**Bitao ZHANG, Youguo PI**

Colleague of Automation Science and Engineering, South China University of Technology, Guangzhou, P.R. China

# Integration of Fuzzy and Sliding Mode Control Based on Fractional Calculus Theory for Permanent Magnet Synchronous Motor

***Abstract***. *The design of integration of fuzzy and sliding mode control (SMC) for the velocity control of permanent magnet synchronous motor (PMSM) is proposed in this paper. A fractional order proportion plus integration sliding surface is chosen. A fuzzy controller is designed using the sliding surface and its derivation as inputs, which is independent on system model. Some simulations and experiments results show that the synthesized performance of the proposed method in this paper based on fractional order manifold is superior to that based on integer order sliding surface.*

***Streszczenie.*** *Przedstawiono projekt zintegrowanego sterowania synchronicznym silnikiem z magnesami stałymi wykorzystujący sterowanie ślizgowe i logikę rozmytą. Symulacje i eksperymenty potwierdziły skuteczność zaproponowanej metody. (**Zintegrowane sterowanie silnikiem synchronicznym wykorzystujące sterowanie ślizgowe i logikę rozmytą**)*

**Keywords:** Velocity control, fractional order sliding manifold, fuzzy control, permanent-magnet synchronous motor (PMSM).
**Słowa kluczowe:** in the case of foreign Authors in this line the Editor inserts Polish translation of keywords.

## Introduction

Permanent-magnet synchronous motor (PMSM) drives play a vitally important role in high performance motion control applications. The field-oriented control, or vector control, is used in the design of PMSM servo drives to achieve smooth starting and acceleration. However, several electromechanical parameters variations (due to temperature variation, saturation and skin effects) and external load disturbances in practical applications lead to degradation of the drive performance [1]. In order to address these drawbacks, several advanced control techniques have been proposed in recent years [2-5]. However, some of these methods are complex and depend on system models. It is difficult to be applied on real-time control system such as machine tool.

Using the differentiation and integration of fractional order or non-integer order in systems control is gaining more and more interests from the systems control community. Due to adding the extra degree of freedom, several fractional order controllers can achieve better control performance than integral order controller [6-9]. However, fractional calculus is usually introduced into classic PID controllers but not other controllers.

It is the motivation of this study to use the advantage feature of strong robustness of sliding mode control to obtain high performance for PMSM drive. In this paper, a scheme of integration of fuzzy and sliding mode control based on fractional calculus theory for PMSM drive is proposed. Fuzzy controller is the main controller, the inputs of which are the sliding mode surface and its differential. Fractional calculus is introduced to sliding mode control to design a fractional order $PI^r$ sliding mode surface. Memory function is specialized in fractional order operator, which makes the operator value decay with time. And this characteristic is good for attenuating the chattering occurring in conventional sliding mode control. Moreover, this method is independent on system model.

The remainder of this paper is organized as follows. First, the design of integration of fuzzy and sliding mode control is introduced in detail. Next, analyze the performance of fractional order sliding mode control based on Bode diagram. Then, some simulations and experiment are carried out to demonstrate the effectiveness of the proposed method on a PMSM servo drive plant. Finally, the conclusions have been draw.

## Design of integration of fuzzy and Sliding Mode Controller

The control objective of the proposed control scheme is to get the state $x(t)$ to track the specific states $x^*(t)$. It means that the control objective is required to drive the tracking error asymptotically to zero for any arbitrary initial conditions and uncertainties. It is well known that the crucial of SMC design are the construction of the sliding surface $s(x), s(x) \in R$ and design of control law.

### Design of fractional order sliding manifold

In conventional SMC design, a sliding surface is chosen by the following equation:

(1) $$s(x) = x(t)(k_p \frac{d}{dt} + k_i)^{n-1}$$

Where: $k_p, k_i$ is positive constant, $n$ is positive integer.

