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Analytical Field Expressions due to Inclined Lightning Channel

Abstract.- *In order to have a fast determination of electromagnetic fields peaks associated with inclined lightning channel, the analytical electromagnetic field expressions were proposed, assuming a pulse shaped return stroke current at channel base and by considering the channel angle and the observation point angle effects on the electromagnetic fields. Also, the return stroke velocity effects on the peaks of electromagnetic fields were processed. The results were compared with the perpendicular lightning channel case. The proposed fields' expressions are very useful in determining the peak values of electromagnetic fields, since they are considered as critical parameters in estimation of lightning induced voltage peak on the power line.*

Streszczenie. *Zaproponowano analityczną metodę wyznaczania pola elektromagnetycznego towarzyszącego odchylonemu kanałowi wyładowania. Przyjęto impulsowy kształt uderzeniowego prądu. Przeprowadzono obliczenia dla różnych kątów kanału wyładowania. (**Analityczne wyrażenie na pole elektromagnetyczne wywołanie wyładowaniem odchylonym pod kątem**)*

Keywords: Electromagnetic Fields, lightning, inclined channel.
Słowa kluczowe: pole elektromagnetyczne, wyładowanie.

Introduction

Evaluation of electromagnetic field due to lightning channel is an important topic for considering indirect effects of lightning on the power systems[1], while the coupled electromagnetic fields with power lines can cause high voltage on the power lines. The most common assumption about the lightning channel is that it makes a perpendicular channel with the ground surface[2]. However, in reality, the lightning strikes the ground surface with an angle less than 90 degrees with respect to the ground[3]. Therefore, in order to estimate the electromagnetic fields fast due to the inclined lightning channel in the time domain, the general field's expressions are proposed and the results are compared with previous field expressions associated with perpendicular lightning channel. To facilitate offering analytical electromagnetic fields expressions, it is assumed that the channel base current is in step shape[4] and the current model is the transmission line (TL) model[1]. The proposed method can be used for the fast calculation of electromagnetic field values associated with inclined lighting channel, whilst the proposed equations can support perpendicular channel by changing the degree of the channel angle. Moreover, the proposed field expressions can be especially valuable in determining the field behaviour against the variations in the channel and the observation point parameters. The basic assumptions in this study are:

- i- The ground surface is flat.
- ii- The lightning channel is straight, without any branches.
- iii- The ground conductivity is perfect.
- iv- The corona effect is ignored.

The return stroke current

The lightning return stroke current can be categorized into two parts[1, 5]: 1) the return stroke current at the channel base; 2) the return stroke current along the lightning channel. The return stroke current at channel base can be expressed by different current functions, but in this study the step current is applied to arrive at the analytical expressions for the electromagnetic fields.

Based on the lightning current wave shape at different heights from the ground surface along the channel, several models were proposed which are classified into four groups[6]: 1) the "gas-dynamic" models, 2) the "electromagnetic" models, 3) the "current-distributed" models, and 4) the "engineering" models. In this paper, the transmission line model (TL)[1] from the "engineering" current group was employed. The return stroke current along lightning channel based on TL model is expressed by equation (1). Note that, the return stroke velocity is assumed to be a constant value along the lightning channel[1, 7].

$$\text{(1)} \qquad i(z',t) = \begin{cases} i(0, t-\frac{z'}{v}), & z' \le vt \\ 0, & z' > vt \end{cases}$$

Where: v is the lightning return stroke wave-front velocity, z' is the vertical space variable, i(z',t) is the return stroke current at height z' along lightning channel, i(0,t) is the channel base current.

