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Mathematical model of the electromagnetic accelerator

***Abstract**. The model of the electromagnetic accelerator is offered as a solenoid with ferromagnetic core. The mathematical model is formed on the basis of the unique magnetic flux without dividing into a basic flux and leakage flux. The mathematical model of accelerator is formed in the coordinates of contour magnetic fluxes and currents of branches as a system differentially eventual equations.*

***Streszczenie.** Analizowano model przyśpieszeniomierza elektromagnetycznego w postaci solenoid z rdzeniem ferromagnetycznym. Analizowano strumień magnetyczny bez podziału na strumień cewki i strumień rozproszony. (**Model matematyczny przyśpieszeniomierza elektromagnetycznego**)*

Keywords: electromagnetic energy, solenoid, magnetic flux, eddy current.
Słowa kluczowe: energia elektromagnetyczna, solenoid, strumień magnetyczny

Introduction

The modeling of electromagnetic and kinetic energies transformation is an important task to decide during the developing of electrical engineering devices. In this article the mathematical model of the electromagnetic accelerator is examined as a solenoid with ferromagnetic core. In such accelerator utilizes an electromagnetic force to propel an core, when we apply voltage in the form of rectangular impulse. Mathematical model of electromagnetic accelerator is formed as a circuit model on the basis of the unique magnetic flux without dividing into basic and leakage fluxes. This model is developed based on the following assumptions:

The mathematical formulation of the problem

– taking into account the magnetic field symmetry of the accelerator, the probed area is broken on such elementary volumes as cylinders (Fig. 1). These volumes are represented in the chart of magnetic core as lumped magnetic resistances;

– eddy currents in ferromagnetic core are equivalently presented as independent currents in concentric hollow cylinders;

– the temperature influence on electric and magnetic conductivity is neglected.

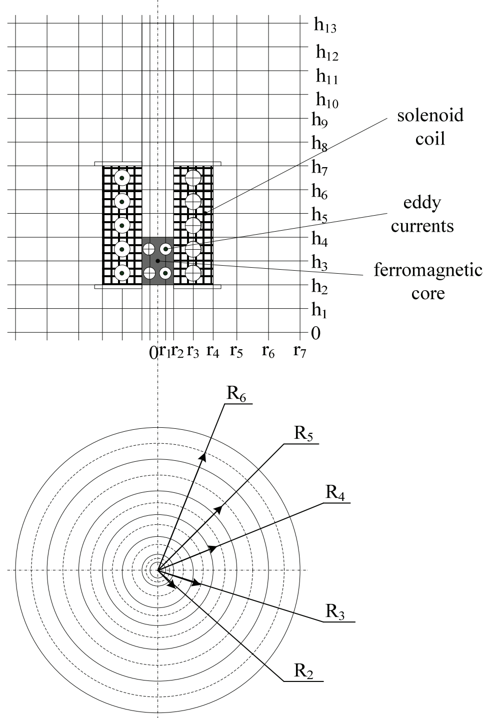

Fig. 1. Space division of the probed area into elementary volumes

The mathematical formulation of the problem

Magnetic flux which penetrates the volumes flows in vertical and horizontal directions. The magnetic resistances of these volumes are determined as follows

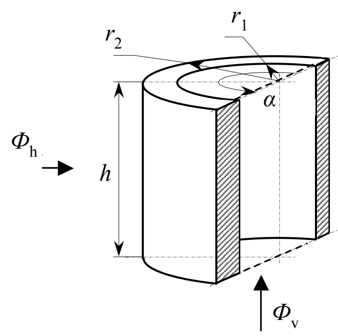

Fig. 2. The elementary volume and its parameters

(1) $$R_{mv} = h \Big/ \left(\mu_o \mu \int_{\alpha_1}^{\alpha_2} \int_{r_1}^{r_2} r\, d\alpha\, dr \right) = \frac{2h}{\left(\mu_o \mu \pi \left(r_2^2 - r_1^2\right)\right)};$$

(2) $$R_{mh} = \int_{R_1}^{R_2} dR / \left(\mu_o \mu h \alpha R\right) = ln\left(R_2 / R_1\right) / \left(\mu_o \mu h \alpha\right);$$

where h, α, r_1, r_2, R_1, R_2 – geometrical sizes of elementary volume; μ – material relative magnetic permeability of elementary volume; μ_0 – permeability of vacuum.