In this study, the order of sliding mode surface($s(x)$) is expended from integer to non-integer. Here, the sliding mode surface is redefined as:

(2) $$s(x) = x(t)(k_p p + k_i)^r, \qquad p = {}_aD_t(\cdot)$$

Where: $a$ and $t$ are the limits and $r$ is the order. ${}_aD_t^r$ is a non-integer (fractional) order fundamental operators defined by

(3) $$ {}_aD_t^r = \begin{cases} \dfrac{d^r}{dt^r} & r > 0 \\ 1 & r = 0 \\ \int_a^t dt^{(-r)} & r < 0 \end{cases}$$

The commonly used definitions for fractional derivatives are Grunwald–Letnikov, Riemann–Liouville and Caputo definitions [10]. In this study the following Caputo definition is adopted for fractional derivative, which allows utilization of initial values of classical integer order derivatives with known physical interpretations.

(4) $$ {}_aD_t^r f(t) = \begin{cases} \dfrac{1}{\Gamma(n-r)} \int_a^t \dfrac{f^{(n)}(\tau)}{(t-\tau)^{r+1-n}} d\tau, & n-1 < r < n \\ \dfrac{d^n}{dt^n} f(t), & r = n \end{cases}$$

Where: $\Gamma(z)$ is the Gamma function with the definition below

(5) $$\Gamma(z)=\int_0^{\infty} t^{z-1}e^{-t}dt$$

From (4), it can be find that direct implementation of fractional-order transfer functions is not simple as that of an ordinary differential equation. Fractional order controllers are infinite dimensional linear filters and all existing implementation schemes are based on the finite dimensional approximations. There are many different methods to find such approximations. Charef [11], Oustaloup [12], and Chen, Y.Q [13] approximations are the well-known approximations of the fractional-order transfer functions. In this study, Chen, Y.Q [13] approximation based on time domain is adopted.

*Design of fuzzy controller*

Due to the parameters variation and unmeasured perturbations, system is not easy to model. But the conventional sliding mode control methods depend on system model, which limits the application to the complex system. In order to apply the sliding mode control approach to non-modeling system, a controller integrated fuzzy and sliding mode control theory is designed. The main objective of the control scheme is to drive the output tracks the reference input rejecting the external disturbances and uncertainties.

The configuration of the proposed fuzzy controller is depicted in Fig.1. The inputs of fuzzy controller are fractional order sliding surface($s(x)$) and its derivative($\dot{s}(x)$). The membership functions of the linguistic terms Negative Big (NB), Negative Middle (NM), Negative Small(NS), Zero (ZE), Positive Small (PS), Positive Middle (PM) and Positive Big (PB) are assigned to the inputs and outputs. The membership functions for the inputs and outputs are triangle type shown in Fig. 2.

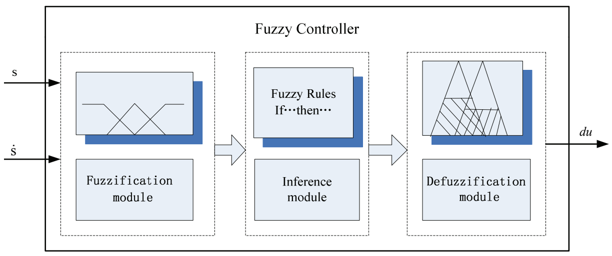



Fig.1. Configuration of the fuzzy controller.

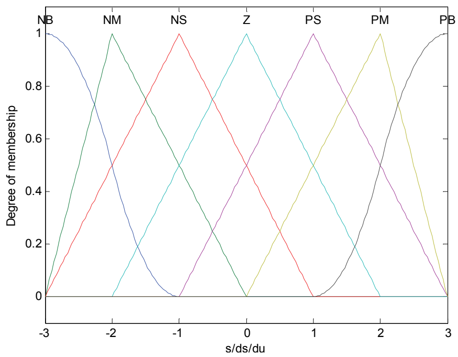



Fig. 2. Fuzzy sets assigned to inputs *s*, *ds* and output *du*

The outputs of fuzzy controller are computed by a mechanism of If-Then rules as following form:

Rule(i): **IF** $x_i$ is $F(x_i)$ and $\dot{x}_i$ is $F(\dot{x}_i)$ **THEN** $y_i$ is $F(y_i)$

In this study, the fuzzy rules are shown in table.1.