Electromagnetic fields due to the vertical lightning channel

As a result of vertical lightning channel, the electromagnetic fields can be estimated by equations (2) to (4)[4].

$$\text{(2)}\ E_z = \zeta_0 \frac{I_p}{4\pi\beta}\left\{\frac{\psi}{E_{ch}} + \frac{\psi}{E_{im}} - \frac{2}{\sqrt{z^2+r^2}}\right\} \qquad \text{for } t > \frac{\sqrt{r^2+z^2}}{c}$$

$$\text{(3)}\ E_r = \zeta_0 \frac{I_p}{4\pi\beta r}\left\{\frac{\beta ct - z}{E_{ch}} - \frac{\beta ct + z}{E_{im}} + \frac{2z}{\sqrt{z^2+r^2}}\right\} \qquad \text{for } t > \frac{\sqrt{r^2+z^2}}{c}$$

$$\text{4)}\ H_\varphi = \frac{I_p}{4\pi r}\left\{\frac{\beta ct - z}{E_{ch}} + \frac{\beta ct + z}{E_{im}}\right\} \qquad \text{for } t > \frac{\sqrt{r^2+z^2}}{c}$$

$$\text{(5)}\ H_\varphi = E_r = E_z = 0 \qquad \text{for } t \le \frac{\sqrt{r^2+z^2}}{c}$$

where: E_r is horizontal electric field, E_z is vertical electric field, H_φ is magnetic field, I_p is the current peak at channel base, ζ_0 is free space characteristic impedance (376.73 Ω), v is return stroke velocity, c is light speed in the free space, r is radial distance from lightning channel to observation point, z is observation point height,
t is time,

$$\beta = \frac{v}{c}$$

$$\gamma = \sqrt{\frac{1}{1-\beta^2}}$$

$$E_{ch} = \sqrt{(\beta ct - z)^2 + \left(\frac{r}{\gamma}\right)^2}$$

$$E_{im} = \sqrt{(\beta ct + z)^2 + (\frac{r}{\gamma})^2}$$

$$\psi = \frac{1}{\gamma^2}$$

Electromagnetic fields due to the inclined lightning channel

The electromagnetic fields associated with the inclined lightning channel can be estimated by using Maxwell's equations[8] , Lorentz gauge[9] and FDTD method[10-11] as proposed by equations (5) to (9). The geometry of the problem is illustrated in Fig.1. Note that, the y-axis is set on the image of the lightning channel on the ground surface.

$$B_{\varphi}(x,y,z,\theta,\phi,t_k) = 10^{-7} \times$$

$$\{-\cos\phi I_p\left[\frac{\frac{\sin\theta\beta\gamma^2 A_1 z}{\cos\theta}+\sin\theta\cos\theta(y^2-z^2)-yz-y\beta\gamma^2 A_1+2\,z\,y\cos^2\theta}{A_4\times\sqrt{x^2+y^2-2\frac{\sin\theta\beta\gamma^2 A_1 y}{\cos\theta}+z^2-2z\beta\gamma^2 A_1+\frac{\beta^2\gamma^4 {A_1}^2}{\cos^2\theta}}}\right]+$$

$$2\sin\phi\cos\theta I_p x\left[\frac{\frac{2\beta\gamma^2 A_1}{\cos\theta}-2\,y\sin\theta-2z\cos\theta}{\{4(x^2+y^2+z^2)-A_2^2\}\times\sqrt{\left\{\frac{\beta^2\gamma^4 {A_1}^2}{\cos^2\theta}+\frac{A_2\beta\gamma^2 A_1}{\cos\theta}+x^2+y^2+z^2\right\}}}\right]+$$

$$\cos\phi I_p\left[\frac{\frac{\sin\theta\beta\gamma^2 A_3 z}{\cos\theta}+\sin\theta\cos\theta(y^2-z^2)-yz-y\beta\gamma^2 A_3+2\,z\,y\cos^2\theta}{A_4\times\sqrt{x^2+y^2-2\frac{\sin\theta\beta\gamma^2 A_3 y}{\cos\theta}+z^2-2z\beta\gamma^2 A_3+\frac{\beta^2\gamma^4 {A_3}^2}{\cos^2\theta}}}\right]-$$

