_l^{(m)}$ – vector of nodal values of normal derivative $\partial_n A$ on boundary S_l of domain Ω_m, $\boldsymbol{A}_\mathrm{s}$ – vector of nodal values of magnetic vector potential (z-th component) A_s of the source magnetic field $\boldsymbol{B}_\mathrm{s}$ on boundary S_1.

Approximate model

As it was mentioned in Introduction, the standard BEM model leads to unacceptable errors if the thickness of the shield, d, is too small. In such a case some elements of matrices $\boldsymbol{H}_l^{(1)}$ and $\boldsymbol{G}_l^{(1)}$ can be numerically very inaccurate, the main matrix is ill conditioned, and the system of equations is relatively large. To avoid those disadvantages, an approximate approach is proposed. If the layer of the shield is smooth enough, by the Taylor series up to the linear terms values of potential A on boundary S_1 can be approximately expressed as a linear function of values of potential A and its normal derivative on S_2, i.e.:

(4) $$A_2 \approx A_1 - \left.\frac{\partial A}{\partial n}\right|_1 d \,.$$

Assuming a suitable discretization, in which each node lying on S_1 has a counterpart lying on S_2, the above relationship leads to the following approximate relationship between the vectors of unknowns:

(5) $$\boldsymbol{Q}_1^{(1)} = \boldsymbol{w}(\boldsymbol{A}_1 - \boldsymbol{A}_2) \,.$$

where $\boldsymbol{w}$ is a diagonal matrix, whose diagonal elements are reciprocals of thickness of the shield at the nodes of field approximation, i.e.

(6) $$\boldsymbol{w} = \mathrm{diag}\left(\frac{1}{d_1}, \frac{1}{d_2}, \frac{1}{d_3}, \ldots\right) .$$

To tell the truth, approximation (5) assumes a linear dependence of A in the normal direction throughout domain the layer. This assumption seems to be justified if thickness d is small enough. It implies that

(7) $$\boldsymbol{Q}_2^{(1)} = -\boldsymbol{Q}_1^{(1)} \,.$$

Eqs. (5) and (7) allow eliminating the BEM equation for the layer of the magnetic shield, yielding the final system of equations as follows:

(8) $$\begin{bmatrix} \boldsymbol{H}_1^{(0)} + \frac{1}{\mu_\mathrm{r}}\boldsymbol{G}_1^{(0)}\boldsymbol{w} & -\frac{1}{\mu_\mathrm{r}}\boldsymbol{G}_1^{(0)}\boldsymbol{w} \\ -\frac{1}{\mu_\mathrm{r}}\boldsymbol{G}_2^{(2)}\boldsymbol{w} & \boldsymbol{H}_2^{(2)} + \frac{1}{\mu_\mathrm{r}}\boldsymbol{G}_2^{(2)}\boldsymbol{w} \end{bmatrix} \begin{Bmatrix} \boldsymbol{A}_1 \\ \boldsymbol{A}_2 \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{A}_\mathrm{s} \\ \boldsymbol{0} \end{Bmatrix} .$$

System of equations (8) has half the number of equations and unknowns than Eq. (3), and no nearly singular integrals occur in it (for sufficiently regular boundary). If thickness d is constant, matrix $\boldsymbol{w}$ can be replaced by a scalar $1/d$, and Eq. (8) can be simplified to the form given in [6].

Numerical results

Both models, the standard BEM and approximate BEM, were implemented as a Mathematica 7.0 program. Second order boundary elements (quadratic) were used to map sufficiently exactly the geometry of possible configurations. In all cases a uniform external magnetic field was assumed.

To make sure all procedures evaluating the coefficients in the system of equations had been implemented correctly, the model was tested on an example that can be easily solved theoretically – a cylindrical magnetic shield of constant thickness d in a uniform transverse magnetic field. The results are shown in Fig. 2 to 6. If shield's thickness d is large enough (Fig. 2a), the standard BEM procedure gives sufficiently accurate results (see Figs. 3 and 4). However, for small enough thickness (Fig. 2b) the standard BEM procedure leads to large errors, which are observed as a dispersion in nodal values of field on boundaries (see Figs. 5 and 6). The dispersion is especially large for $\partial_n A$. The results confirm that thin layers (here, the thin magnetic shield) require a special treatment.

