

Calculation of the transient currents in transformers using field-circuits methods

Abstract. The Field-Circuit Method (FCM) and the new Modified Time Stepping Finite Element Method (MTSF) as well as classical Time Stepping Finite Element Method (TSF) have been used for the transient current calculations in transformers. The execution times for MTSF and TSF methods have been compared. The novel method (MTSF) is about two times faster than the classical (TSF) method. Two constructions of the 1-phase transformer have been studied. The first one is an amorphous modular transformer (T1) and the second one is the transformer with laminated carbon steel core (T2). The calculated inrush currents were compared with the measured ones.

Streszczenie Do obliczeń przebiegów czasowych prądów w transformatorach wykorzystano: Metodę Polowo-Obwodową (MPO), Zmodyfikowaną Metodę Kroku Czasowego (ZMKC) oraz klasyczną Metodę Kroku Czasowego. Porównano czasy obliczeń dla klasycznej Metody Kroku Czasowego (MKC) oraz dla ZMKC wykorzystującej dwuwymiarową analizę polową. ZMKC jest około dwa razy szybsza od MKC. Analizowano dwa 1-fazowe transformatory. Pierwszy to transformator amorficzny budowy modułowej T1, natomiast drugi to transformator tradycyjny z rdzeniem z blachy elektrotechnicznej T2. Otrzymano dobrą zgodność obliczeń z pomiarami. (Obliczenia przebiegów czasowych prądów w transformatorach przy użyciu metod polowo-obwodowych).

Słowa kluczowe: Metoda Kroku Czasowego, Metoda Polowo-Obwodowa, prąd załączania transformatora.

Keywords: Modified Time Stepping Method, Field-Circuit Method, inrush current of the transformer.

Introduction

The small power transformers are switch on/off in their operation. In the switching on moments a non-sinusoidal transient inrush current arises. In some cases, the magnitude of the inrush current is several times higher than the operational load current. Its value depends mostly on the voltage magnitude of the supplying source and the residual flux in the transformer core, as well as the current flux derivative called dynamic inductance L_d [5].

The benefits of the computer simulations and the current determination are well recognized. Knowledge of their values is also important for correct determination of the shelters parameters [1]. A accurate approximation of the inrush current requires detailed information regarding the transformer parameters [2]. If its physical model is approachable, the equivalent circuit parameters can be simply obtained from its measurements. However, during the transformer designing they can be obtained from computer simulations, e.g. from magnetic field calculations [3, 4].

The transients of transformers were analyzed in many works e.g. [3, 5]. Not always the residual (remanent) flux is considered for the transformer soft magnetic material core. In some cases, the hysteresis effects were also taken into account [3]. In this work we carried out the calculations using the equivalent circuit parameters, which have been obtained from numerical analysis. We included different values of the residual flux.

Using magnetic field analysis, the non-linear characteristic of the dynamic inductance, as a function of magnetizing current, has been determined. Also, the leakage inductances have been computed. In the computations the material characteristics have been included and the magnetic residual flux has been indirectly taken into account for the initial value of the magnetizing current fixing.

In this work have been compared commercial programs with program created by authors as part of the grant of Polish Ministry of Science and Higher Education. In this algorithm is used modified Time Stepping Method. This modification consists in using calculated magnetic flux values in every step to determine inductance.

Analyzed objects

To investigate the inrush currents, the new construction of 1-phase transformer with amorphous modular core (T1) and a conventional construction of 1-phase small power transformer (T2) have been chosen [5] (Fig. 1).

Each column of the amorphous transformer consists of two hollow cylinders (toroids). Its yokes have rectangular shape with two rounded thinner sides (Fig. 1a) [6, 7]. Contrary to the T1, the T2 transformer core is assembled from packages of thin sheets high-grade steel. In Fig. 1, the main dimensions of the transformers and the assumed Cartesian coordination systems are shown.