$$2\sin\phi\cos\theta I_p x\left[\frac{\frac{2\beta\gamma^2 A_3}{\cos\theta}-2\,y\sin\theta-2z\cos\theta}{\{4(x^2+y^2+z^2)-A_2^2\}\times\sqrt{\left\{\frac{\beta^2\gamma^4 {A_3}^2}{\cos^2\theta}+\frac{A_2\beta\gamma^2 A_3}{\cos\theta}+x^2+y^2+z^2\right\}}}\right]\} \quad (5)$$

$$\frac{dE_z(x,y,z,\theta,\phi,t_k)}{dt} = \frac{1}{4\pi\varepsilon_0}\left\{-I_p\times\frac{-z+\beta\gamma^2 A_1}{\left[x^2+y^2+z^2-2\frac{y\beta\gamma^2 A_1\sin\theta}{\cos\theta}-2z\beta\gamma^2 A_1+\frac{\beta^2\gamma^4 {A_1}^2}{\cos^2\theta}\right]^{\frac{3}{2}}}+I_p\times\frac{-z+\beta\gamma^2 A_3}{\left[x^2+y^2+z^2-2\frac{y\beta\gamma^2 A_3\sin\theta}{\cos\theta}-2z\beta\gamma^2 A_3+\frac{\beta^2\gamma^4 {A_3}^2}{\cos^2\theta}\right]^{\frac{3}{2}}}\right\}$$
(6)

$$\frac{dE_r(x,y,z,\theta,\phi,t_k)}{dt} = \frac{1}{4\pi\varepsilon_0}\left\{\frac{\sin\phi\, xI_p+\cos\phi\, I_p\left(y-\frac{\beta\gamma^2 A_1\sin\theta}{\cos\theta}\right)}{\left[x^2+y^2+z^2-2\frac{y\beta\gamma^2 A_1\sin\theta}{\cos\theta}-2z\beta\gamma^2 A_1+\frac{\beta^2\gamma^4 {A_1}^2}{\cos^2\theta}\right]^{\frac{3}{2}}}+\frac{-\sin\phi\, xI_p-\cos\phi\, I_p\left(y-\frac{\beta\gamma^2 A_3\sin\theta}{\cos\theta}\right)}{\left[x^2+y^2+z^2-2\frac{y\beta\gamma^2 A_3\sin\theta}{\cos\theta}-2z\beta\gamma^2 A_3+\frac{\beta^2\gamma^4 {A_3}^2}{\cos^2\theta}\right]^{\frac{3}{2}}}\right\}$$
(7)

$$E_r(x,y,z,\theta,\phi,t_k) = E_r(x,y,z,\theta,\phi,t_{k-1}) + \Delta t\times\frac{dE_r(x,y,z,\theta,\phi,t_k)}{dt}$$
(8)

$$E_z(x,y,z,\theta,\phi,t_k) = E_z(x,y,z,\theta,\phi,t_{k-1}) + \Delta t\times\frac{dE_z(x,y,z,\theta,\phi,t_k)}{dt}$$
(9)

where: ε_0 is the permittivity of free space, μ_0 is the permeability of free space, c is the light speed in free space, B_{φ} is the magnetic flux density, Θ is the channel angle respect to z axis, Φ is the observation point angle respect to channel image on the ground surface and it is $\arccos(\frac{y}{\sqrt{x^2+y^2}})$,

$x = r\sin\phi$,

$y = r\cos\phi$,
$r = \sqrt{x^2+y^2}$,
$A_1 = -\beta z + ct_k - E_{ch}$,
$A_2 = -2y\sin\theta - 2z\cos\theta$,
$A_3 = -\beta z - ct_k + E_{im}$,
$A_4 = -2\,yz\sin\theta\cos\theta + \cos^2\theta\,[y^2-z^2] + x^2 + z^2$,
Δt is sampling time,
$t_k = k\Delta t + \frac{\sqrt{r^2+z^2}}{c}$ for k=1,2,...