Table 1 Rules-base of the fuzzy controller

| du | | s | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | PB | PM | PS | ZE | NS | NM | NB |
| ds | NB | ZE | NS | NM | NB | NB | NB | NB |
| | NM | PS | ZE | NS | NM | NB | NB | NB |
| | NS | PM | PS | ZE | NS | NM | NB | NB |
| | ZE | PB | PM | PS | ZE | NS | NM | NB |
| | PS | PB | PB | PM | PS | ZE | NS | NM |
| | PM | PB | PB | PB | PM | PS | ZE | NS |
| | PB | PB | PB | PB | PB | PM | PS | ZE |

Here, a center average defuzzification, a Mamdani implication in the rule base and a product inference engine are used in designing the defuzzification module. Fuzzy controller output(*du*) can be calculated by the center of area defuzzification as:

(6) $$du=\frac{\sum \gamma_j \mu_j}{\sum \mu_j}$$

where: $\gamma_j$ is the vector containing the output fuzzy center of the membership function of output (*du*). $\mu_j$ represents the membership value of the output to output fuzzy set *j*.

## Performance analysis

In this section, some performances of the proposed controller are analyzed including robustness, convergence property and control performance.

*Robustness and convergence property analysis*

First of all, the proposed fractional sliding surface is existent. It means that wherever the initial states, fuzzy control output drives the initial states converge to sliding manifold. From the fuzzy rules and logic inference system, it can be seen that the control law keeps the following condition

(7) $$s\times\dot{s}<0$$

being satisfied. According to Lyapunov stability theorem, the proposed fuzzy controller is stable.

Then, when the sliding mode occurs, states converge to balance point fast. Due to systems with memory are typically more stable than their memoryless counterpart, fractional order differential equations are, at least, as stable as their integer order counterpart [14]. The proposed fractional order sliding manifold satisfies the following condition

(8) $$\left|\arg(-\frac{k_i}{k_p})\right|=\pi>r\pi/2(0<r<1)$$

According to fractional order differential stability theorem [15], the proposed fractional manifold is asymptotic stable and the components of the states decay towards 0 like $t^{-r}$ .

*Control performance analysis*

The transformation of integer and fractional order sliding surface(2) (In this study, *n* is set to 0.)is expressed as follow

(9) $$G(s)=\begin{cases} k_p+\frac{k_i}{s} & r=1 \\ k_p+\frac{k_i}{s^r} & 0<r<1 \end{cases}$$

In order to make comparison between the performance of integer and fractional order system, the Bode diagrams of (9) is shown in Fig.3. From Fig.3, the performance of integer and fractional order system can be summarized as Table.2.