For the T1 transformer, the turn number of the windings is $N=232$, while for the conventional one (T2), the winding is wounded with $N=182$ turns.

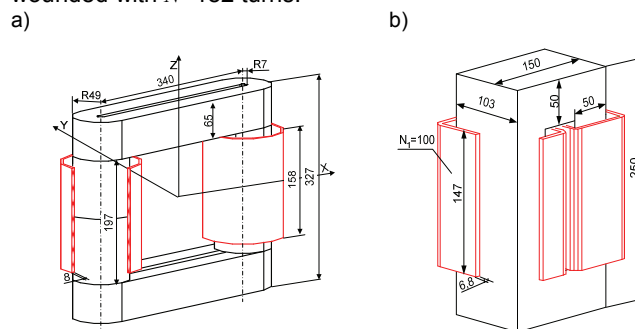


Fig.1. Outline of the analyzed transformers with: a) amorphous core - T1, b) traditional core - T2.

Mathematical models Field-Circuit Method (FCM)

Generally, the inrush current of the transformer was analyzed by using Field-Circuit Method (FCM). In this model the calculations of the transients are based on the transformer equivalent circuit (Fig. 2), which is described by the following system of the differential equations [3, 8]:

$$(1) \quad \begin{cases} L_s \frac{di}{dt} = u - R \cdot i + R_{Fe} \cdot (i_\mu - i) \\ L_d(i_\mu) \frac{di_\mu}{dt} = -R_{Fe} \cdot (i_\mu - i) \end{cases}$$

In the expressions (1), the leakage inductance value is $L_s=732 \mu\text{H}$ for T1 and $L_s=177 \mu\text{H}$ for T2, the RMS value of the excitation voltage $U=220 \text{ V}$, the coil resistance value - $R=0.24 \Omega$ for T1 and $R=0.136 \Omega$ for T2. The core losses resistances of $R_{Fe}=1913 \Omega$ (for T1) and $R_{Fe}=2186 \Omega$ (for T2) have been determined. The currents i and i_μ are unknown functions. The nonlinear dynamic inductance $L_d(i_\mu)$ should be determined by the finite element (FEM) calculations.

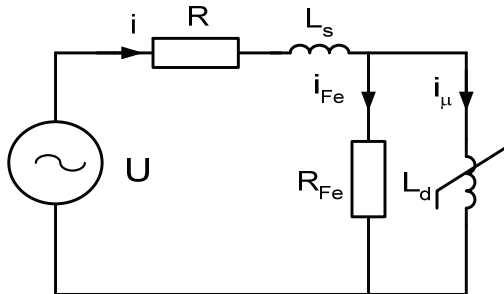
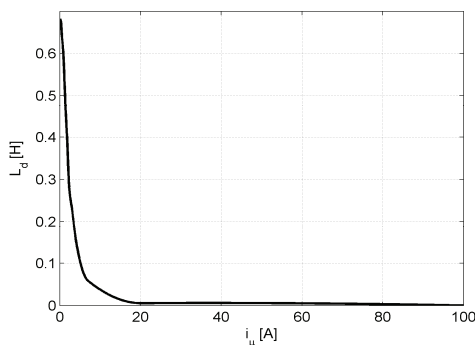


Fig.2. Equivalent circuit of the transformer under no load state.

As was mentioned in the section 1, the residual flux value B_r and the flux magnetic path l_μ influence on the current value i_μ

$$(2) \quad i_\mu(0) = \frac{B_r \cdot l_\mu}{\mu \cdot N}$$

a)



b)

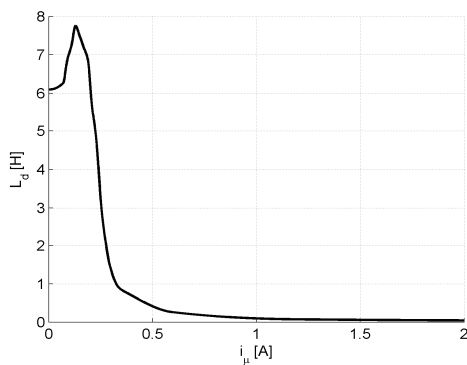


Fig.3. Dynamic inductance L_d vs. coil excitation current: a) T1 transformer, b) T2 construction.