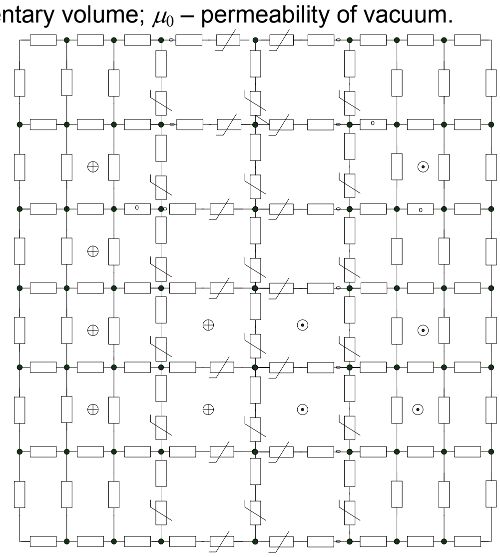

Fig. 3. The model scheme of the magnetic circuit

The mathematical model of the accelerator is formed in the coordinates of loop magnetic fluxes and currents of branches as a system of differential-finite equations:

(3) $$\Gamma_m \vec{U}_m(\Phi) - \mathrm{W}_t \vec{i} = 0\,;$$

(4) $$\mathrm{W}\, d\vec{\Phi}_\kappa / dt + \mathrm{R}\vec{i} - \vec{u} = 0\,;$$

where Γ_m – $(q \times s) \times p_m$-dimensional second incidence matrix of the magnetic circuit (here $q \times s$ - dimension of the magnetic circuit grid); p_m - number of the graph edges of the magnetic circuit; $\overline{U}_m(\Phi) - p_m$-dimensional column vector of the magnetic tensions of branches; $\overline{\Phi}_\kappa$ – column vector of loop magnetic fluxes; $\mathbf{W}$ – n x $(q$ x $s)$-dimensional matrix turns the elementary loops of a magnetic circuit; $\vec{i} = (i_1, i_2, \ldots i_n)_t$ – column vector of the solenoid current and eddy currents; $\vec{u} = (u_1, 0, \ldots 0)_t$ – column vector of the solenoid tensions and equivalent winding voltages; $\mathbf{R} = \mathrm{diag}(R_1, R_2, \ldots R_n)$ – resistance diagonal matrix.

To integrate the differential equations let us use the implicit Euler method, and in order to solve the nonlinear equations - Newton method. So, on the $k+1$-th step of integration on $p+1$-th iteration cycle we have discrete model (3), (4) written in the following vector form:

(5) $$\begin{Vmatrix} \Gamma_m \left(\partial \vec{U}_m / \partial \vec{\Phi}\right)^{(p)}_{k+1} \Gamma_{m,t} & -\mathrm{W}^{(p)}_{t\,k+1} \\ a_0 h^{-1} \mathrm{W}^{(p)}_{k+1} & \mathrm{R} \end{Vmatrix} \times \begin{Vmatrix} \Delta\vec{\Phi}^{(p)}_{k,k+1} \\ \Delta\vec{i}^{(l)}_{k+1} \end{Vmatrix} =$$

$$= \begin{Vmatrix} \Gamma_m U_m\left(\vec{\Phi}^{(p)}_{\kappa\,k+1}\right) - \mathrm{W}^{(p)}_{t\,k+1} \vec{i}^{(p)}_{k+1} \\ q^{-1} \mathrm{W}^{(p)}_{k+1} \left(\alpha_0 \vec{\Phi}^{(p)}_{k,k+1} + \sum_{s=1}^{n} \alpha_s \vec{\Phi}^{(p)}_{k,k+1-s} \right) + \mathrm{R}\vec{i}^{(p)}_{k+1} - \vec{u}^{(p)}_{k+1} \end{Vmatrix}$$

where: α_o, α_s – coefficients of the BDF method; k – number of integration step ; q – integration step width of the BDF method; p – ordinal number of iteration step.