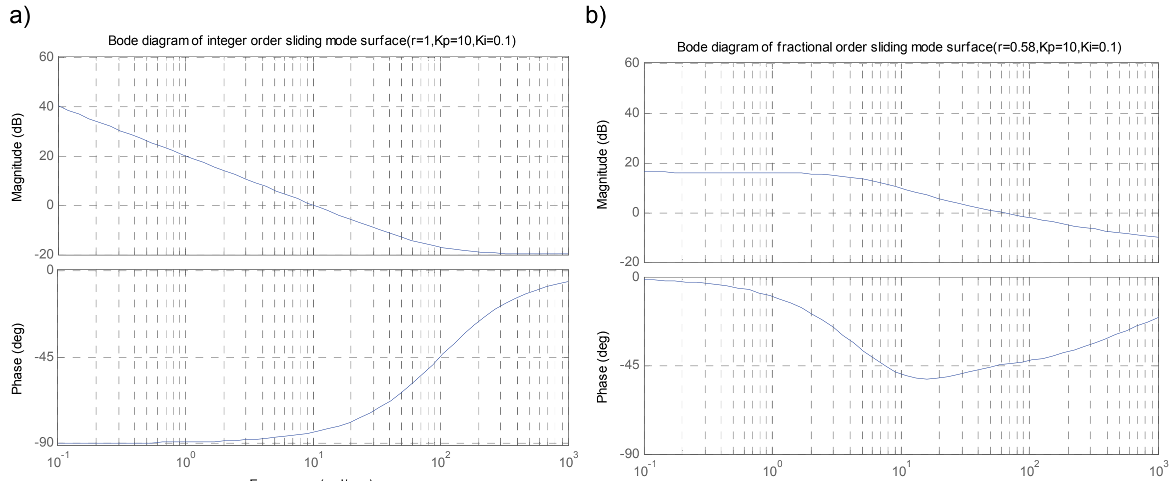



Fig.3 Bode diagram of sliding surface :(a)integer order(b)fractional order

Table.2 Performance comparison between integer and fractional order systems

| Performance | | Frequency band | | |
|---|---|---|---|---|
| | | Low band | Medium band | High band |
| Systems | Integer order | Steady state error coefficients:<br>Position: $K_p = \infty$<br>Velocity: $K_v = const$<br>Acceleration: $K_a = 0$ | Cross-over frequency:<br>$\omega_c = 10 rad/\sec$<br>Band: narrow<br>Phase margin: small | Attenuation ratio:<br>Small<br>Robustness: weak |
| | Fractional order | Steady state error coefficients:<br>Position: $K_p = const$<br>Velocity: $K_v = 0$<br>Acceleration: $K_a = 0$ | Cross-over frequency:<br>$\omega_c = 60 rad/\sec$<br>Band: broad<br>Phase margin: big | Attenuation ratio:<br>Big<br>Robustness:<br>Strong |

According to table.2, it can be find that, steady state error is existent in fractional order system for step response. Fortunately, the steady state error is adjusted by fuzzy controller, which depends on fuzzy rules and logic inference. And this steady state error is good for attenuating the chattering existed in conventional sliding mode control[16]. The Phase margin of fractional order system is bigger than that of integer order system, which means the stability of fractional order system is stronger than that of integer order system. Finally, the attenuation ratio of fractional order system in high frequency band is bigger than that of integer order system. It means that the robustness, rejecting high frequency disturbance, of fractional order system is stronger that of integer order system. So, when the system reaches the sliding mode state, the fuzzy controller with fractional order sliding manifold achieves better performance than that with integer order sliding surface.

**Simulations and experiments**

## Simulations and results discussion

A comparison of tracking performance is carried out between the integer order sliding surface and proposed fractional order manifold on a second class of system. Simulates are based on"matlab7.1". The controlled system and sliding surface, including fractional order and integer order, are chosen as(10)and(11) respectively.

Considering a second class system

$$\ddot{x} = k\dot{x} - x + u \tag{10}$$

where $k \in R^-$.

A candidate sliding manifold is chosen as follow(when *r* is integer, sliding manifold becomes conventional integer order sliding surface.)

$$s(x) = x + 550 {}_0D_t^r x \tag{11}$$

The unit step responses of the proposed fractional order control scheme and the conventional integer sliding surface are shown in Fig. 4. From Fig.4, it can be seen that, the rise time of fuzzy controller based on integer order sliding surface is a little faster than that based on fractional order sliding surface. However, there is almost no chattering with the fuzzy controller based on fractional order sliding surface.

The simulations results of robustness rejecting the parameter variation are shown in Fig.5. It can be seen that the proposed control scheme is robust rejecting the parameter variation.