The leakage inductance L_s value of the transformer winding has been determined by using the field analysis under short-circuit state. Resistance R_{Fe} has been determined from measurements under no-load state, whereas the dynamic inductance L_d curve has been created using the field models. The assumed excitation current values in the models has changed from 0.2 to 100 A. In Fig. 3 the dynamic inductance verso excitation current is presented.

Time Stepping FEM (TSF) and its modification –MTSF method

In the second mathematical model (called MTSF) the governing expressions for the magnetic field is represented by Maxwell's equation with the magnetic vector potential \vec{A} . If the eddy currents in the iron core are neglected, the magnetic field can be expressed by the partial differential equation (PDE)

$$(3) \quad \nabla \times \left(\frac{1}{\mu(\vec{B})} \nabla \times \vec{A} \right) = 0$$

where $\mu(\vec{B})$ is the nonlinear permeability of the material.

In the area of the windings, the magnetic field can be governed by the equation

$$(4) \quad \nabla \times \left(\frac{1}{\mu(\vec{B})} \nabla \times \vec{A} \right) = \vec{J}$$

where \vec{J} is the total current density.

The Galerkin's approach is the most popular method for matrix of elements formulation. In this proposed model, the weighting functions are the same as the shape functions for this particular weighted residual method. According to the Galerkin's method, the magnetic vector potential can be expressed as

$$(5) \quad A = \sum_{j=1}^3 N_j A_j,$$

where N_j are the element shape functions and the A_j are the approximations of the vector potential at the nodes of the elements. Thus, the formulation for the field, in the current carrying regions, is expressed by:

$$(6) \quad \iint_s \left[\frac{\partial N_i}{\partial x} \frac{\partial}{\partial x} \frac{1}{\mu} \sum_{j=1}^3 N_j A_j + \frac{\partial N_i}{\partial y} \frac{\partial}{\partial y} \frac{1}{\mu} \sum_{j=1}^3 N_j A_j + N_i \left(\frac{i}{S} \right) \right] ds = 0$$

For the other subregions of the transformer the functional is expressed

$$(7) \quad \iint_s \left[\frac{\partial N_i}{\partial x} \frac{\partial}{\partial x} \frac{1}{\mu} \sum_{j=1}^3 N_j A_j + \frac{\partial N_i}{\partial y} \frac{\partial}{\partial y} \frac{1}{\mu} \sum_{j=1}^3 N_j A_j \right] ds = 0$$

Taking into account the average length path l of the flux in the coil and its cross-section S , the electric circuit can be described by the:

$$(8) \quad u = \frac{l}{S} \left[\iint_{S_1} N_i \frac{\partial A}{\partial t} ds - \iint_{S_2} N_i \frac{\partial A}{\partial t} ds \right] + Ri + L_s \frac{di}{dt} = 0$$

The integrals in (8) refer the region S_1 with the positive direction of the excitation current and the region S_2 with the negative current direction.

Contrary to the commercial computational methods of the time variable FEM [9], we have solved the equation (8) with our software. For the field calculation with the discretized functionals we use the FEMM software. In this modeling method the 2D field calculations have been done. In the case of amorphous transformer, according to its geometry, it is difficult to obtain real magnetic field distribution in 2D. Therefore, depth of this object was fitted for magnetic flux value.

Our method characterizes simplicity and multitask system. Thus the computational platform don't need to execute so much iterations like in the classic TSF algorithm. Contrary to the TSF method, the values of the dynamic inductance, which concern the field values in the step "i", are stored in the separate matrix, which is located in the RAM memory. Thus, we don't need to compute integrals within the eq. 8 in each step of the computational process. It is a great advantage of the MTSF, because only at several time steps the problem must be solved.

