**Mohammad Ali TAGHIKHANI**

Imam Khomeini International University

# Modeling of Heat Transfer in Layer-Type Power Transformer

***Abstract**. Transformers are important and expensive elements of a power system. To prescribe the limits of short-term and long-term loading capability of a transformer, it is necessary to estimate the hottest spot temperature (HST) of transformer. This paper proposes the steady state temperature distribution of the power transformer windings. Finite element method is used for numerical solution. The selected model for simulation is a 50KVA, 20 kV/400V oil natural, and air natural cooling (ONAN) transformer.*

***Streszczenie.** Aby określić długoterminową i krótkoterminową obciążalność transformatora musimy znać najwyższą lokalną jego temperaturę. W artykule zaprezentowano metodę określania rozkładu temperatury w uzwojeniu transformatorów – olejowego i powietrznego. (**Modelowanie przepływu ciepła w transformatorze mocy**)*

**Keywords:** finite element method; hottest spot temperature; power transformer; temperature distribution.
**Słowa kluczowe:** metoda elementów skończonych, transformatory mocy, nagrzewanie.

## Introduction

Thermal impact leads not only to long-term oil/paper-insulation degradation; it is also a limiting factor for the transformer operation [1], [2]. Therefore, the knowledge of the temperature, especially the hottest spot temperature, is of high interest. If the temperature rise goes beyond the permissible value, in order to preserve the insulation from deterioration, the load of transformer must be reduced or an auxiliary transformer is used. For an oil-immersed transformer, the oil surrounds the transformer body. Oil is a nearly incompressible fluid and density changes due to temperature rise, therefore oil moves in the transformer. The heat transferred by convection is the most important method of heat transfer. The analytical solution of convection equation is normally difficult and sometimes it is impossible due to the complexity of the geometry.

The basic criterion for transformer loading is the hottest spot temperature of the solid insulation. It must not exceed the prescribed value in order to avoid insulation faults. A procedure of hottest spot temperature calculation is given in the International Standards [3]-[5]. The algorithm for calculating the hottest spot temperature of a directly loaded transformer using data obtained in a short circuit heating test is given in [6]-[8]. Heat transfer theory results from winding to oil are exposed in [9], [10]. The usage of average heat transfer coefficient is typical in a transformer designing process to calculate needed number (area) of cooling surfaces.

In this paper, a procedure for obtaining the temperature distribution in the power transformer is proposed. For this reason Energy and Navier-Stokes equations are solved using finite element method. Therefore, a code has been provided under MATLAB software. The model can be used for temperature calculation on the arbitrary change of current and outside air temperature. In the paper, Thermal model is in section two and Results and discussion of the proposed work have been provided in Section three.

## Thermal Model

Fig.1 shows cross section of a 50KVA, 20kV/400V power transformer in two dimensions. Dimensions and specifications of the power transformer have been summarized in Table 1. Table 2 shows the losses. Energy equation for Newtonian incompressible fluid such as oil in two dimensions is [11]

$$-k\cdot\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+\rho\cdot C_p\cdot\left(V_x\frac{\partial T}{\partial x}+V_y\frac{\partial T}{\partial y}\right)=0 \tag{1}$$

where: $T$: temperaturę, $Vx$: velocity in x direction, $Vy$: velocity in y direction, $k$: thermal conductivity, $\rho$ : density, $C_p$: special heat, $\mu$:: viscosity

In (1), oil properties vary with temperature [1], [12]-[14]. The temperature dependence of oil properties are given:

$$\mu=a_1\cdot\exp\left(\frac{a_2}{T+273}\right) \tag{2}$$

$$C_p=a_3+a_4\cdot T \tag{3}$$

$$\rho=a_5+a_6\cdot T \tag{4}$$

$$k=a_7+a_8\cdot T \tag{5}$$

$$\beta=a_9 \tag{6}$$

Where $\beta$ is volumetric expansion coefficient. The nine constants for transformer oil have been listed in Table 3. It is generally valid for all transformer oils that the variation of the oil viscosity with temperature is much higher than the variation of other oil properties [12]-[14]. Thus, all oil physical properties except the viscosity can be replaced by a constant. However, in this paper has been considered the influence of all oil properties. In (1) velocity is unknown, and then we must solve Navier-Stokes equations. Navier-Stokes equations in two dimensions for incompressible fluid are [11]

Continuity Equation:

$$\frac{\partial V_x}{\partial x}+\frac{\partial V_y}{\partial y}=0 \tag{7}$$

Momentum Equations:

$$\mu.\left(\frac{\partial^2 V_x}{\partial x^2}+\frac{\partial^2 V_x}{\partial y^2}\right)-\rho.\left(V_x\frac{\partial V_x}{\partial x}+V_y\frac{\partial V_x}{\partial y}\right)-\frac{\partial P_{rel}}{\partial x}+(\rho-\rho_0).g_x=0 \tag{8}$$

$$\mu.\left(\frac{\partial^2 V_y}{\partial x^2}+\frac{\partial^2 V_y}{\partial y^2}\right)-\rho.\left(V_x\frac{\partial V_y}{\partial x}+V_y\frac{\partial V_y}{\partial y}\right)-\frac{\partial P_{rel}}{\partial y}+(\rho-\rho_0).g_y=0 \tag{9}$$

where: $P_{rel}$: relative pressure (related to air), $g_x$ : acceleration due to gravity in x direction, $g_y$ : acceleration due to gravity in y direction, $\rho_0$: density at ambient temperature.