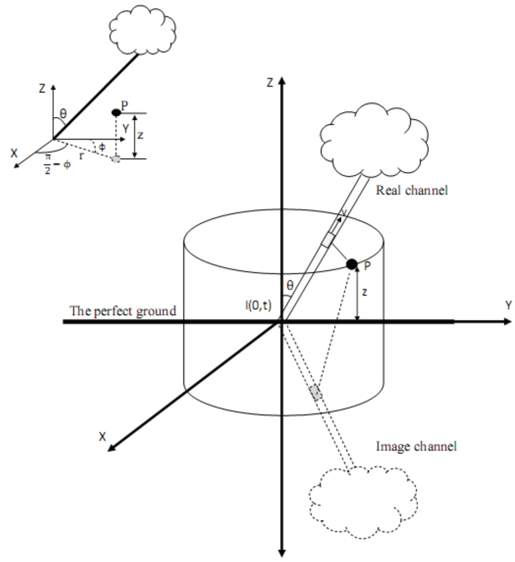

Fig.1.The geometry of problem

It should be noted that all the components of the electromagnetic fields in the time periods are smaller or equal to zero($\leq\frac{\sqrt{r^2+z^2}}{c}$)[12]. In order to increase the accuracy of the results, the small value for Δt should be considered[11].

Results and discussion

The magnetic flux density at different channel angles (θ) and various observation point angles (ϕ) are demonstrated in Fig.2, having set the observation point at r=50m and z=10 m. As Fig.2 reveals that by decreasing channel angle value (θ) with respect to z-axis and under constant value of φ, the magnetic flux density values drop. Alternatively, under constant value of θ, the φ value shows a reverse effect on the values of magnetic flux density. Accordingly by increasing the observation point angle to 90 degrees, the magnetic flux density values are reduced. Additionally, according to Fig.2, the magnetic flux densities have an initial time shift equal to $\frac{\sqrt{r^2+z^2}}{c}$, so the electromagnetic values in the time period less than $\frac{\sqrt{r^2+z^2}}{c}$ are equal to zero[12]. Fig.3 demonstrates that by increasing the channel angle (θ) at constant value of ϕ, the vertical electric field is decreased. Also, by increasing the observation point angle (ϕ) up to 90 degrees, the values of vertical electric fields drop. In addition, Fig.4 illustrates that the horizontal electric field reflects the changes in θ and ϕ. That is, by increasing the channel angle with respect to z-axis at constant value of ϕ, the values of horizontal electric fields increase as well. Note that, the horizontal electric field is as an effective factor to estimate the lightning induced voltage [13-15]. On the other hand, Fig.4 reveals that by increasing observation point angle (ϕ) up to 90 degrees under constant value of θ, the values of horizontal electric fields drop. Moreover, by comparing the Figs.2 with Fig.4, it is observed that the proposed method can also cover electromagnetic fields due to vertical lightning obtained from equations (1) to (4), when θ is set at zero.

Additionally, it is important to mention that the difference in radial distance from the upward and downward charges to the observation point is considered in the proposed equations.

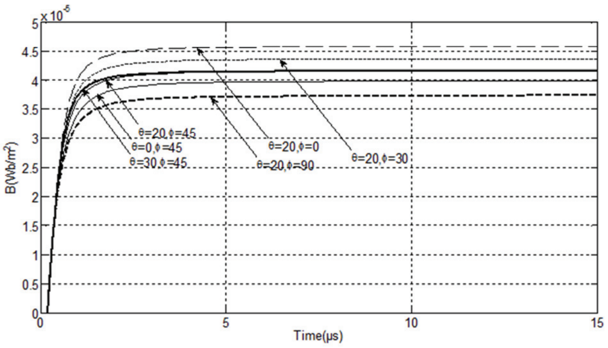

Fig.2.The magnetic flux density changes versus different channel and observation point angles (r=50m, z=10m,$v = 1.2 \times 10^8\ \frac{m}{s}, I_p = 10kA$)