Calculation results

Amorphous transformer (T1) - inrush currents

In this paper the supplying voltage for the primary winding under the no load state of the transformer has been assumed. However, the magnetic flux density distribution has been calculated for many values of the current excitation. For example, in Fig. 4 the flux density is presented. The field analysis is devoted to the dynamic inductance determination.

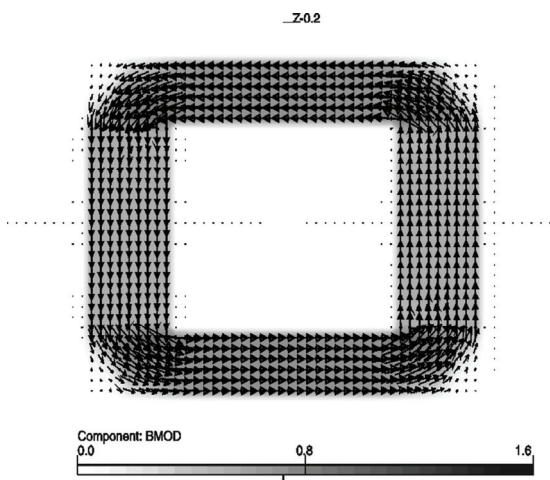
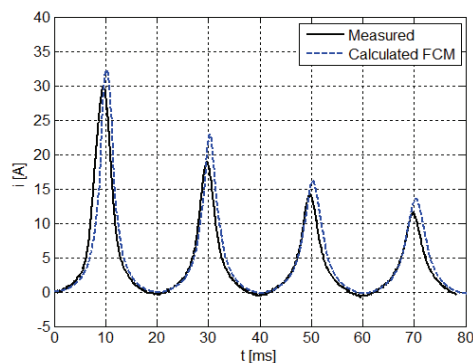


Fig.4. Flux density distribution for $l=1.5$ A.

a)



b)

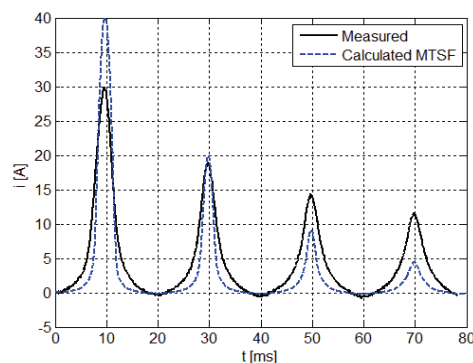
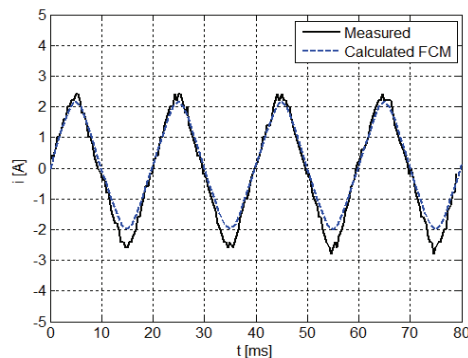


Fig.5. Inrush current waves for T1 transformer ($\varphi=0$): a) FCM, b) MTSF

a)



b)