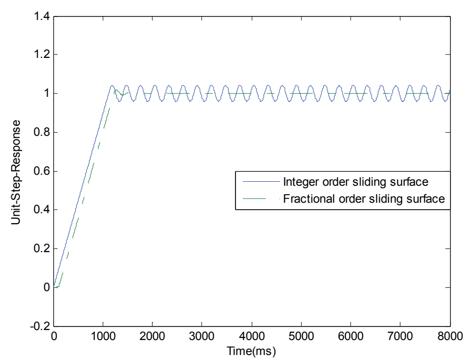



Fig. 4. The unit step response of fuzzy controller with fractional and integer order sliding surface (k=-60)

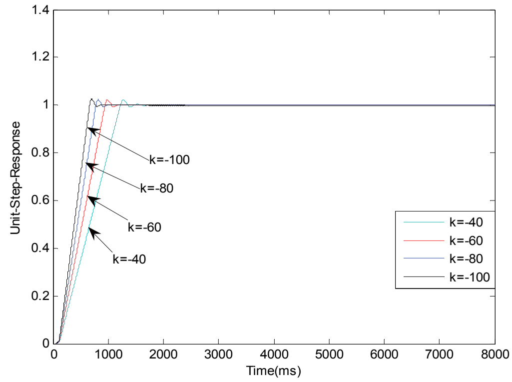



Fig.5.Unit step response with different system parameter(k)

*Experiment and results discussion*

Some experiments are designed to illustrate the control performance of the proposed control scheme. These experiments are based on the velocity-loop control of the permanent magnet synchronous motor(PMSM) drive system. Then, in order to find the impact of system order($r$) on control performance, some different values of system orders are given for experiments. The experiments are carried out using the Ti Code Composer Studio (CCS) software in PC. The data is sampled, used this software, from encoder to control board, and then load these data into "matlab" package to analyze the results. And the control board is based on"TMSC320F2812" DSP. The configuration of the velocity-loop control of the PMSM drive system and plant are shown in Fig.6 and Fig.7 respectively.

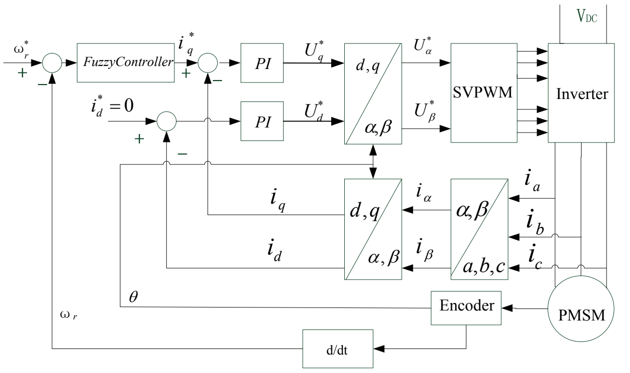



Fig. 6. Configuration of the velocity-loop control of the PMSM drive

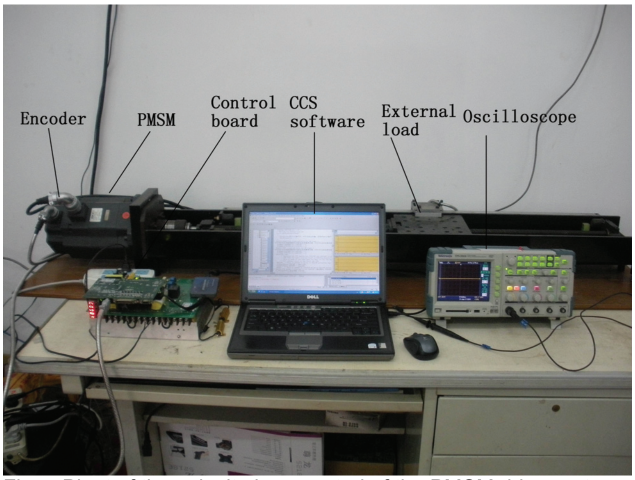
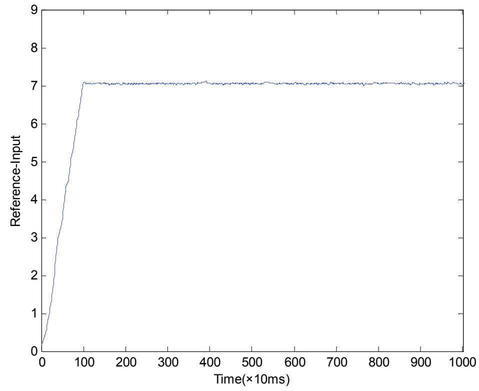