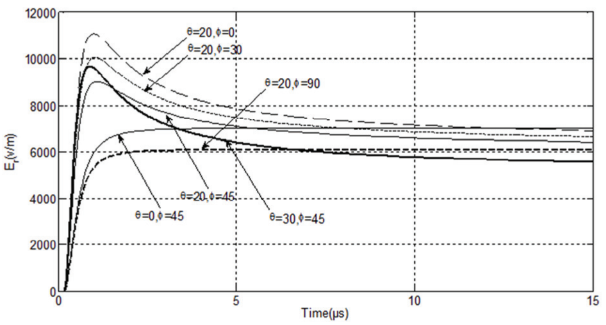

Fig.3.The vertical electric field changes versus different channel and observation point angles (r=50m, z=10m,$v = 1.2 \times 10^8\ \frac{m}{s}, I_p = 10kA$)

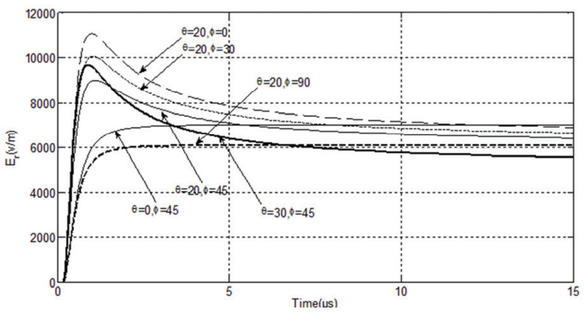

Fig.4.The horizontal electric field versus different channel and observation point angles (r=50m, z=10m,$v = 1.2 \times 10^8\ \frac{m}{s}, I_p = 10kA$)

Fig. 5 demonstrates that the electromagnetic field components act reversely towards an increase in the channel angle with respect to z-axis at two different radial distances from the lightning channel. It is observed that by increasing θ, the peak values of horizontal electric field have a rising trend up to 25 and 35 degrees for r=200m ,50 m, respectively. while the peak values of magnetic flux density and vertical electric field have a downward trend. Also, it is illustrated that by increasing the radial distance from the lightning channel under constant values of θ and ϕ, the peak values of electromagnetic fields are reduced. Furthermore, Fig.6 illustrates the observation point angle (ϕ) has an effect on the peak values of electromagnetic fields under constant value of θ for the two different radial distances from the lightning channel. According to Fig.6, by increasing the observation point angle (ϕ), the peak values of magnetic flux density and vertical electric fields have a downward trend around 90 degrees; however after that, they have a rising trend between 90 to 180 degrees with almost symmetrical shape with respect to the first period. Likewise, it can be perceived from Fig.6 that the peak values of electromagnetic fields at close radial distances from the lightning channel are higher than those ones over long distances. On the other hand, the horizontal electric field has a downward trend reverse to the increasing observation point angle (ϕ) up to around 90 degrees; however, after 90 degrees at closed distance from lightning channel (r=50m) it shows a rising trend which is not symmetrical compared to the first period. Moreover, the peak values of horizontal electric field at r=200m after 90 degrees has a mild downward trend compared to the first period. Consequently, the peak values of horizontal electric fields at closed distances from lightning channel behave in a complete two separate ways compared to the far distances from the lightning channel. Note that, the electromagnetic fields changes vs. different values of observation point angles(ϕ) can be more effective for the calculation of the lightning induced voltage on the power lines when the most coupling models are based on electromagnetic fields at different points along the power line[4, 15-17]. Fig.7 shows that by increasing the return stroke current velocity along the lightning channel, the peak values of electric field components have a downward trend, while the peak values of magnetic flux density has a rising trend. Also, it displays that the declining trend of the peak values for the horizontal electric field is higher than the similar values for vertical electric field. Bear in mind that, the return stroke current velocity is assumed to be constant along the lightning channel and it is usually between c/3 to 2c/3[7].