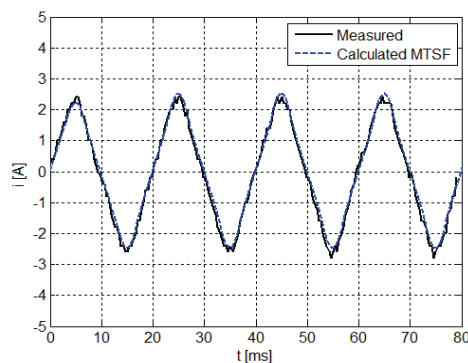
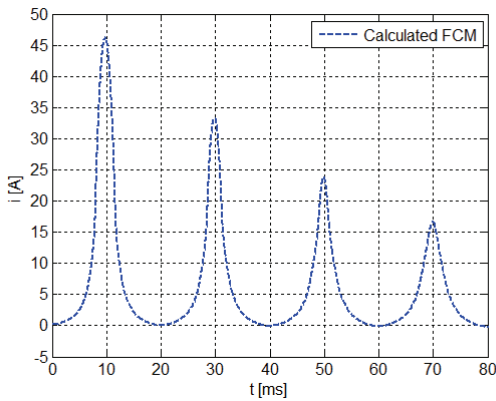


Fig.6. Inrush current waves for T1 transformer ($\varphi=90^\circ$): a) FCM, b) MTSF

In Figs. 5 and 6 the comparison of the calculated and measured inrush current waves for T1 transformer, under excitation phase $\varphi=0$ and $\varphi=90^\circ$ have been presented. The FCM method and the MTSF one give similar results. However, the first one is about 1.5 times faster. In the case of the $\varphi=90^\circ$, both models give almost the same inrush current waves (Fig. 6).

For our investigations, in the analyzed amorphous core the maximal value of the residual magnetic flux density B_r reaches only 0.2 T. In this paper are presented transient inrush current waves for two values of the residual flux density: $B_r=0.1$ T and $B_r=0.2$ T (Figs. 7a and 7b). Increasing the residual flux density value causes increasing of the inrush current peak value. In the case of $B_r=0.1$ T the current peak value is 50% higher than for the net value of the B_r . In the second case ($B_r=0.2$ T), the current peak reaches $i=65$ A.

a)



b)

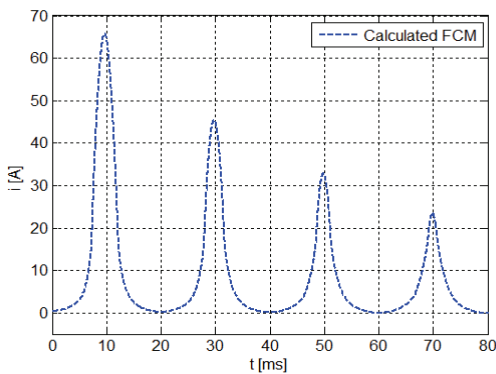


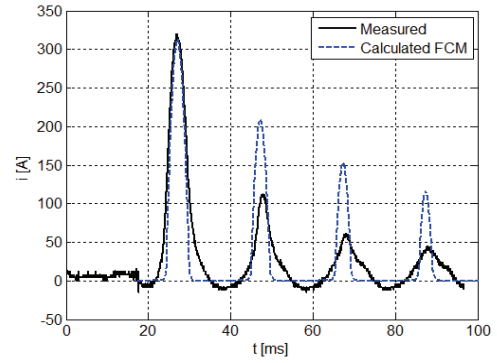
Fig.7. Inrush current waves for T1 transformer: a) for $B_r=0.1$ T, b) $B_r=0.2$ T

Inrush currents for the conventional transformer (T2)

We also calculated the inrush currents for the conventional construction of the transformer T2. The calculations have been executed for three different values of the core residual flux density. The assumed switching on phase $\varphi=0$ has been chosen. The comparison of the calculation results for two computing methods shows, that the MTSF method is more adequate for transient calculations, (Figs. 8 and 9).