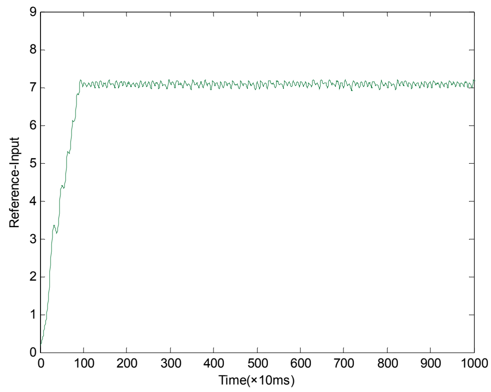

Fig.7. Plant of the velocity-loop control of the PMSM drive system

The experiments results of step response are shown in Fig.8. From Fig.8, it can be seen that the transient state of step response based on fractional order manifold is smooth and steady. But there are some swings in the acceleration phase of step response based on integer order sliding manifold. Moreover, the amplitude of chattering is smaller with fractional order surface than that with integer order surface. This is congruent according to theory. The experiments results show that the proposed control scheme based on fractional calculus achieves better tracking performance than convention control method based on integer order calculus.

a) (r=-0.38)




b) (r=-1)




Note: Since the TMSC320F2812 is a fixed point DSP, the real speed of reference input shown in figure is 500r/min

Fig.8. Experiments results of fuzzy controller with different sliding surface:(a)fractional order(b)integer order

The impacts of different order on control performance are described in Fig.9, which shows that the smaller absolute value of order is, the shorter rise time of step response is. However, when the absolute value of order reduces or increases to a certain limit, chattering of the speed intensifies in the transient state. Moreover, when the system state reaches sliding mode state, the bigger the absolute value of order is, the bigger chattering amplitude occurs.

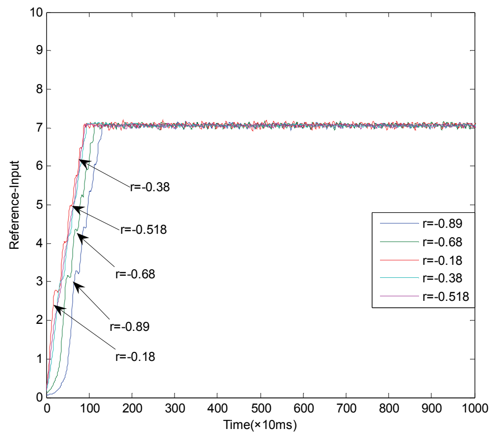



Fig. 9. Impacts of different fractional order(r) sliding manifolds

## Conclusions

A fuzzy control scheme based on fractional order sliding surface has been proposed in this paper. The theoretical analyses and simulations show that the proposed control method achieves better tracking performance compared with the conventional fuzzy control based on integer order sliding manifold. And the proposed control scheme is robust rejecting the parameter variation. Experiments results based on PMSM drive system also demonstrate the tracking error of velocity loop of servo converges to zero and the stability of the system can be guaranteed. Moreover, the proposed control approach is independent on system model.

The contribution of this study includes: 1) propose a fractional order sliding surface and analyze its performance. 2) Successfully apply the proposed control scheme on the speed control of PMSM.

## REFERENCES


[1] Fayez F. M. El-Sousy.Hybrid H∞-Based Wavelet-Neural-Network Tracking Control for Permanent-Magnet Synchronous Motor Servo Drives.*IEEE Transactions on Industrial Electronics*, 57(2010), no. 9, 3157-3166,.