a) b)

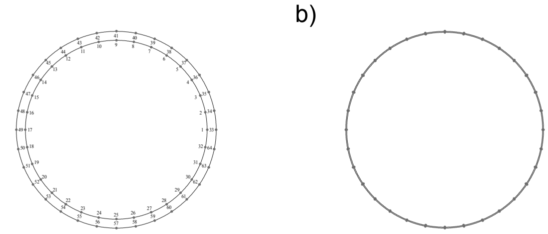

Fig. 2. Cylindrical magnetic shield (cross-section): a) thick (d/R_2 = 0.1), b) thin (d/R_2 = 0.01)

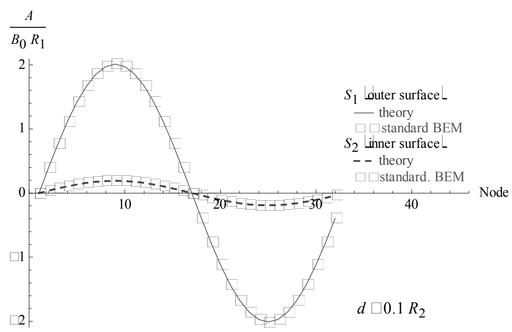

Fig. 3. Nodal values of magnetic potential A on boundaries S_1 and S_2 for thick magnetic shield (μ_r = 100) – theory and standard BEM

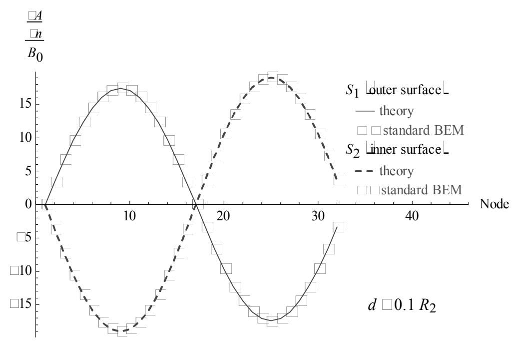

Fig. 4. Nodal values of normal derivative $\partial_n A$ on boundaries S_1 and S_2 for thick magnetic shield (μ_r = 100) – theory and standard BEM

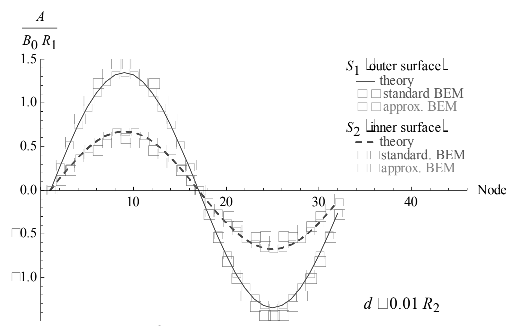

Fig. 5. Nodal values of magnetic potential A on boundaries S_1 and S_2 for thin magnetic shield (μ_r = 100) – theory, standard BEM and approximate BEM

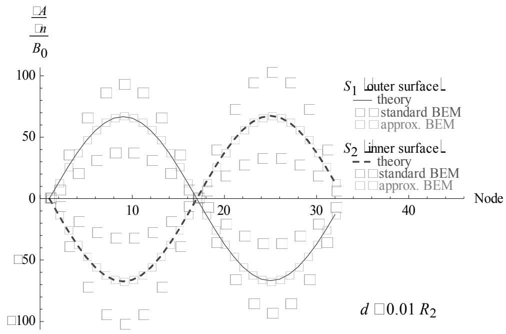

Fig. 6. Nodal values of normal derivative $\partial_n A$ on boundaries S_1 and S_2 for thin magnetic shield (μ_r = 100) – theory, standard BEM and approximate BEM

To verify if variable thickness is correctly incorporated in the approximate model, again a cylindrical shield of internal radius R is considered, but this time d/R changes around 0.01 from 0.002 to 0.018 (Fig. 7). Because no exact analytical solution exists in this case, the model was verified by using μ_r = 1. In such a case the exact solution is $\boldsymbol{A} = \boldsymbol{A}_s$, and Figs. 8 and 9 confirm the correctness of the approximate model. Finally, Figs. 10 and 11 show the boundary nodal values of A and its normal derivative for an arbitrary case μ_r = 100.