The heat conduction equation for core and windings is written

$$k_x\cdot\frac{\partial^2 T}{\partial x^2}+k_y\cdot\frac{\partial^2 T}{\partial y^2}+Q=0 \tag{10}$$

where $k_x$ and $k_y$ are thermal conductivity in x and y directions respectively. The term $Q$ is the volumetric heat source function and has been modified here to take care of variation of electrical resistance of copper with temperature. The heat source term *Q* can be of the form [12]-[14]

(11) $$Q = Q_0 \cdot [1 + \alpha_c \cdot (T - T_{amb})]$$

where $\alpha_c$ is the temperature coefficient of electrical resistance of copper wire.

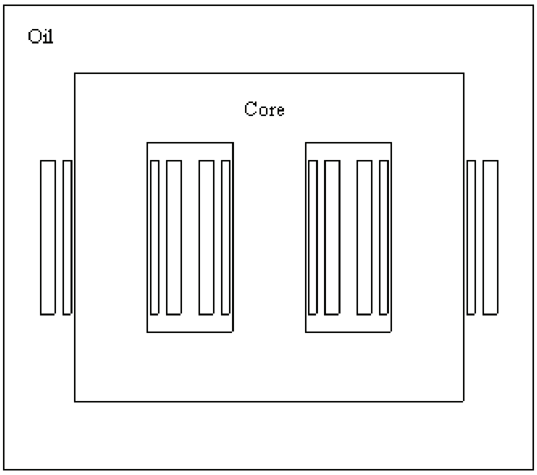

Fig. 1. Three phase power transformer in two dimensions

Table 1. Dimensions and specifications of the proposed power transformer

| | |
|---|---|
| Core diameter (cm) | 10 |
| Width of each window (cm) | 12 |
| Height of window (cm) | 28 |
| Thickness of LV winding (cm) | 1.2 |
| Thickness of HV winding (cm) | 2 |
| Rated power | 50KVA |
| HV voltage | 20kV |
| LV voltage | 400V |
| HV current | 1.44A |
| LV current | 72A |

Table2. Power transformer losses

| Losses (w) | Value |
|---|---|
| Core | 158 |
| DC of LV windings | 384 |
| DC of HV windings | 534 |
| Eddy currents of LV windings | 64 |
| Eddy currents of HV windings | 1 |

Table3. Oil constants

| Oil constant | Transformer oil |
|---|---|
| $a_1$ | 0.0000013573 |
| $a_2$ | 2797.3 |
| $a_3$ | 1960 |
| $a_4$ | 4.005 |
| $a_5$ | 887 |
| $a_6$ | -0.659 |
| $a_7$ | 0.124 |
| $a_8$ | -0.0001525 |
| $a_9$ | 0.00086 |

With this representation, the function *Q* becomes temperature dependent, distributed heat source. Thermal conductivities are unequal in different directions. Thermal conductivity has been treated as a vector quantity, having components in both radial and axial direction. Resultant thermal conductivity of the system is [12]

(12) $$K = \sqrt{k_x^2 + k_y^2}$$

where

$$k_x = \frac{\log \frac{r_n}{r_1}}{\left(\frac{\log \frac{r_2}{r_1}}{k_1} + \frac{\log \frac{r_3}{r_2}}{k_2} + \dots + \frac{\log \frac{r_n}{r_{n-1}}}{k_n}\right)}$$

$$k_y = \frac{k_{cu} k_{in} (t_{cu} + t_{in})}{(t_{in} k_{cu} + t_{cu} k_{in})}$$

Term *K* represents resultant thermal conductivity of insulation and conductor system. In this paper, the bottom oil temperature rise over ambient temperature has been calculated as

(13) $$\theta_u = \theta_{fl} \cdot \left(\frac{I_r^2 R_l + 1}{R_l + 1}\right)^n$$

Where $\theta_u$ is the bottom oil temperature rise over ambient temperature, $\theta_{fl}$ is the full load bottom oil temperature rise over ambient temperature obtained from an off-line test; $R_l$ is the ratio of load loss at rated load to no-load loss. The variable $I_r$ is the ratio of the specified load to rated load.

(14) $$I_r = \frac{I}{I_{rated}}$$

The exponent *n* depends upon the cooling state. The loading guide recommends the use of *n*=0.8 for natural convection and *n*=0.9-1.0 for forced cooling.