[2] Changliang Xia, Chen Guo, and Tingna Shi. A Neural-Network-Identifier and Fuzzy-Controller-Based Algorithm for Dynamic Decoupling Control of Permanent-Magnet Spherical Motor. *IEEE Trans Ind Electron*, 57(2010), no. 8, 2868-2878.

[3] S. Li and Z. Liu,Adaptive speed control for permanent-magnet synchronousmotor system with variations of load inertia,*IEEE Trans. Ind.Electron*, 56(2009), no. 8, 3050–3059.

[4] A.R. Ghafari-Kashani J. Faiz M.J. Yazdanpanah. Integration of non-linear H∞and sliding mode control techniques for motion control of a permanent magnet synchronous motor, *IET Electr. Power*, 4(2010),no. 4, pp. 267–280.

[5] Teresa Orlowska-Kowalska, Mateusz Dybkowski, and Krzysztof Szabat. Adaptive Sliding-Mode Neuro-Fuzzy Control of the Two-Mass Induction Motor Drive Without Mechanical Sensors. *IEEETrans. Ind. Electron,* 57(2010), no. 2, 553–564.

[6] HongSheng Li, Ying Luo, and YangQuan Chen,A Fractional Order Proportional and Derivative (FOPD) Motion Controller: Tuning Rule and Experiments ,*IEEE Transactions on Control Systems Technology*, 18(2010), no. 2, 516-520.

[7] Ying Luo, YangQuanChen, Hyo-SungAhn, YouGuoPi, Fractional order robust control for cogging effect compensation in PMSM position servo systems: Stability analysis and experiments, *Control Engineering Practice*, 18(2010),no.2, 1022–1036.

[8] Ching-Hung Lee, Fu-Kai Chang,Fractional-order PID controller optimization via improved electromagnetism-like algorithm, *Expert Systems with Applications*, 37(2010), 8871–8878.

[9] Concepcio A. Monjea, Blas M. Vinagreb, Vicente Feliuc, YangQuan Chen,Tuning and auto-tuning of fractional order controllers for industry applications *Control Engineering Practice*, 16(2008), 798–812,.

[10] I. Podlubny, Fractional Differential Equations, Academic Press, NewYork, 1999.

[11] A. Charef, IEE Proc. *Control Theory*. *Appl*. 153(2006),no.6, 714-723,.

[12] A. Oustaloup, F. Levron, B. Mathieu, F.M. Nanot, IEEE Trans. Circuits Syst. I: Fund. *Theory Appl*. 47(2000).no.1, 18-25.

[13] Chen, Y. (2008a). Impulse response invariant discretization of fractional orderintegrators/differentiators compute a discrete-time finite dimensional ztransfer function to approximate sr with r a real number. Category: Filter Designand Analysis, MATLAB Central. http://www.mathworks.com/matlabcentral/fileexchange/loadFile.do objectId D 21342 objectType D FILE.

[14] D. Matignon, Stability results for fractional differential equations with applications to control processing, Computational Engineering in Systems and Application multi-conference, IMACS, in: *IEEE-SMC Proceedings*, Lille, France, 2(1996), 963–968,.

[15] E. Ahmed, A.M.A. El-Sayed, H.A.A. El-Saka, Equilibrium points, stability and numerical solutions of fractional-order predatorprey and rabies models, *J. Math. Anal*. 325( 2007), 542–553.

[16] Baik, I. C., Kim, K. H. & Youn, M. J. Robust nonlinear speed control of PM synchronous motor using boundary layer integral sliding mode control technique. *IEEE Transactions on Control Systems Technology*, 8（2000）,47-54.



***Authors**: dr Bitao.Zhang ,Colleague of Automation Science and Engineering, South China University of Technology, Guangzhou, P.R. China, E-mail: z.bitao@mail.scut.edu.cn; prof. Youguo.Pi, E-mail: billjhontaoi@163.com.*