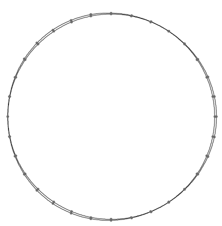

Fig. 7. Assumed thin cylindrical magnetic shield of variable thickness

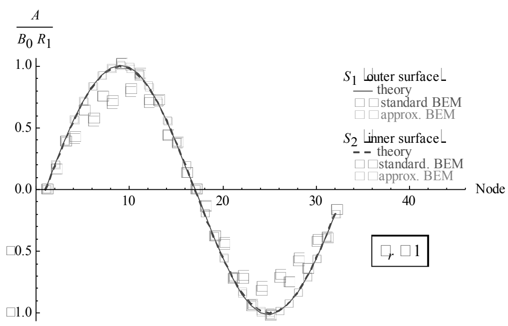

Fig. 8. Nodal values of magnetic potential A on boundaries S_1 and S_2 for thin non-magnetic layer of variable thickness (μ_r = 1) – theory, standard BEM and approximate BEM

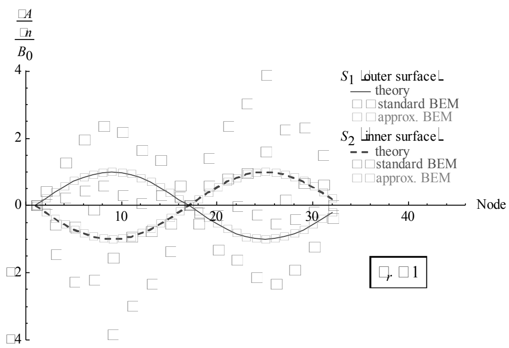

Fig. 9. Nodal values of normal derivative $\partial_n A$ on boundaries S_1 and S_2 for thin non-magnetic layer of variable thickness (μ_r = 1) – theory, standard BEM and approximate BEM

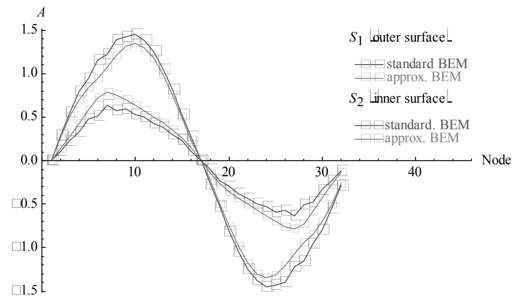

Fig. 10. Nodal values of magnetic potential A on boundaries S_1 and S_2 for thin magnetic layer of variable thickness (μ_r = 100) – theory, standard BEM and approximate BEM

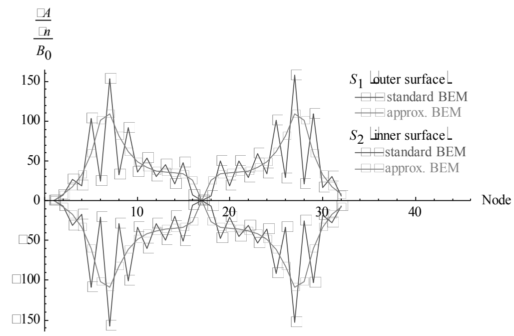

Fig. 11. Nodal values of normal derivative $\partial_n A$ on boundaries S_1 and S_2 for thin magnetic layer of variable thickness (μ_r = 100) – theory, standard BEM and approximate BEM

Concluding remarks

The method of modeling a thin magnetic layer with use of approximate local analytical solution has the following advantages:

- no nearly singular integrals due to thin layer,
- half the number of equations of standard BEM model,
- good results for thin layers,
- can be used in modeling thin layers of other type, e.g. weak-conductive.

However, there are some disadvantages:

- approximate relationship for the layer is used, which can be justified only partially,
- cannot be applicable if $\partial_n A$ varies much at corresponding points lying on boundaries S_1 and S_2.

Owing to the disadvantages the method requires further testing.

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***Author**: dr inż. Paweł Jabłoński, Częstochowa University of Technology, Electrical Faculty, al. Armii Krajowej 17, 42-200 Częstochowa, E-mail: paweljablonski7@gmail.com.*