**Results and Discussion**

A software program has been provided for finite element solution of energy and Navier-Stokes equations using MATLAB 6.5. Depth cross section of the proposed power transformer has been shown in Fig.2. We assume inlet oil temperature is temperature base. Therefore, we have

(15) $$\Delta T = \frac{T - T_{b,oil}}{T_{b,oil}} \times 100$$

where $T_{b,oil}$ is the bottom oil temperature.

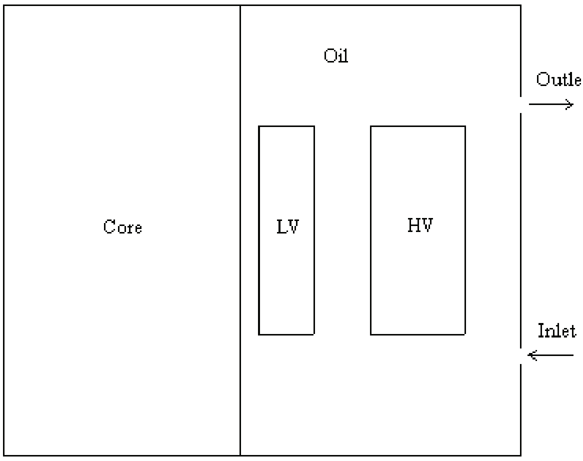

Fig.2. Depth cross section of the proposed power transformer

Fig.3 shows temperature distribution from LV winding in one per unit (p.u.) load with oil natural cooling (ON). It can be pointed out that for LV winding, maximum temperature location is around 80% of winding height from the bottom and at about 50% of radial thickness of the layer. Temperature distribution from HV winding has been shown in Fig.4. It can be observed that the maximum temperature occurs in the neighborhood of 55% of the axial and 50% of the radial thickness of the layer. Fig.5 shows HST rise over bottom oil temperature versus load at $T_{amb}$=25(°C). Table 4 shows comparison of the proposed method with finite integral transform used in [12] and experimental measurement in [15].

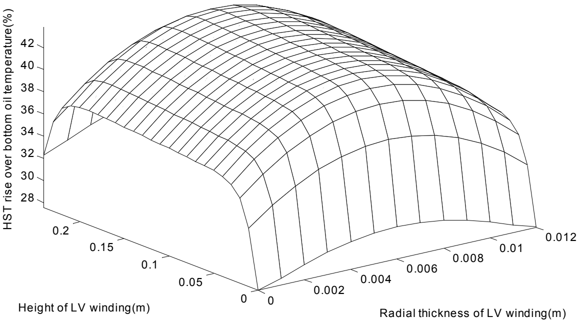

Fig.3. Temperature distribution (1 p.u.) of LV winding at $T_{amb}$=25(°C)

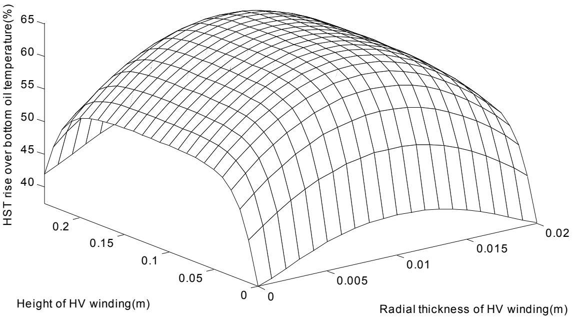

Fig.4.Temperature distribution (1 p.u.) of HV winding at $T_{amb}$=25(°C)

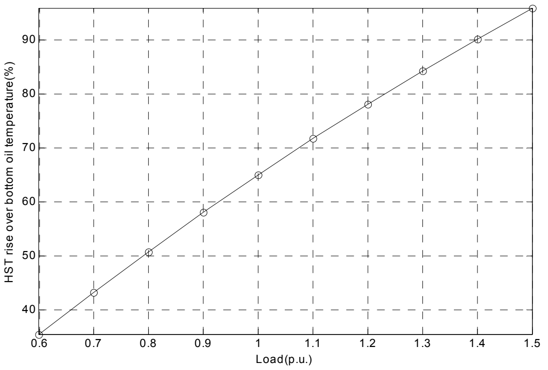

Fig.5. HST rise over bottom oil temperature versus load at $T_{amb}$=25(°C)

Table4. HST (°C) Magnitude at $T_{AMB}$=25(°C)

| Load (p.u.) | Proposed | Analytical [12] | Experimental [15] |
|---|---|---|---|
| 0.6 | 51.72 | 53 | 50 |
| 1 | 82.54 | 80 | 81 |

## Conclusion

In this paper, we solved the heat transfer partial differential equations to find temperature distribution in power transformer windings. The results seem to correspond reasonably well with results of calculations and actual tests [12]-[15]. The authors wish to point out that the IEEE loading guide offers relations for the calculation of the HST based on per-unit load. The formulations tend to ignore the possibilities of two transformers that are rating identical but have a different winding structure and varying heat loss/unit volume.

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***Author**: M.A.Taghikhani, Engineering Department, Imam Khomeini International University, Qazvin, Iran, E-mail: taghikhani@ikiu.ac.ir*