The next approximations will be calculated using the following expression:

(6) $$\left(\vec{\Phi}^{(p+1)}_{k,k+1}, \vec{i}^{(p+1)}_{k+1}\right)_t = ;$$
$$= \left(\vec{\Phi}^{(p)}_{k,k+1}, \vec{i}^{(p)}_{k+1}\right)_t - \left(\Delta\vec{\Phi}^{(p)}_{k,k+1}, \Delta\vec{i}^{(p)}_{k+1}\right)_t .$$

Ferromagnetic core is magnetized in magnetic field of the solenoid. Magnetization of the ferromagnetic core is calculated by the expression

(7) $$J = (\mu - 1) H_v$$

where: H_v – magnetic intensity in the ferromagnetic core along the solenoid axis.

Between the ferromagnetic core and magnetic field of solenoid the interaction force appears. To calculate the interaction force the magnetized core is represented as circular current I_{magn} that flows through the lateral surface of the core

(8) $$I_{magn} = J h_c\,;$$

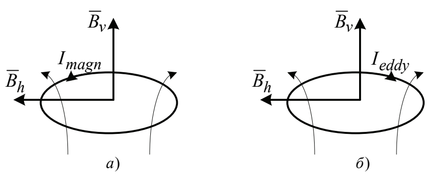

Fig. 4. Representation of interaction of the solenoid magnetic field with magnetization current I_{magn} and eddy current I_{eddy}

From the Fig. 4 it can be seen that the axial component of solenoid magnetic field B_v causes the Ampere force, which tries to stretch (squeeze) the current ring. Therefore, it is not a reason of the core movement. The horizontal component of solenoid magnetic field B_h causes the Ampere force, directed along the axis of solenoid. Therefore, the ferromagnetic core movement is a result of interaction between the horizontal component of the solenoid magnetic field, current magnetization and eddy currents.

Thus, the electromagnetic force that causes the core motion is determined by the expression

(9) $$F_{eм} = -\left(B_h I_{magn} l_1 + B_h I_{eddy} l_2\right)$$

where: $l_1 = 2\pi R_1$ – length of the ring magnetization current; $l_2 = 2\pi R_2$ – length of the ring eddy current.

Motion equation of the ferromagnetic core looks as

(10) $$m\, d^2 x / dt^2 = F_{eм} - kmg$$

where: m – mass of core; k – coefficient of friction.

It should be noted that when the core moves, its space position changes. Movements of the core requires recalculation of the matrix elements W , R , $\partial \overline{U}_m / \partial \overline{\Phi}$ at each integration step because they are functions of coordinates.

Obtained results of calculation of transient processes

To verify the adequacy of the accelerator mathematical model the test problem was calculated. Calculation results are presented in corresponding figures 5-9.

In Fig. 5 the distribution of the solenoid magnetic field calculated under the following conditions is shown: solenoid is switched on direct voltage; ferromagnetic core is located at the origin of h1 (Fig.1); core is fixed. Magnetic fluxes of the branches are presented in the Fig. 5 in the form of layers. Odd layers are magnetic flux horizontal branches, and even layers – magnetic flux vertical branches. The colour of rectangular segment reflects the amount of magnetic flux branches. The value of the magnetic flux changes from low (blue) to high (red).

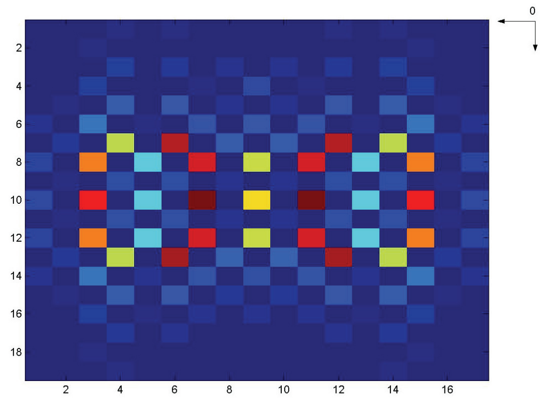

Fig. 5. Distribution of the magnetic field

In the Fig. 6 the change of the coordinate beginning of ferromagnetic core during its movement is shown. This movement is caused by switching of the solenoid by direct voltage. Electromagnetic force changes its direction when it crosses the middle of core solenoid. The core is stopped in the center of solenoid after several oscillations. The same results were observed during the physical experiment, when the solenoid was connected to the DC voltage source. The system with such position of the ferromagnetic core has minimum of energy.