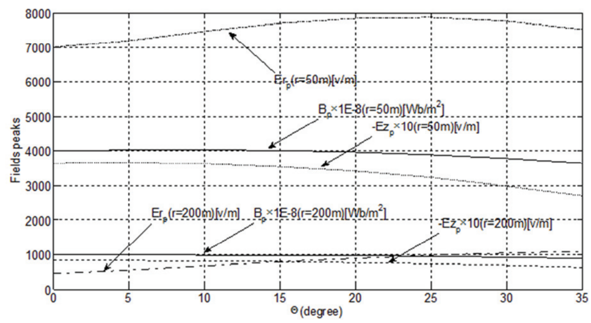

Fig.5.The electromagnetic fields peak values versus different channel angles at two radial distances respect to lightning channel ($\phi = 60^0$, z=10m, $v = 1.2 \times 10^8 \frac{m}{s}$, $I_P = 10kA$)

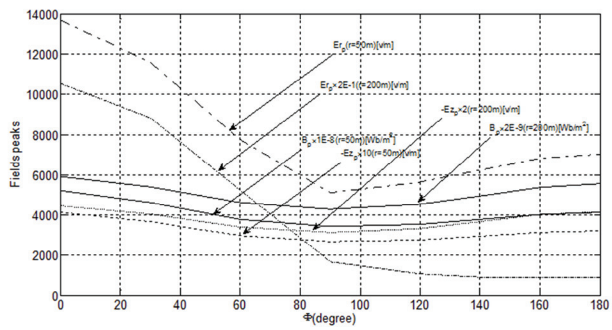

Fig.6.The electromagnetic fields peak values versus different observation point angles at two radial distances respect to lightning channel ($\theta = 30^0$, z=10m, $v = 1.2 \times 10^8 \frac{m}{s}$, $I_P = 10kA$)

As a result, the proposed method indicates some valuable points to be considered as follows:

i- The electromagnetic fields can be estimated by analytical expression directly in the time domain.

ii- The difference in radial distances between the upward and downward charges to the observation point is well thought-out.

iii- The field expressions are considered on different channel angles (θ) and observation point angles (ϕ).

iv- The proposed method can support vertical lightning channel case when θ=0.

v- The proposed equations can be very valuable for prediction of the peak values of electromagnetic fields, while the pulse current is applied as basic assumption.

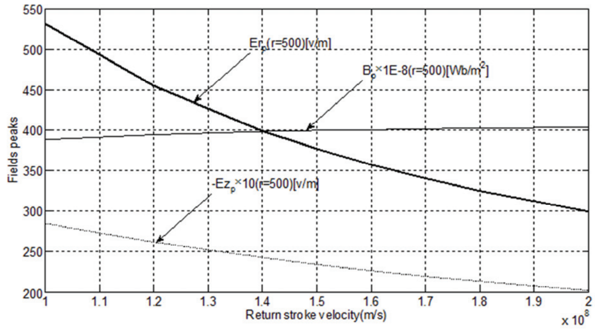

Fig.7.The electromagnetic fields peak values versus different return stroke velocities ($\theta = 20^0, \phi = 30^0$, r=500m, z=10 , $I_P = 10kA$)

Conclusion

In this paper, the general equations for evaluation of electromagnetic fields due to inclined lightning channel are proposed, while the lightning channel usually has an impact on the ground surface with a special angle more than zero with respect to z-axis. Therefore, in order to provide analytical fields expressions, the step return stroke current is applied. Besides, the behaviour of electromagnetic fields versus some effective parameters is considered. Moreover, the proposed fields' equations can support vertical lightning channel case, when θ is set at zero. The proposed method is very beneficial for a quick estimation of electromagnetic fields associated with inclined lightning channel which are more effective on the induced voltage and the selection of suitable protection level on the power lines.

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