It can be observed a fine attenuation of the current waves in the case $B_r=1.2$ T. The MTSF calculation method gives more accurate results comparing to the simplified field-circuit method (Fig. 8).

a)



b)

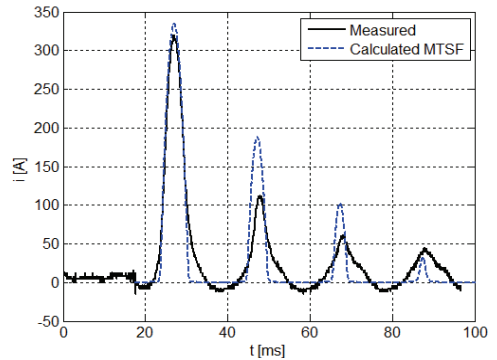
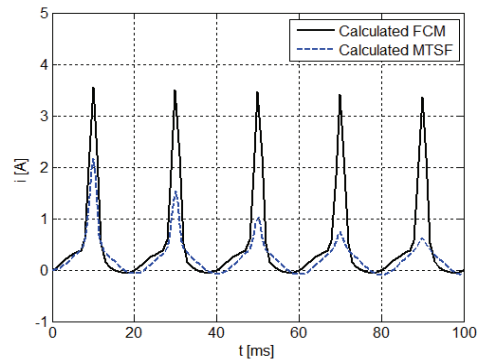


Fig.8. Inrush current waves for T2 transformer for $B_r=1.2$ T: a) FCM, b) MTSF

a)



b)

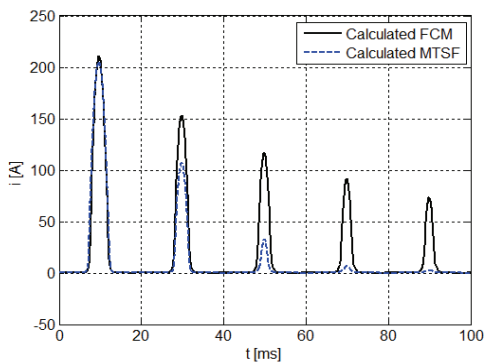


Fig.9. Inrush currents for T2 transformer: a) $B_r=0$ T, b) $B_r=0.8$ T

Fig. 9 shows the calculated current waves for two values of the residual flux density: $B_r=0$ T and $B_r=0.8$ T. We can observe that the residual flux density strongly influenced on the inrush current. The current values in the first times period of the current wave are about one hundred times greater than those simulated without the residual magnetism.

The calculations of the currents have been executed with a TSF method, as well. The method is included in many commercial FEM applications. To compare the calculation times for all the methods, we also computed the problem using FCM model. The MTSF method was two times faster than the TSF one for the inrush current calculations (Tab. 1). The computations have been done with using the AMD64 3200+ processor and 3GB of RAM.

Table 1. Compared CPU times for analyzed T2 transformer

| | FCM | TSF | MTSF |
|--------------|-----|------|------|
| CPU time [s] | 543 | 1782 | 846 |

Conclusions

The field-circuit method (FCM) and time stepping FEM (MTSF) for simulation of the inrush current waves in the two transformers have been studied. In the case of amorphous transformer T1 both methods give similar results. However, for T2 transformer analysis, the more accurate results arise from the MTSF method. First of all its is due to time discretization within numerical process.

The MTSF has been compared with the commercial TSF, [10]. The CPU time is about two times shorter for the MTSF, which validates the algorithm [11].

The influence of the residual flux on the maximum values of the current waves is higher in the case of conventional transformer, compared to the amorphous one (Figs. 6 and 7). It is mostly due to the air gaps length in the modular construction. Thus, the saturation of the transformer T1 core is considerable lower than the saturation of the T2 transformer core.

The calculation method presented in this work has been validated by measurements of the single-phase transformers. We observed relatively good conformity between computed and measured current waves [Figs. 5, 6, 8]. The differences between calculation and measurement results arise from the simplifications in the field analysis and measurement errors. The main difficulty is precisely determination of residual flux value which has significant influence on inrush current. For example, the residual flux is difficult for testing and contributes to the errors of our method, as well. Additionally, it is impossible to determine

exactly the air gap length in the prototype tests. Moreover, we can see that the inrush current for amorphous modular transformer has maximal value several times lower compared to the conventional one.

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