The fig.7 shows the magnetic field distribution in time when the ferromagnetic core is at the center of solenoid.

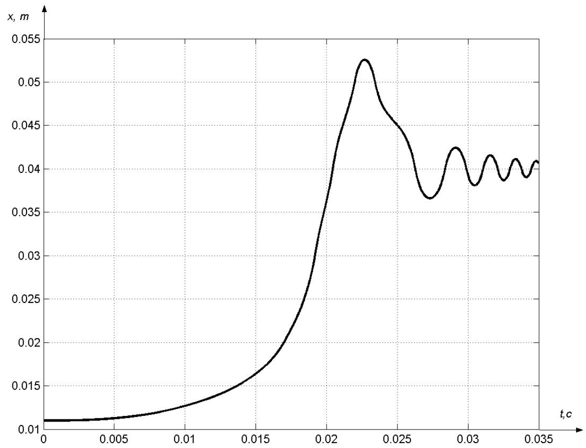

Fig. 6. Coordinates of ferromagnetic core during its movement when we apply by direct voltage

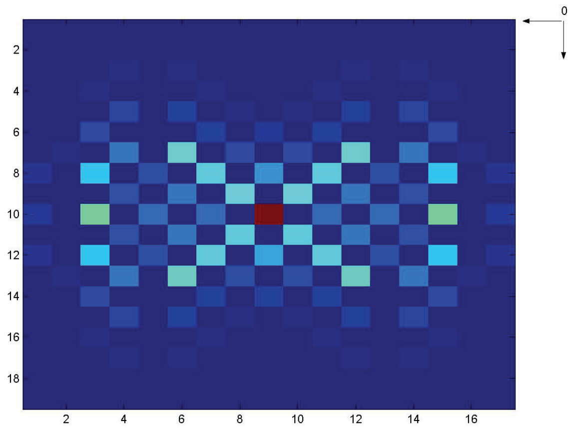

Fig. 7. Distribution of the magnetic field when the core is at the center of solenoid

When we apply voltage in the form of rectangular impulse the ferromagnetic core will be pulled out of the solenoid. The change of the coordinate of the ferromagnetic core in this case is shown in the Fig. 8.

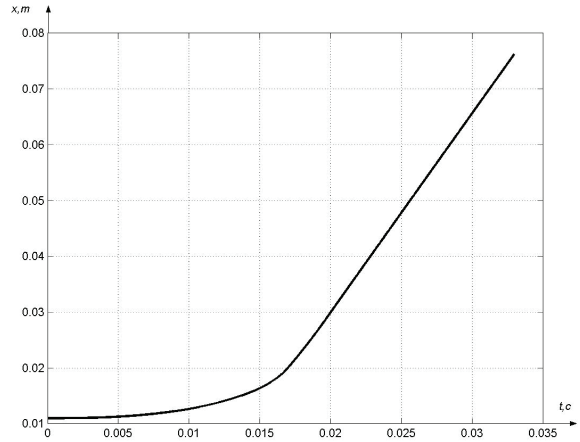

Fig. 8. Coordinates of the core in case of rectangular pulse voltage applying

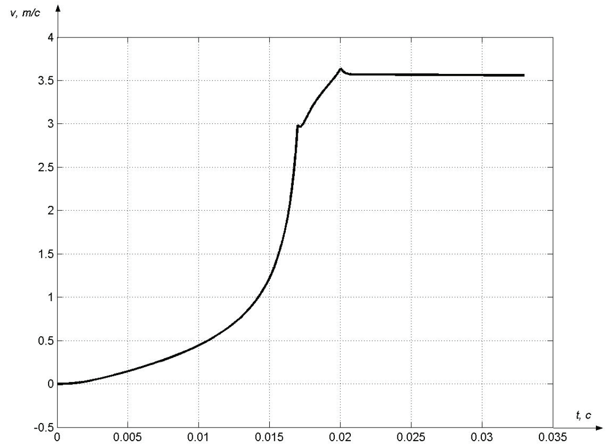

Fig. 9. Speed of the ferromagnetic core

Conclusions

So, the proposed model of the accelerator can be used for optimization of its parameters in order to obtain the maximum acceleration of the core.

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